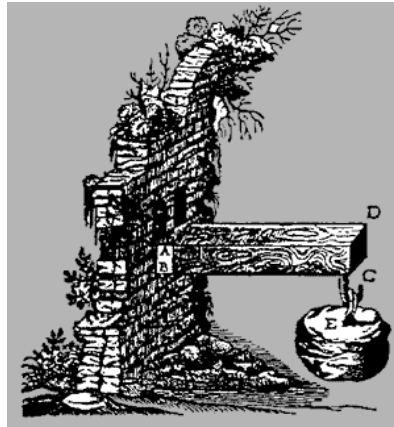


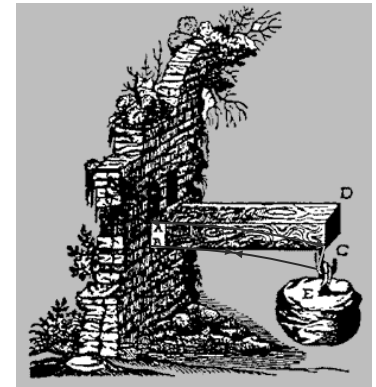
lecture
eight

beams:
bending and shear stress



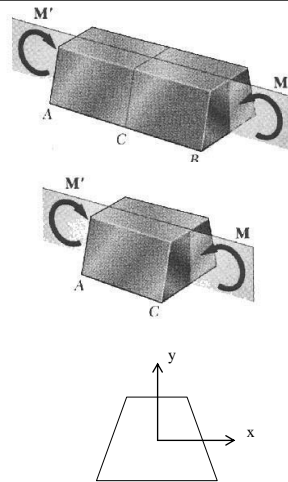
Beam Bending

- Galileo
 - relationship between stress and depth²
- can see
 - top squishing
 - bottom stretching
- what are the stress across the section?



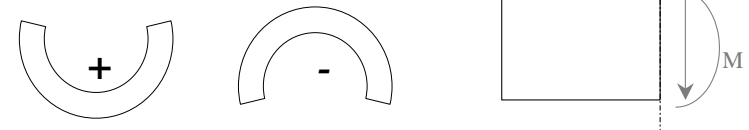
Pure Bending

- bending only
- no shear
- axial normal stresses from bending can be found in
 - homogeneous materials
 - plane of symmetry
 - follow Hooke's law



Bending Moments

- sign convention:



- size of maximum internal moment will govern our design of the section

Normal Stresses

- *geometric fit*
 - plane sections remain plane
 - stress varies linearly

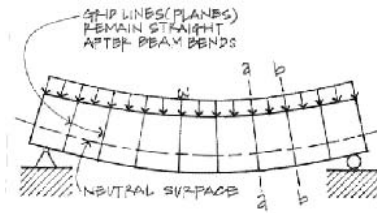


Figure 8.5(b) Beam bending under load.

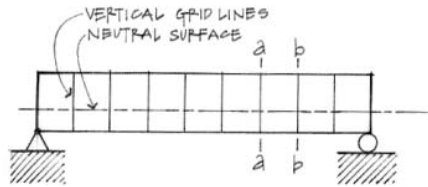
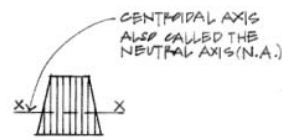


Figure 8.5(a) Beam elevation before loading.



Beam cross section.

Neutral Axis

- stresses vary linearly
- zero stress occurs at the centroid
- neutral axis is line of centroids (n.a.)

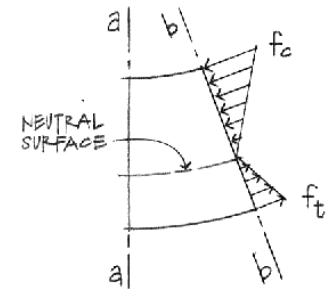


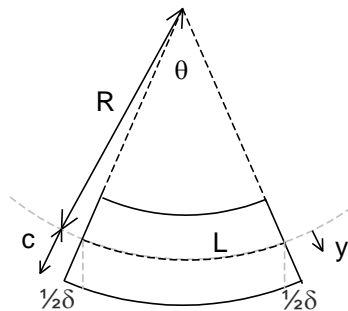
Figure 8.8 Bending stresses on section b-b.

Derivation of Stress from Strain

- pure bending = arc shape

$$L = R\theta$$

$$L_{outside} = (R + y)\theta$$



$$\epsilon = \frac{\delta}{L} = \frac{L_{outside} - L}{L} = \frac{(R + y)\theta - R\theta}{R\theta} = \frac{y}{R}$$

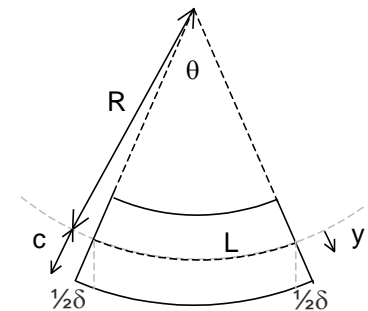
Derivation of Stress

- zero stress at n.a.

$$f = E\epsilon = \frac{Ey}{R}$$

$$f_{max} = \frac{Ec}{R}$$

$$f = \frac{y}{c} f_{max}$$



Bending Moment

- resultant moment from stresses = bending moment!

$$M = \Sigma fy\Delta A$$

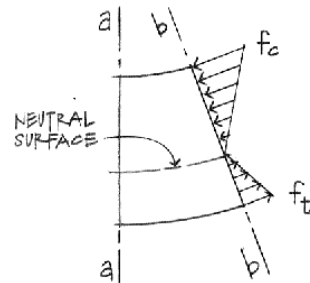


Figure 8.8 Bending stresses on section b-b.

$$= \Sigma \frac{yf_{max}}{c} y\Delta A = \frac{f_{max}}{c} \Sigma y^2 \Delta A = \frac{f_{max}}{c} I = f_{max} S$$

Bending Stress Relations

$$\frac{1}{R} = \frac{M}{EI}$$

curvature

$$f_b = \frac{My}{I}$$

general bending stress

$$S = \frac{I}{c}$$

section modulus

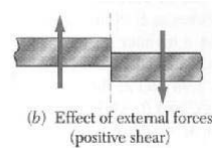
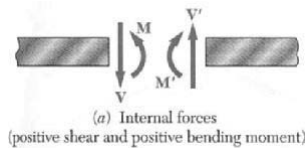
$$f_b = \frac{M}{S}$$

maximum bending stress

$$S_{required} \geq \frac{M}{F_b}$$

required section modulus for design

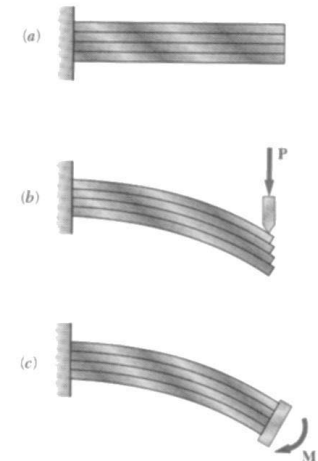
Transverse Loading and Shear



- perpendicular loading
- internal shear
- along with bending moment

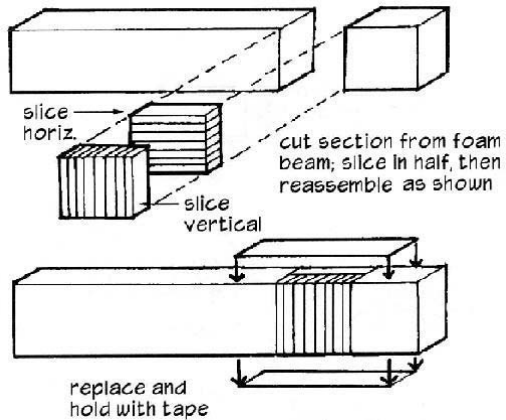
Bending vs. Shear in Design

- bending stresses dominate
- shear stresses exist horizontally with shear
- no shear stresses with pure bending



Shear Stresses

- horizontal & vertical



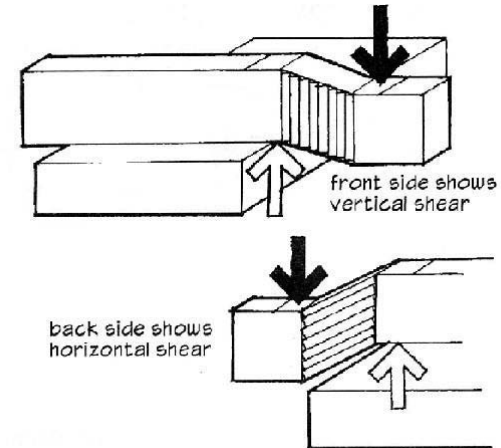
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Shear Stresses

- horizontal & vertical



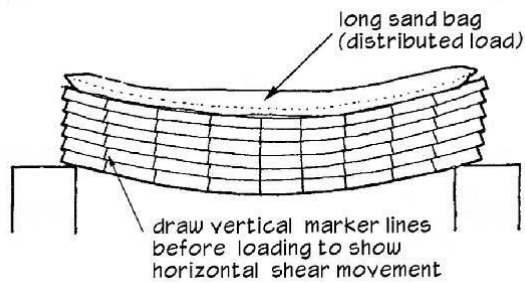
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Beam Stresses

- horizontal with bending



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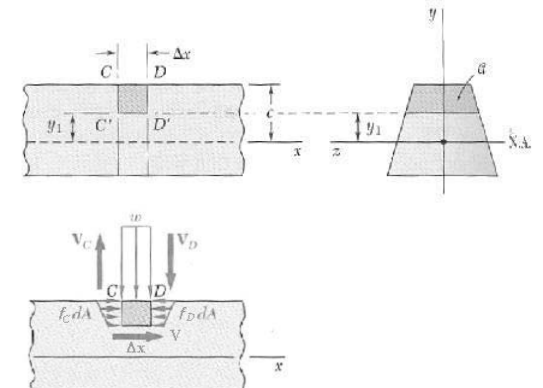
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Equilibrium

- horizontal force V needed

$$V_{longitudinal} = \frac{V_T Q}{I} \Delta x$$



- Q is a moment area

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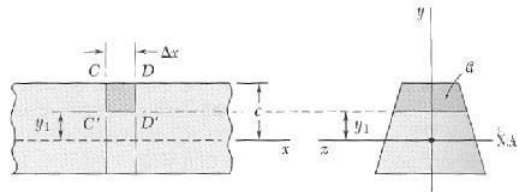
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Moment of Area

- Q is a moment area with respect to the n.a. of area above or below the horizontal

- Q_{max} at $y=0$ (neutral axis)



- q is shear flow:

$$q = \frac{V_{longitudinal}}{\Delta x} = \frac{V_T Q}{I}$$

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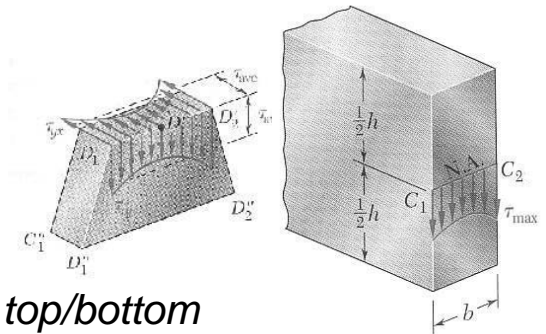
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Shearing Stresses

$$f_v = \frac{V}{\Delta A} = \frac{V}{b \cdot \Delta x}$$

$$f_{v-ave} = \frac{VQ}{Ib}$$



- $f_{v-ave} = 0$ on the top/bottom
- b min may not be with Q max
- with $h/4 \geq b$, $f_{v-max} \leq 1.008 f_{v-ave}$

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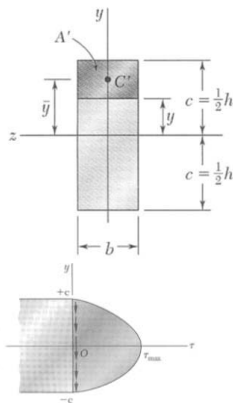
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Rectangular Sections

$$I = \frac{bh^3}{12} \quad Q = A\bar{y} = \frac{bh^2}{8}$$

$$f_v = \frac{VQ}{Ib} = \frac{3V}{2A}$$

- f_{v-max} occurs at n.a.



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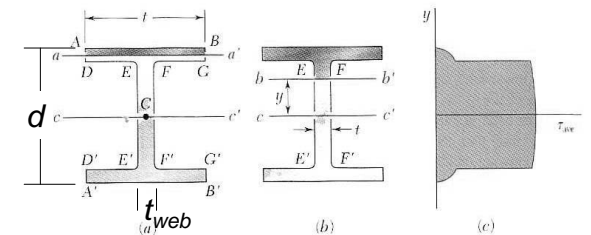
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Steel Beam Webs

- W and S sections

– b varies



– stress in flange negligible

– presume constant stress in web

$$f_{v-max} = \frac{3V}{2A} \approx \frac{V}{A_{web}}$$

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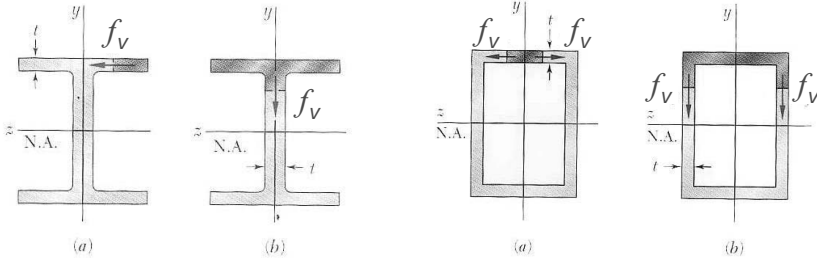
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Shear Flow

- loads applied in plane of symmetry
- cut made perpendicular

$$q = \frac{VQ}{I}$$



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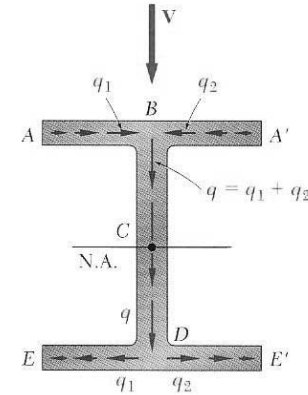
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Shear Flow Quantity

- sketch from Q

$$q = \frac{VQ}{I}$$



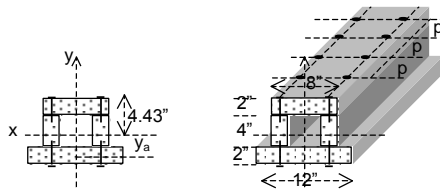
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Connectors Resisting Shear

- plates with
 - nails
 - rivets
 - bolts
- splices



$$\frac{V_{longitudinal}}{p} = \frac{VQ}{I}$$

$$nF_{connector} \geq \frac{VQ_{connected\ area}}{I} \cdot p$$

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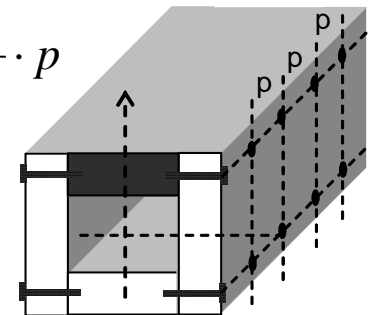
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Vertical Connectors

- isolate an area with vertical interfaces

$$nF_{connector} \geq \frac{VQ_{connected\ area}}{I} \cdot p$$



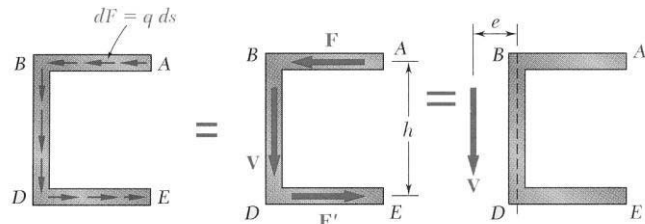
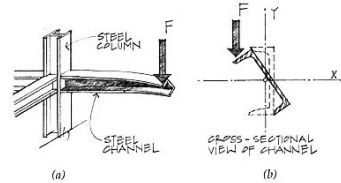
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Unsymmetrical Shear or Section

- member can bend and twist
 - not symmetric
 - shear not in that plane
- shear center
 - moments balance



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