ARCHITECTURAL STRUCTURES:
FORM, BEHAVIOR, AND DESIGN
ARCH 331
DR. ANNE NICHOLS
SUMMER 2013

lecture
SIX


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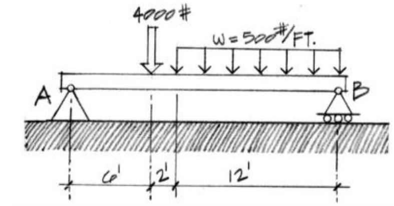
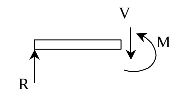
beams – internal forces & diagrams

Beams

- *span horizontally*
  - *floors*
  - *bridges*
  - *roofs*

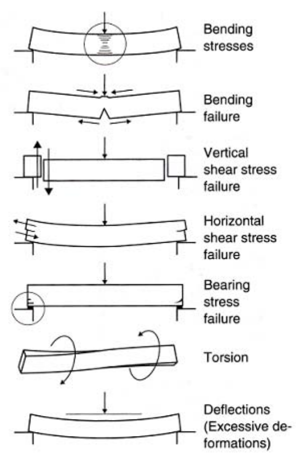


- *loaded transversely by gravity loads*
- *may have internal axial force*
- *will have internal shear force*
- *will have internal moment (bending)*



Beams

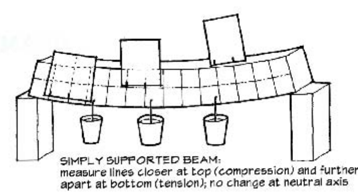
- *transverse loading*
- *sees:*
  - *bending*
  - *shear*
  - *deflection*
  - *torsion*
  - *bearing*
- *behavior depends on cross section shape*

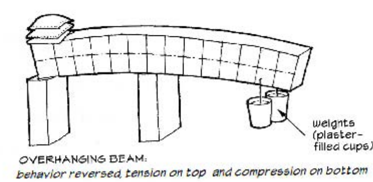


Transverse Loadings

Beams

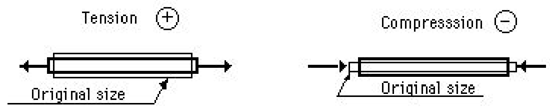
- *bending*
  - *bowing of beam with loads*
  - *one edge surface stretches*
  - *other edge surface squishes*

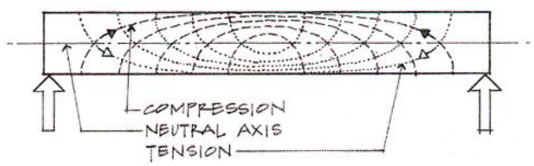




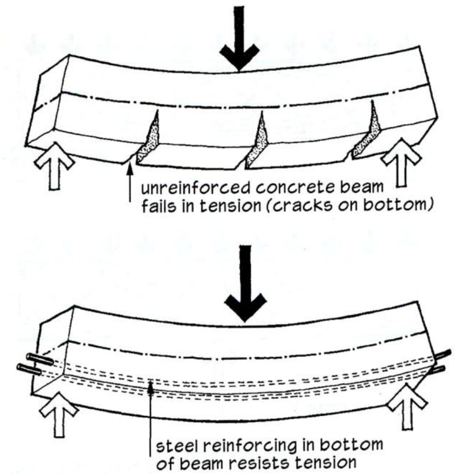
Beam Stresses

- *stress = relative force over an area*
  - *tensile*
  - *compressive*
  - *bending*
    - *tension and compression + ...*



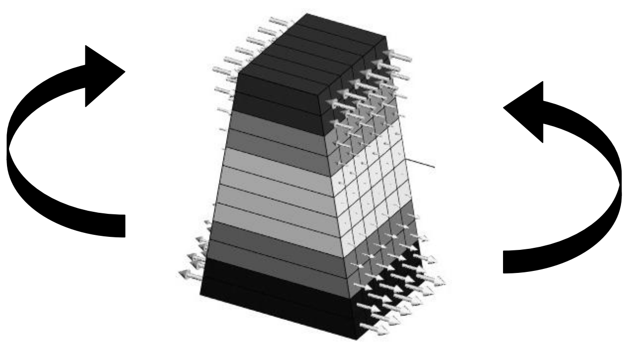


Beam Stresses



Beam Stresses

- *tension and compression*
  - *causes moments*



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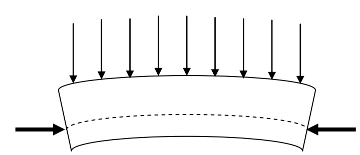
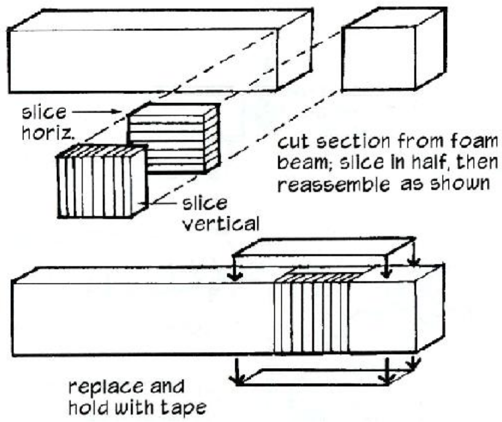
Beam Stresses

- *prestress or post-tensioning*
  - *put stresses in tension area to "pre-compress"*



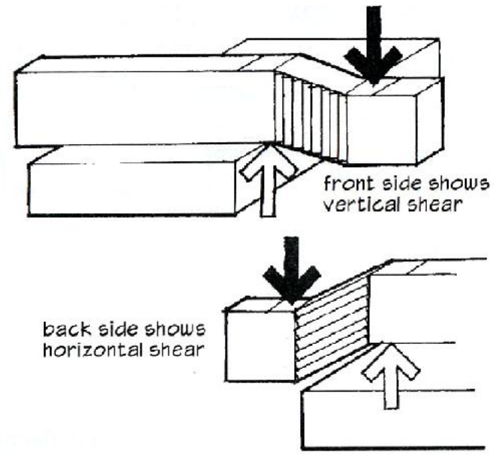
Beam Stresses

- *shear – horizontal & vertical*



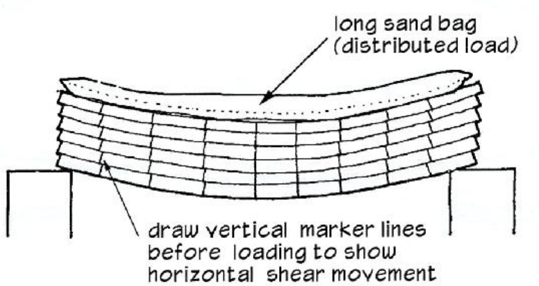
Beam Stresses

- *shear – horizontal & vertical*



Beam Stresses

- *shear – horizontal*



Beam Deflections

- *depends on*
  - *load*
  - *section*
  - *material*

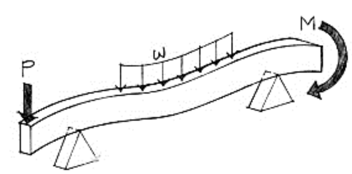
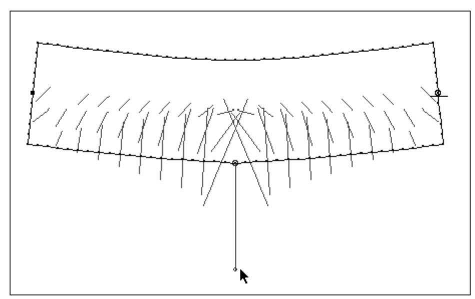
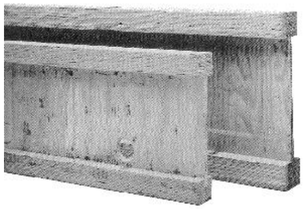
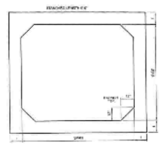
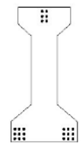


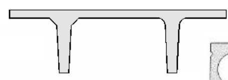
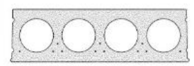
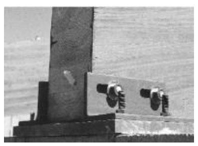
Figure 5.4 *Bending (flexural) loads on a beam.*

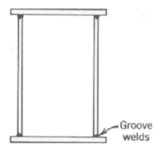


Beam Deflections

- *"moment of inertia"*



(e) Welded box girder.

Beam Styles

- *vierendeel*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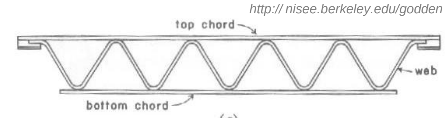
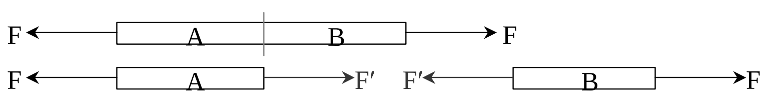
- *open web joists*
- *manufactured*

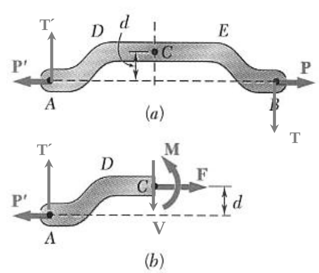


Internal Forces

- *trusses*
  - *axial only, (compression & tension)*

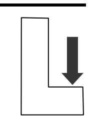


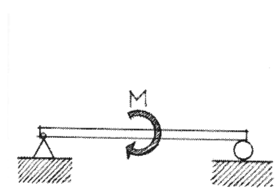
- *in general*
  - *axial force*
  - *shear force, V*
  - *bending moment, M*



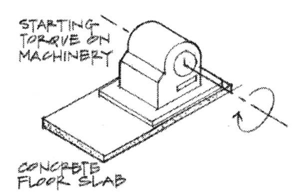
Beam Loading

- *concentrated force*
- *concentrated <u>moment</u>*
  - *spandrel beams*



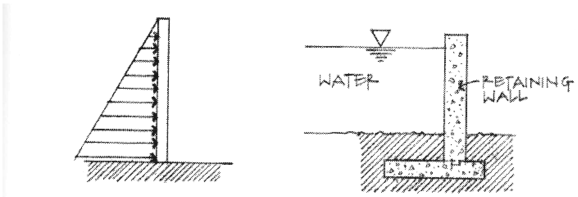


(d) Pure moment.



Beam Loading

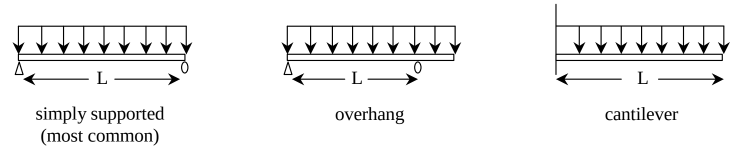
- *uniformly distributed load (line load)*
- *non-uniformly distributed load*
  - *hydrostatic pressure = γh*
  - *wind loads*



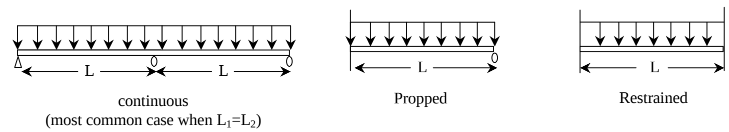
(c) Nonuniformly distributed load.

Beam Supports

- *statically determinate*

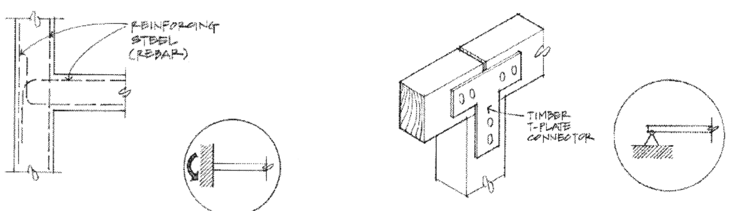


- *statically indeterminate*

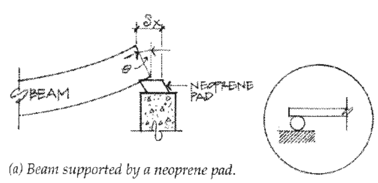


Beam Supports

- *in the real world, modeled type*



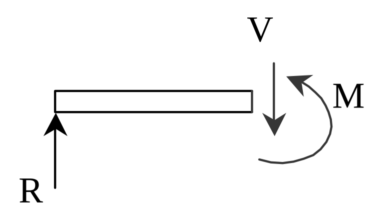
(c) Timber beam–column connection with T-plate.



(a) Beam supported by a neoprene pad.

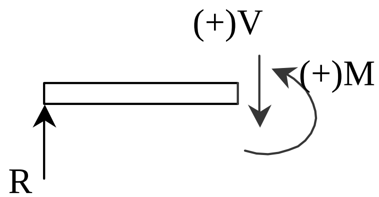
Internal Forces in Beams

- *like method of sections / joints*
  - *no axial forces*
- *section <u>must</u> be in equilibrium*
- *want to know where biggest internal forces and moments are for designing*



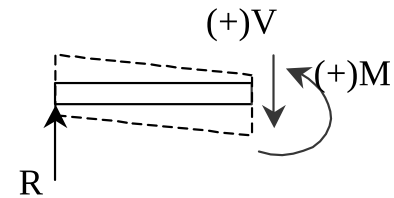
V & M Diagrams

- tool to locate V_{max} and M_{max} (at $V = 0$)
- necessary for designing
- have a different sign convention than external forces, moments, and reactions

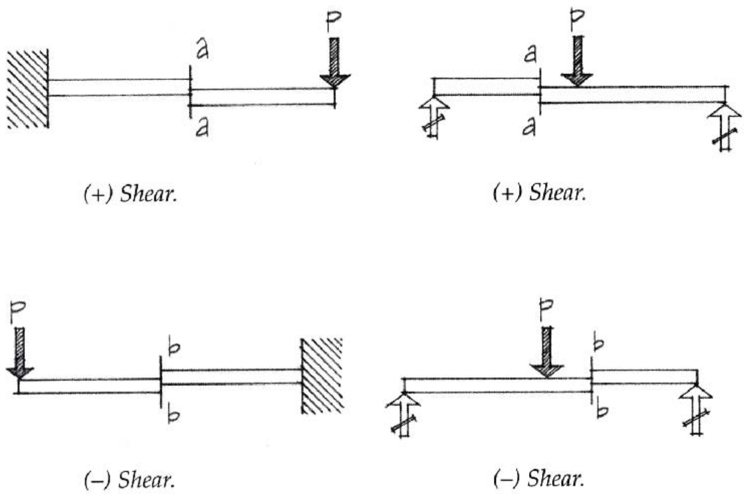


Sign Convention

- shear force, V:
  - cut section to LEFT
  - if $\sum F_y$ is positive by statics, V acts down and is POSITIVE
  - beam has to resist shearing apart by V

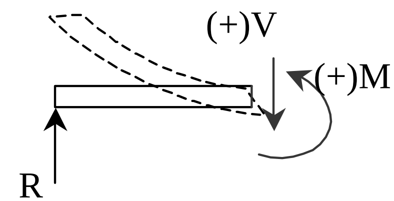


Shear Sign Convention

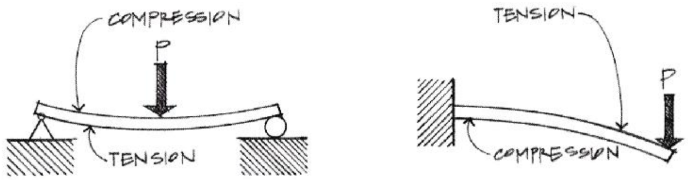


Sign Convention

- bending moment, M:
  - cut section to LEFT
  - if $\sum M_{cut}$ is clockwise, M acts ccw and is POSITIVE – flexes into a “smiley” beam has to resist bending apart by M



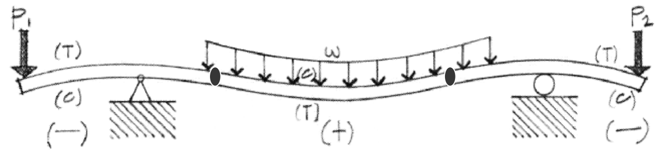
Bending Moment Sign Convention



(+) Moment. (–) Moment.

(+) Moment. (–) Moment.

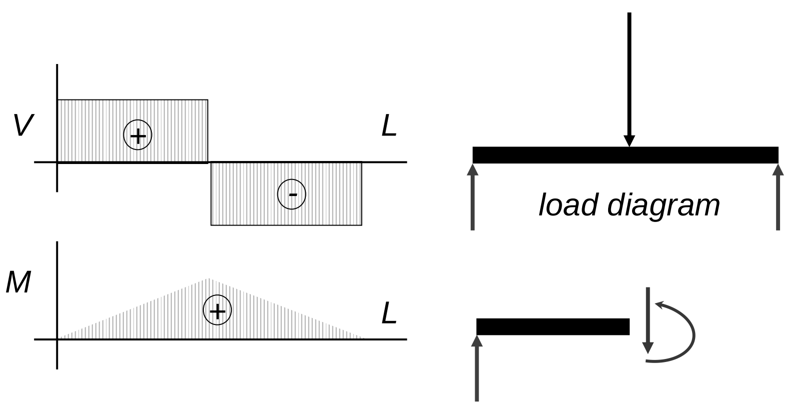
Deflected Shape



- *positive bending moment*
  - *tension in bottom, compression in top*
- *negative bending moment*
  - *tension in top, compression in bottom*
- *zero bending moment*
  - *inflection point*

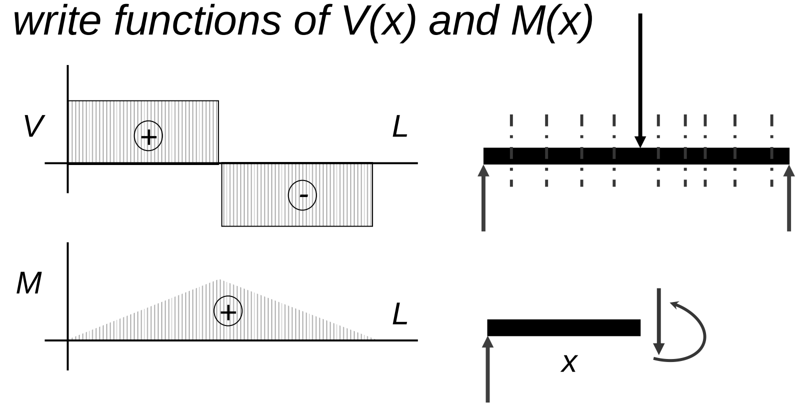
Constructing V & M Diagrams

- *along the beam length, plot V, plot M*



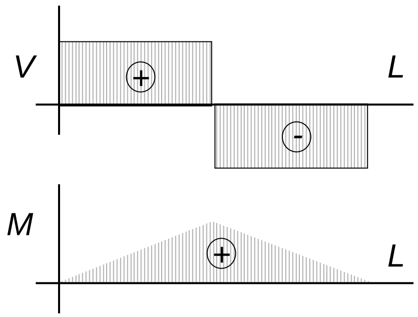
Mathematical Method

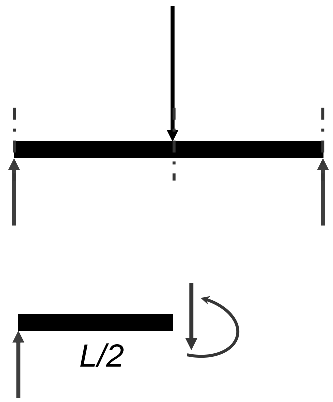
- *cut sections with x as width*
- *write functions of V(x) and M(x)*



Method 1: Equilibrium

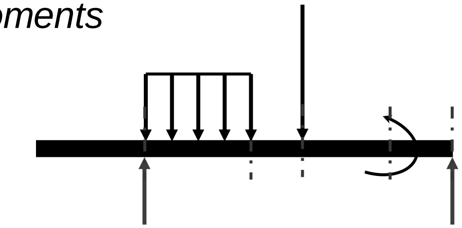
- cut sections at important places
- plot V & M





Method 1: Equilibrium

- important places
  – supports
  – concentrated loads
  – start and end of distributed loads
  – concentrated moments
- free ends
  – zero forces



Method 2: Semigraphical

- by knowing
  – area under loading curve = change in V
  – area under shear curve = change in M
  – concentrated forces cause “jump” in V
  – concentrated moments cause “jump” in M

$$V_D - V_C = -\int_{x_C}^{x_D} w\,dx \qquad M_D - M_C = \int_{x_C}^{x_D} V\,dx$$

Method 2

- relationships

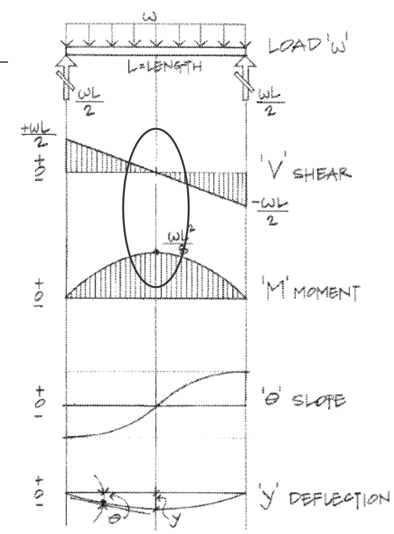
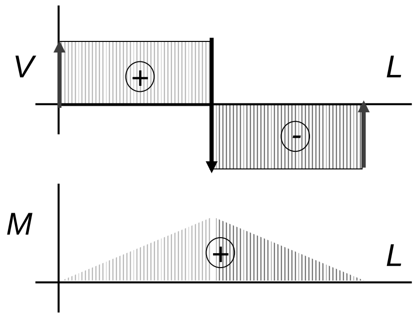
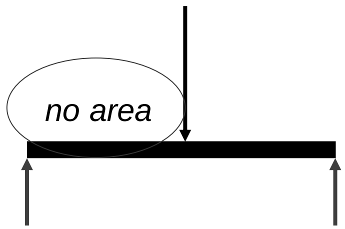


Figure 7.11 Relationship of load, shear, moment, slope, and deflection diagrams.

Method 2: Semigraphical

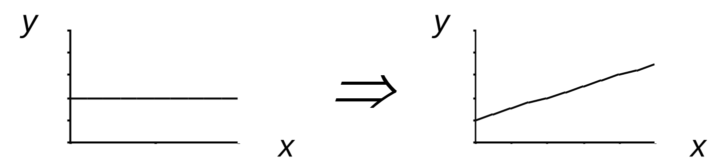
- M_{max} occurs where $V = 0$ (calculus)





Curve Relationships

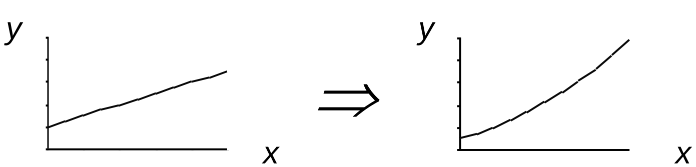
- integration of functions
- line with 0 slope, integrates to sloped



- ex: load to shear, shear to moment

Curve Relationships

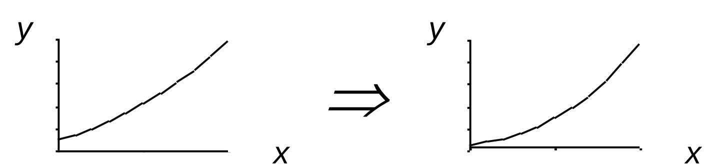
- line with slope, integrates to parabola



- ex: load to shear, shear to moment

Curve Relationships

- parabola, integrates to 3^{rd} order curve



- ex: load to shear, shear to moment

Basic Procedure with Sections

1. *Find reaction forces & moments*

   Plot axes, underneath beam load diagram

V:

2. *Starting at left*
3. *Shear is 0 at free ends*
4. *Shear has 2 values at point loads*
5. *Sum vertical forces at each section*

Basic Procedure with Sections

M:

6. *Starting at left*
7. *Moment is 0 at free ends*
8. *Moment has 2 values at moments*
9. *Sum moments at each section*
10. *Maximum moment is where shear = 0! (locate where V = 0)*

Basic Procedure by Curves

1. *Find reaction forces & moments*

   Plot axes, underneath beam load diagram

V:

2. *Starting at left*
3. *Shear is 0 at free ends*
4. *Shear jumps with concentrated load*
5. *Shear changes with area under load*

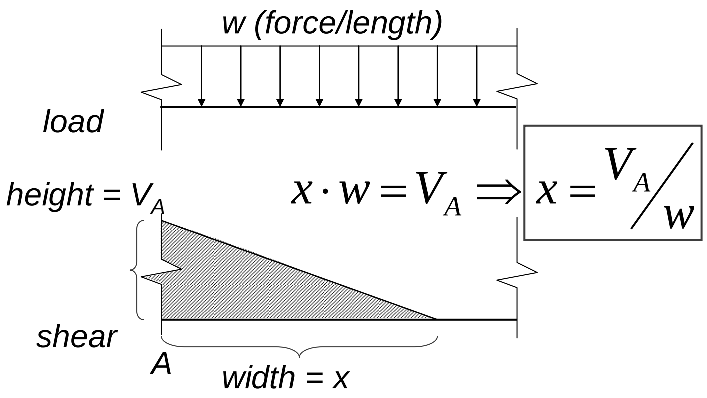
Basic Procedure by Curves

M:

6. *Starting at left*
7. *Moment is 0 at free ends*
8. *Moment jumps with moment*
9. *Moment changes with area under V*
10. *Maximum moment is where shear = 0! (locate where V = 0)*

Shear Through Zero

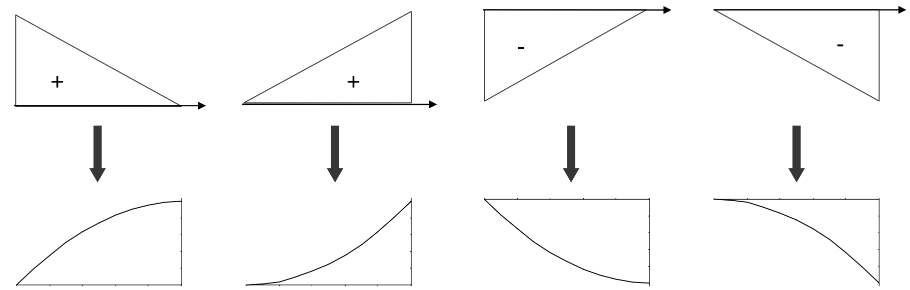
- *slope of V is w (-w:1)*



$$x \cdot w = V_A \Rightarrow x = \frac{V_A}{w}$$

Parabolic Shapes

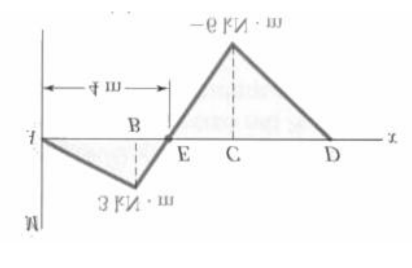
- *cases*

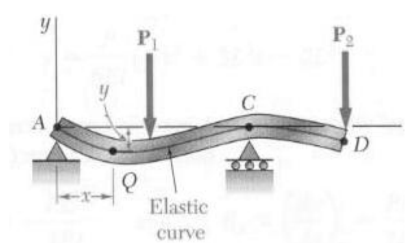


up fast, then slow | *up slow, then fast* | *down fast, then slow* | *down slow, then fast*

Deflected Shape & M(x)

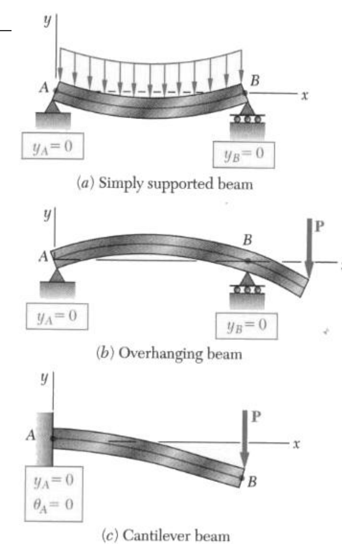
- *-M(x) gives shape indication*
- *boundary conditions must be met*





Boundary Conditions

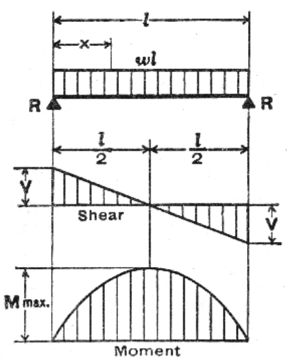
- *at pins, rollers, fixed supports: $y = 0$*
- *at fixed supports: $\theta = 0$*
- *at inflection points from symmetry: $\theta = 0$*
- *y_{max} at* $\frac{dy}{dx} = 0$



Tabulated Beam Formulas

- how to read charts

1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load . . . . $= wl$

R = V . . . . $= \frac{wl}{2}$

V_x . . . . $= w\left(\frac{l}{2} - x\right)$

M max. (at center) . . . . . $= \frac{wl^2}{8}$

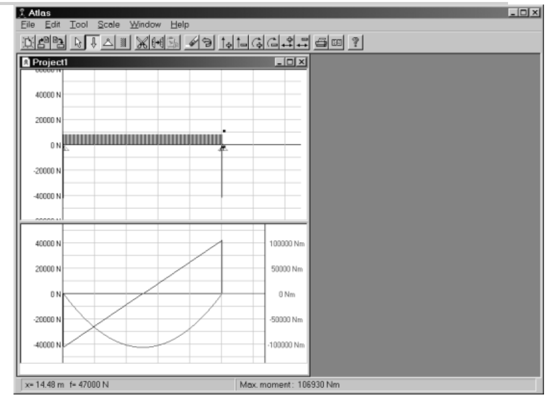
M_x . . . . $= \frac{wx}{2}(l - x)$

Δmax. (at center) . . . . . $= \frac{5\,wl^4}{384\,EI}$

Δx . . . . $= \frac{wx}{24EI}(l^3 - 2lx^2 + x^3)$

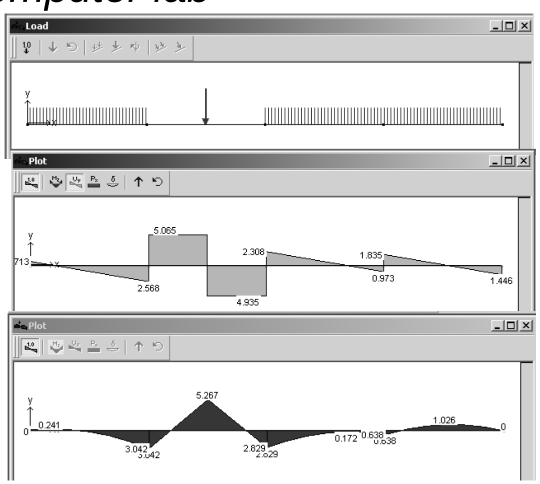
Tools

- software & spreadsheets help
- http://www.rekenwonder.com/atlas.htm



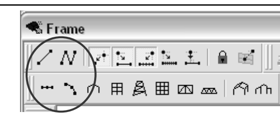
Tools – Multiframe

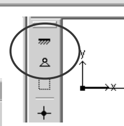
- in computer lab

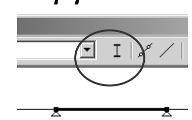


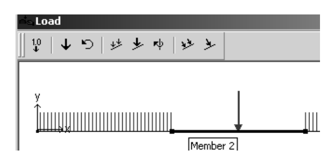
Tools – Multiframe

- frame window
  – define beam members
  – select points, assign supports
  – select members, assign <u>section</u>
- load window
  – select point or member, add point or distributed loads



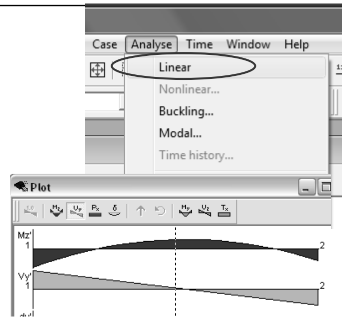


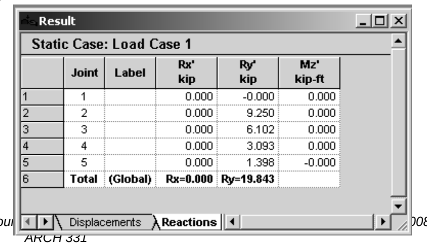




Tools – Multiframe

- *to run analysis choose*
  - *Analyze menu*
    - *Linear*
- *plot*
  - *choose options*
  - *double click (all)*
- *results*
  - *choose options*





	Joint	Label	Rx' kip	Ry' kip	Mz' kip-ft
1	1		0.000	-0.000	0.000
2	2		0.000	9.250	0.000
3	3		0.000	6.102	0.000
4	4		0.000	3.093	0.000
5	5		0.000	1.398	-0.000
6	Total	(Global)	Rx=0.000	Ry=19.843	