ARCHITECTURAL STRUCTURES:
FORM, BEHAVIOR, AND DESIGN
ARCH 331
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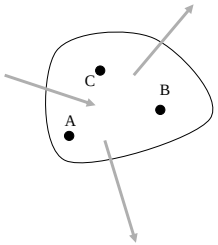
SUMMER 2013

lecture
four

rigid body equilibrium

Equilibrium

- *rigid body*
  - *doesn't deform*
  - *coplanar force systems*
- *static:*



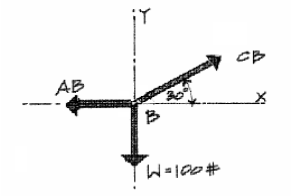
$$R_x = \sum F_x = 0$$

$$R_y = \sum F_y = 0$$

$$\boxed{M = \sum M = 0}$$

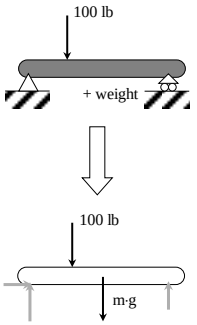
Free Body Diagram

- *FBD (sketch)*
- *tool to see all forces on a body or a point including*
  - *external forces*
  - *weights*
  - *force reactions*
  - *external moments*
  - *moment reactions*
  - *internal forces*



Free Body Diagram

- *determine body*
- *FREE it from:*
  - *ground*
  - *supports & connections*
- *draw all external forces acting ON the body*
  - *reactions*
  - *applied forces*
  - *gravity*



Free Body Diagram

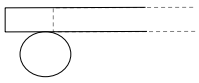
- *sketch FBD with relevant geometry*
- *resolve each force into components*
  - *known & unknown angles – name them*
  - *known & unknown forces – name them*
  - *known & unknown moments – name them*
- *are any forces related to other forces?*
- *for the unknowns*
- *write only as many equilibrium equations as needed*
- *solve up to 3 equations*

Free Body Diagram

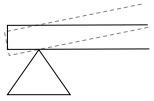
- *solve equations*
  - *most times 1 unknown easily solved*
  - *plug into other equation(s)*
- *common to have unknowns of*
  - *force magnitudes*
  - *force angles*
  - *moment magnitudes*

Reactions on Rigid Bodies

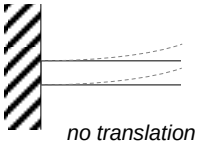
- *result of applying force*
- *unknown size*
- *connection or support type*
  - *known direction*
  - *related to motion prevented*



no vertical motion

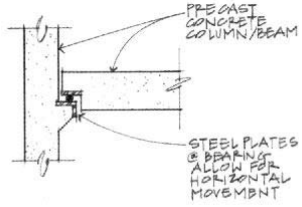


no translation

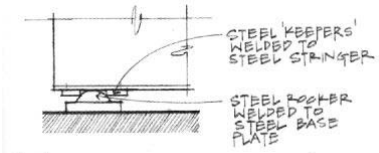


no translation
no rotation

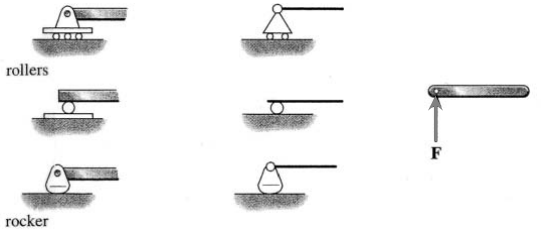
Supports and Connections



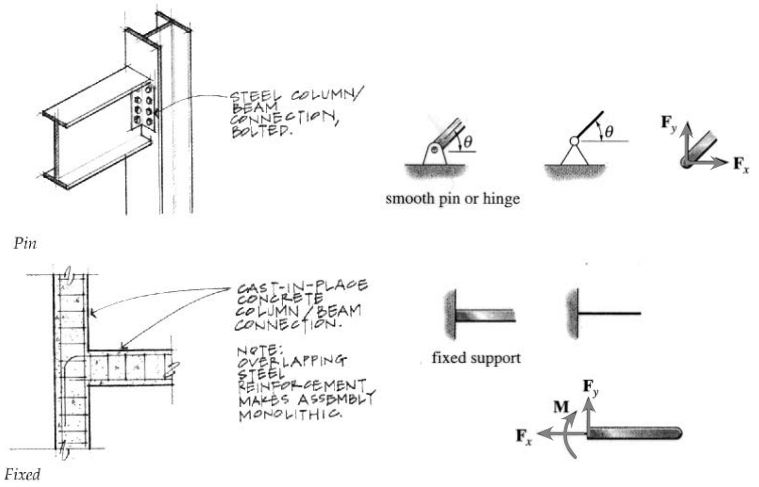
Roller



Rocker

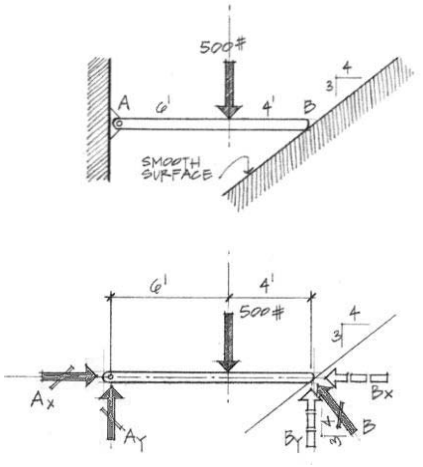


Supports and Connections



FBD Example

- *500 lb known*
- *pin – A_x, A_y*
- *smooth surface – B at 4:3*
- *3 equations*
- *sum moments at*
  - *A?*
  - *B? (B_x)*

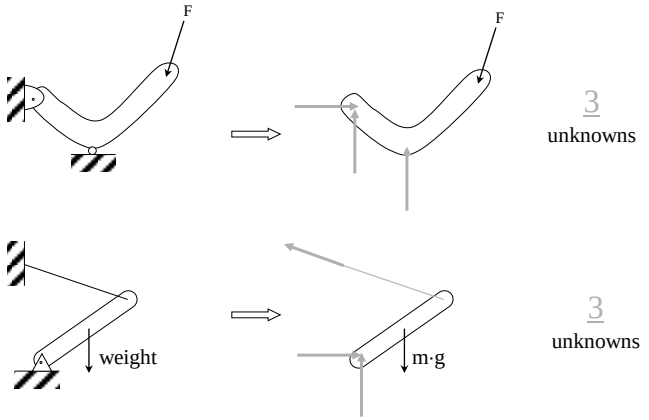


Moment Equations

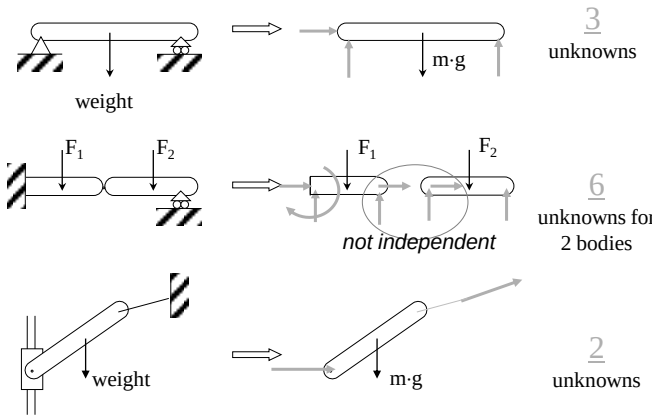
- *sum moments at intersection where the most forces intersect*
- *multiple moment equations may not be useful*
- *combos:*

$$\sum F_x = 0 \qquad \sum F = 0 \qquad \sum M_1 = 0$$
$$\sum F_y = 0 \qquad \sum M_1 = 0 \qquad \sum M_2 = 0$$
$$\sum M_1 = 0 \qquad \sum M_2 = 0 \qquad \sum M_3 = 0$$

Recognizing Reactions

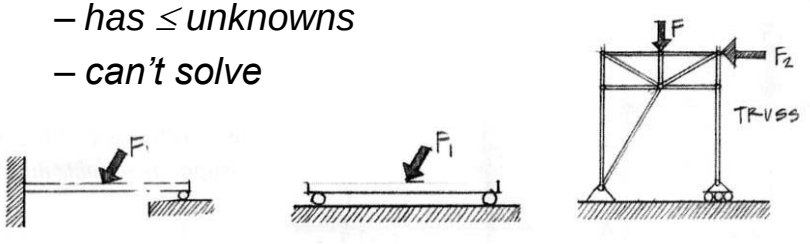


Recognizing Reactions



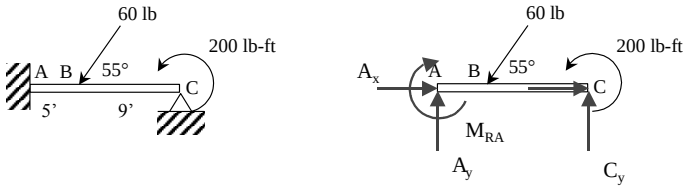
Constraints

- *completely constrained*
  - *doesn't move*
  - *may not be statically determinate*
- *improperly or partially constrained*
  - *has ≤ unknowns*
  - *can't solve*

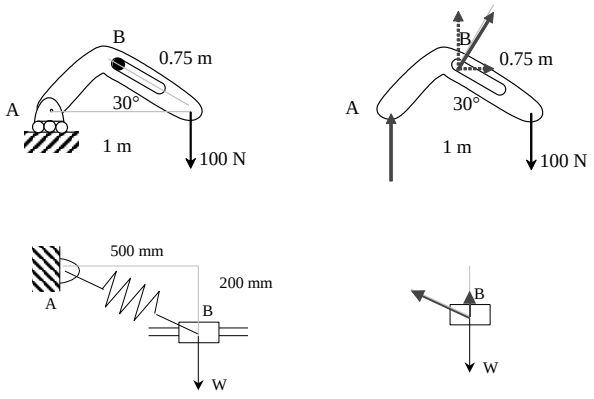


Constraints

- *overconstrained*
  - *won't move*
  - *can't be solved with statics*
  - *statically indeterminate to n^{th} degree*

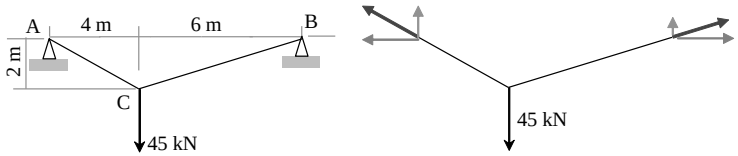


Partial Constraints



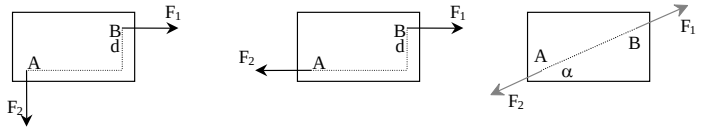
Cable Reactions

- *equilibrium:*
  - *more reactions (4) than equations*
  - *but, we have slope relationships*
  - *x component the same everywhere*



Two Force Rigid Bodies

- *equilibrium:*
  - *forces in line, equal and opposite*

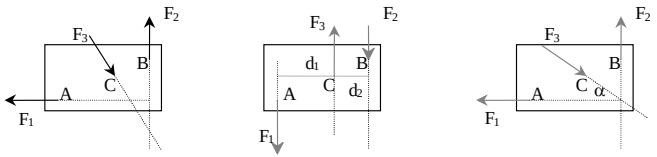


A (no) B (no) C

Three Force Rigid Bodies

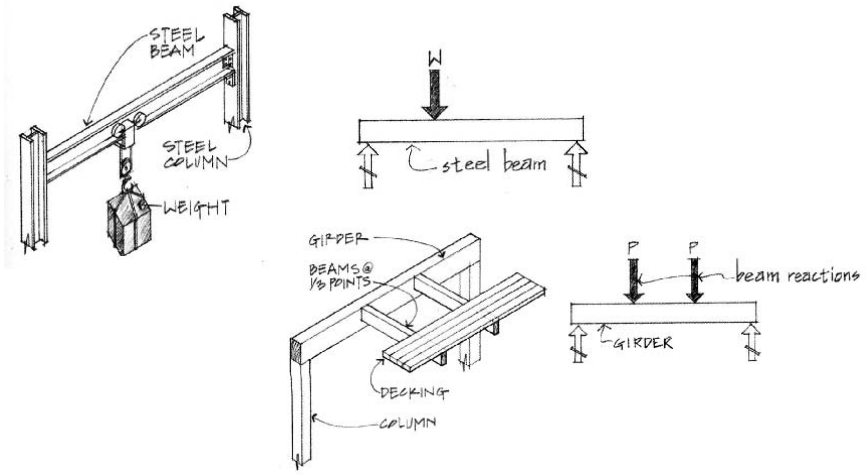
- *equilibrium:*
  - *concurrent or parallel forces*

beams!

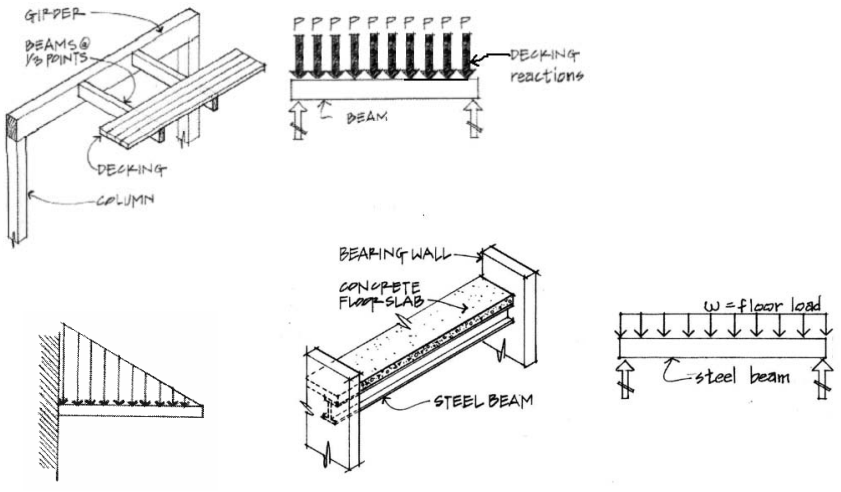


A (no) B C

Concentrated Loads

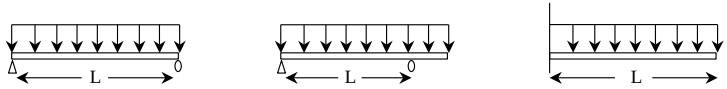


Distributed Loads



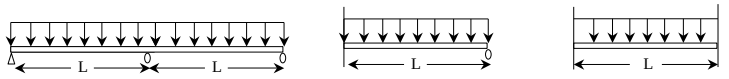
Beam Supports

- *statically determinate*



simply supported (most common) | overhang | cantilever

- *statically indeterminate*

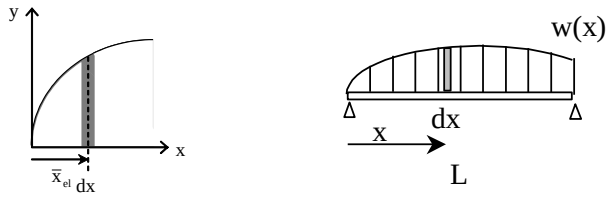


continuous (most common case when $L_1=L_2$) | Propped | Restrained

Equivalent Force Systems

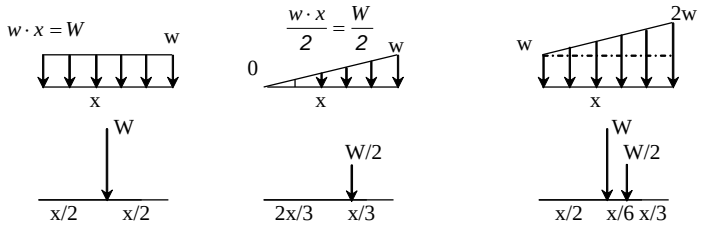
- *replace forces by resultant*
- *place resultant where M = 0*
- *using calculus and area centroids*

$$W = \int_0^L w\,dx = \int dA_{loading} = A_{loading}$$



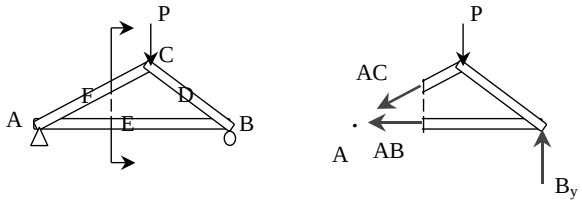
Load Areas

- *area is width x "height" of load*
- *w is load per unit length*
- *W is total load*



Method of Sections

- *relies on internal forces being in equilibrium on a section*
- *cut to expose <u>3 or less</u> members*
- *coplanar forces* $\rightarrow \sum M = 0$ *too*



Method of Sections

- *joints on or off the section are good to sum moments*
- *quick for few members*
- *not always obvious where to cut or sum*