

ARCHITECTURAL STRUCTURES: FORM, BEHAVIOR, AND DESIGN

ARCH 331

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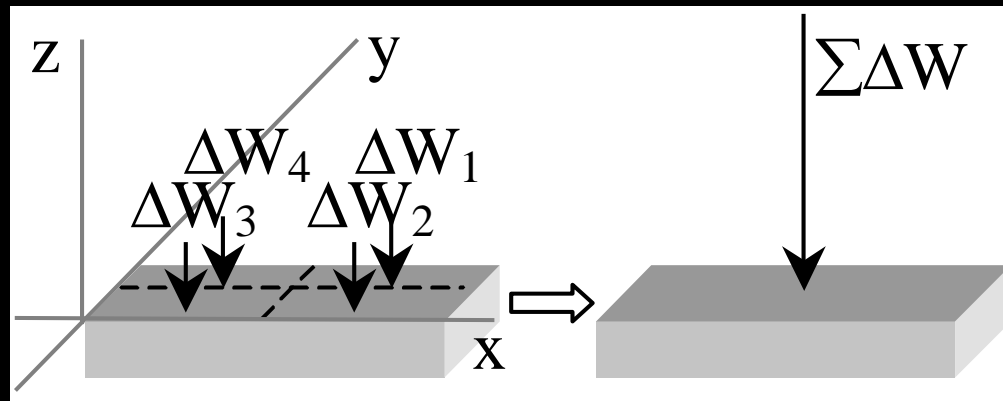
lecture
seven

**beam sections -
geometric properties**



Center of Gravity

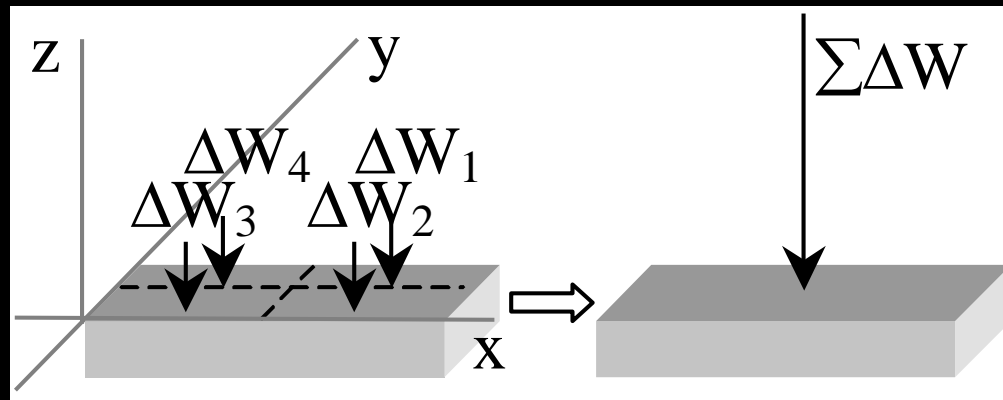
- *location of equivalent weight*
- *determined with calculus*



- *sum element weights* $W = \int dW$

Center of Gravity

- “average” x & y from moment



$$\sum M_y = \sum_{i=1}^n x_i \Delta W_i = \bar{x} W \Rightarrow \bar{x} = \frac{\sum (x \Delta W)}{W}$$

“bar” means average

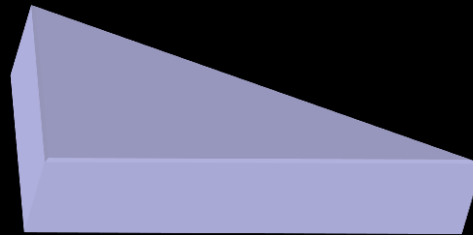
$$\sum M_x = \sum_{i=1}^n y_i \Delta W_i = \bar{y} W \Rightarrow \bar{y} = \frac{\sum (y \Delta W)}{W}$$

Centroid

- “average” x & y of an area
- for a volume of constant thickness
 - $\Delta W = \gamma \Delta A$ where γ is weight/volume
 - center of gravity = centroid of area

$$\bar{x} = \frac{\sum(x\Delta A)}{A}$$

$$\bar{y} = \frac{\sum(y\Delta A)}{A}$$

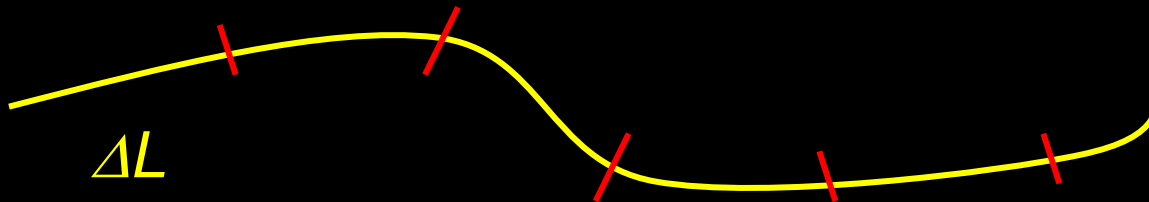


Centroid

- for a line, sum up length

$$\bar{x} = \frac{\sum(x\Delta L)}{L}$$

$$\bar{y} = \frac{\sum(y\Delta L)}{L}$$

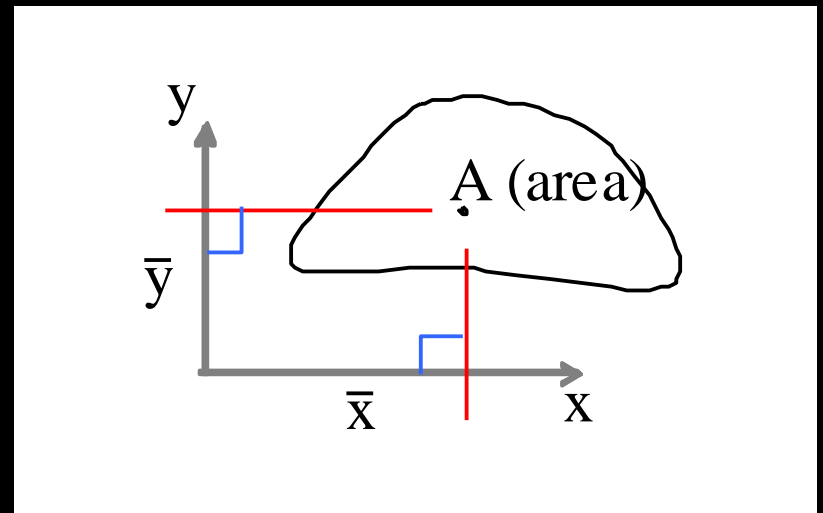


1st Moment Area

- *math concept*
- *the moment of an area about an axis*

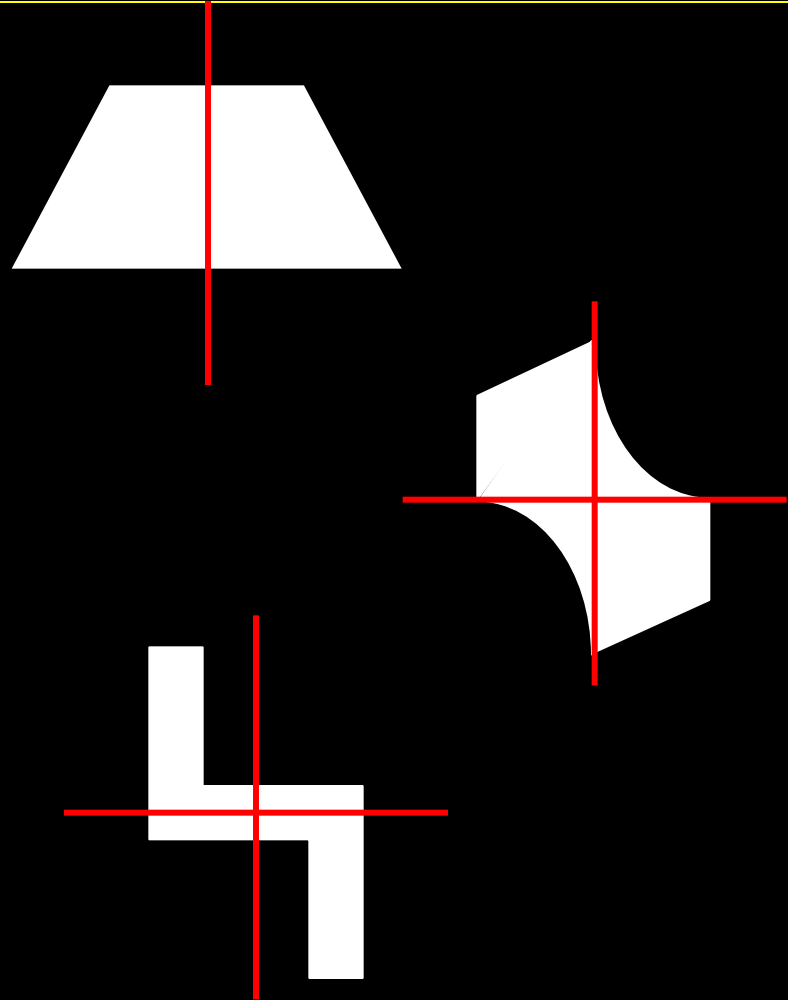
$$Q_x = \bar{y}A$$

$$Q_y = \bar{x}A$$



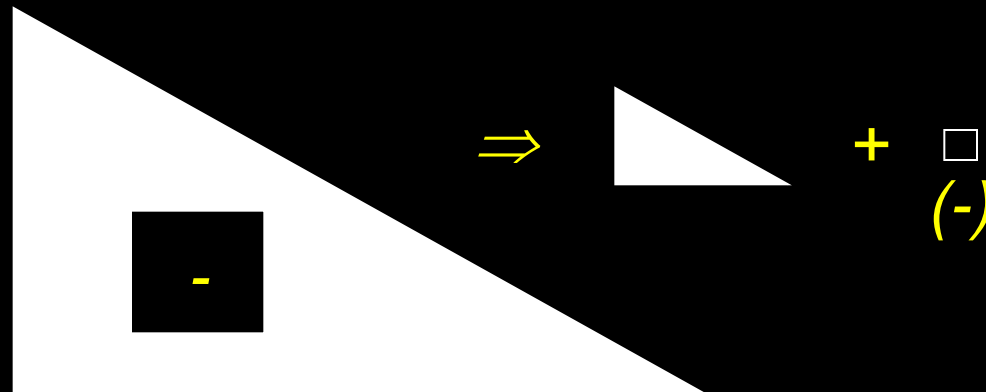
Symmetric Areas

- *symmetric about an axis*
- *symmetric about a center point*
- *mirrored symmetry*



Composite Areas

- *made up of basic shapes*
- *areas can be negative*
- *(centroids can be negative for any area)*



Basic Procedure

1. Draw reference origin (if not given)
2. Divide into basic shapes (+/-)
3. Label shapes
4. Draw table
5. Fill in table
6. Sum necessary columns
7. Calculate \hat{x} and \hat{y}

Component	Area	\bar{x}	$\bar{x}A$	\bar{y}	$\bar{y}A$
Σ					

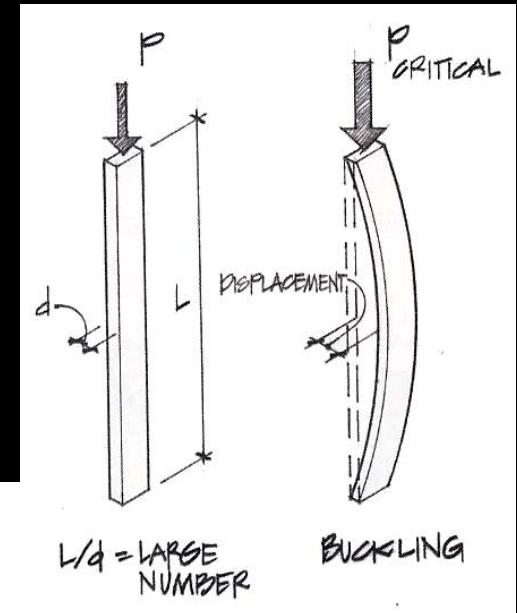
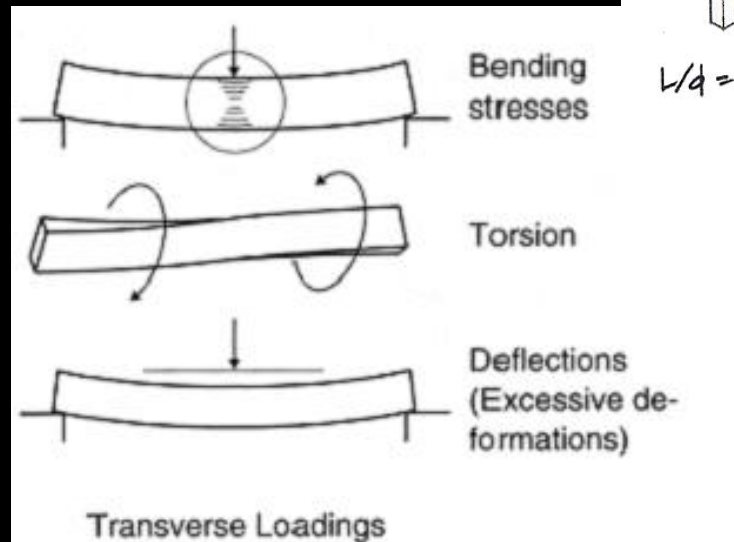
Area Centroids

- *Table 7.1 – pg. 242*

Centroids of Common Shapes of Areas and Lines			
Shape		\bar{x}	\bar{y}
Triangular area		$\frac{b}{3}$	$\frac{h}{3}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$
Semicircular area		0	$\frac{4r}{3\pi}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$
Parabolic area		0	$\frac{3h}{5}$

Moments of Inertia

- 2nd moment area
 - math concept
 - area \times (distance)²
- need for behavior of
 - beams
 - columns

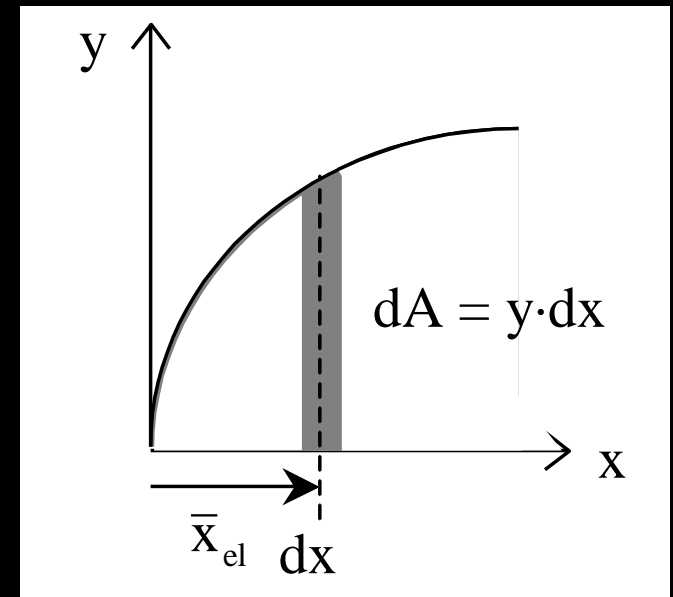


Moment of Inertia

- about any reference axis
- can be negative

$$I_y = \int x^2 dA$$

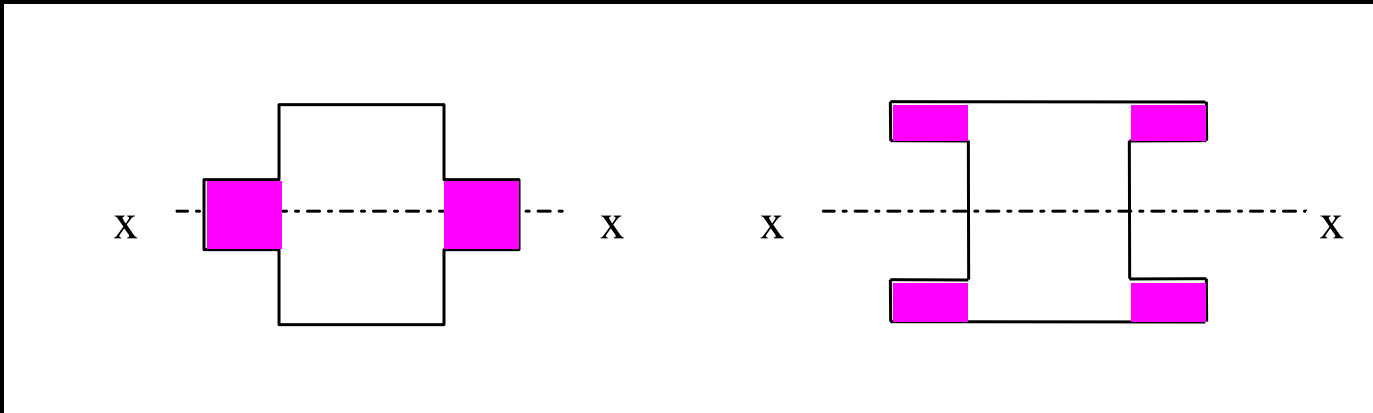
$$I_x = \int y^2 dA$$



- resistance to bending and buckling

Moment of Inertia

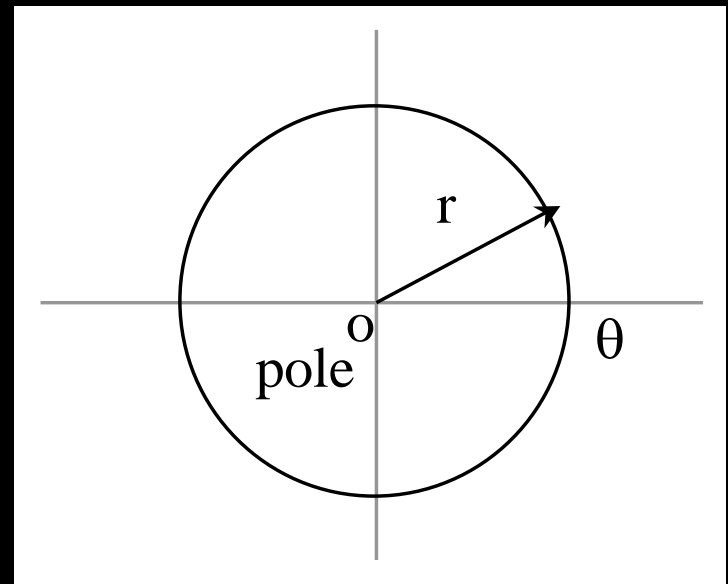
- *same area moved away a distance*
– *larger I*



Polar Moment of Inertia

- *for roundish shapes*
- *uses polar coordinates (r and θ)*
- *resistance to twisting*

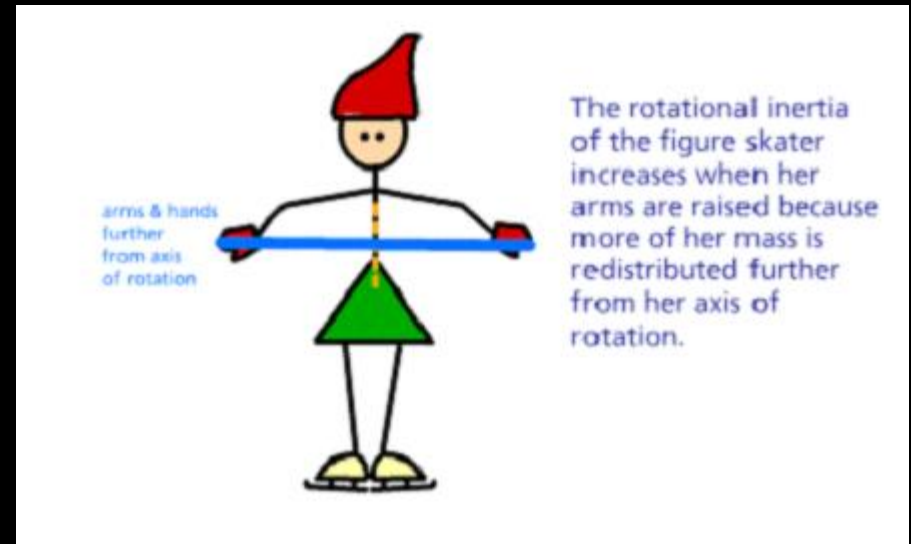
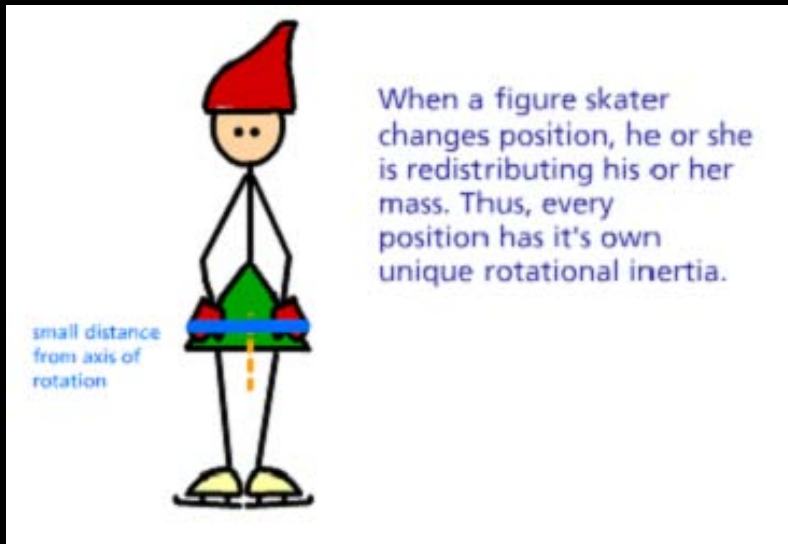
$$J_o = \int r^2 dA$$



Radius of Gyration

- *measure of inertia with respect to area*

$$r_x = \sqrt{\frac{I_x}{A}}$$



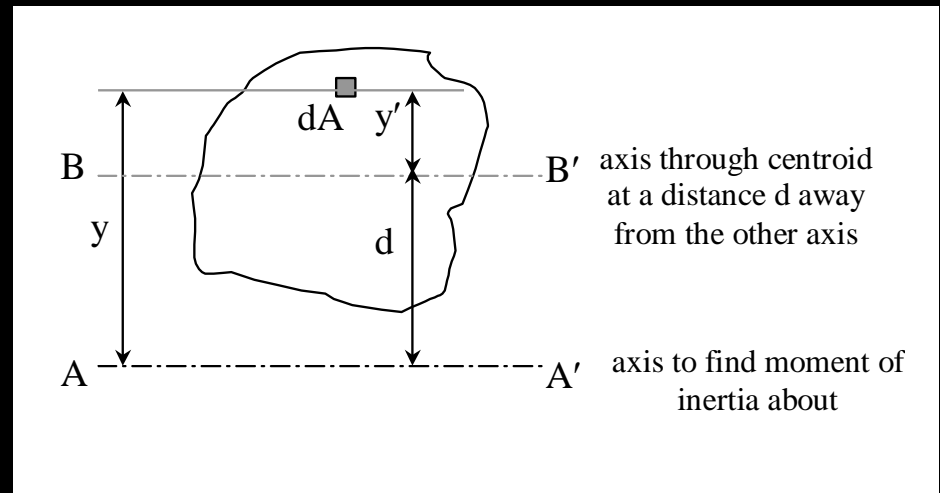
Parallel Axis Theorem

- can find composite I once composite centroid is known (basic shapes)

$$\begin{aligned} I_x &= I_{cx} + Ad_y^2 \\ &= \bar{I}_x + Ad_y^2 \end{aligned}$$

$$I = \sum \bar{I} + \sum Ad^2$$

$$\bar{I} = I - Ad^2$$



Basic Procedure

1. Draw reference origin (if not given)
2. Divide into basic shapes (+/-)
3. Label shapes
4. Draw table with A , \bar{x} , $\bar{x}A$, \bar{y} , $\bar{y}A$, \bar{I} 's, d 's, and Ad^2 's
5. Fill in table and get \hat{x} and \hat{y} for composite
6. Sum necessary columns
7. Sum \bar{I} 's and Ad^2 's

$$\begin{aligned} (d_x &= \hat{x} - \bar{x}) \\ (d_y &= \hat{y} - \bar{y}) \end{aligned}$$

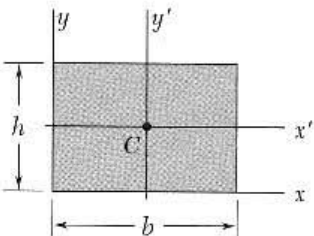
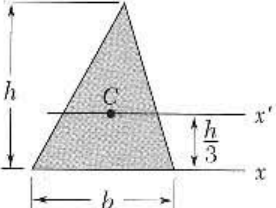
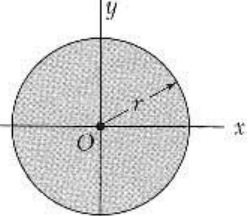
Area Moments of Inertia

- *Table 7.2 – pg. 252: (bars refer to centroid)*

– x, y

– x', y'

– C

Rectangle		$\bar{I}_{x'} = \frac{1}{12}bh^3$ $\bar{I}_{y'} = \frac{1}{12}b^3h$ $I_x = \frac{1}{3}bh^3$ $I_y = \frac{1}{3}b^3h$ $J_C = \frac{1}{12}bh(b^2 + h^2)$
Triangle		$\bar{I}_{x'} = \frac{1}{36}bh^3$ $I_x = \frac{1}{12}bh^3$
Circle		$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$ $J_O = \frac{1}{2}\pi r^4$