ARCHITECTURAL STRUCTURES:
FORM, BEHAVIOR, AND DESIGN

ARCH 331

DR. ANNE NICHOLS

SUMMER 2013

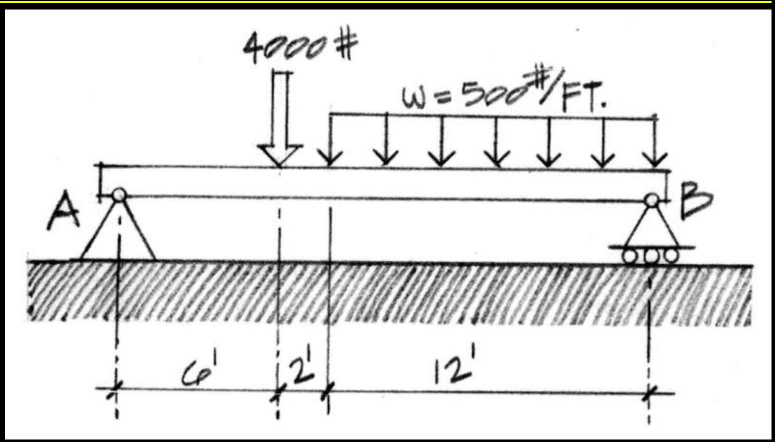
lecture
SIX

http:// nisee.berkeley.edu/godden

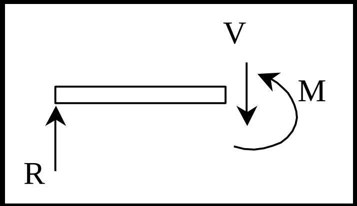
beams – internal forces & diagrams

Beams

- *span horizontally*
  - *floors*
  - *bridges*
  - *roofs*

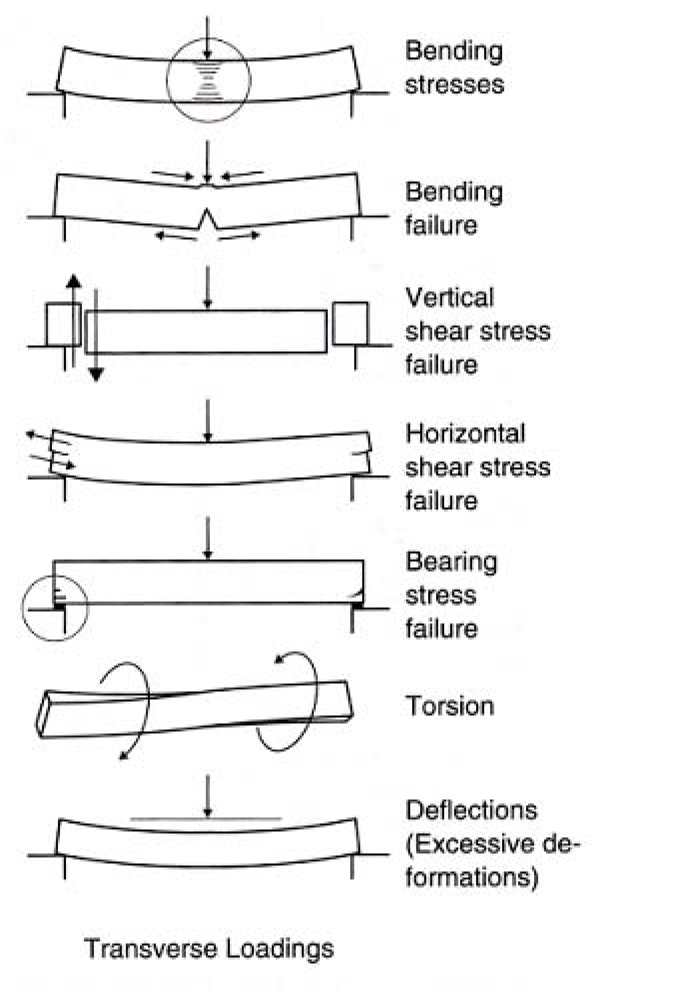


- *loaded transversely by gravity loads*
- *may have internal axial force*
- *will have internal shear force*
- *will have internal moment (bending)*



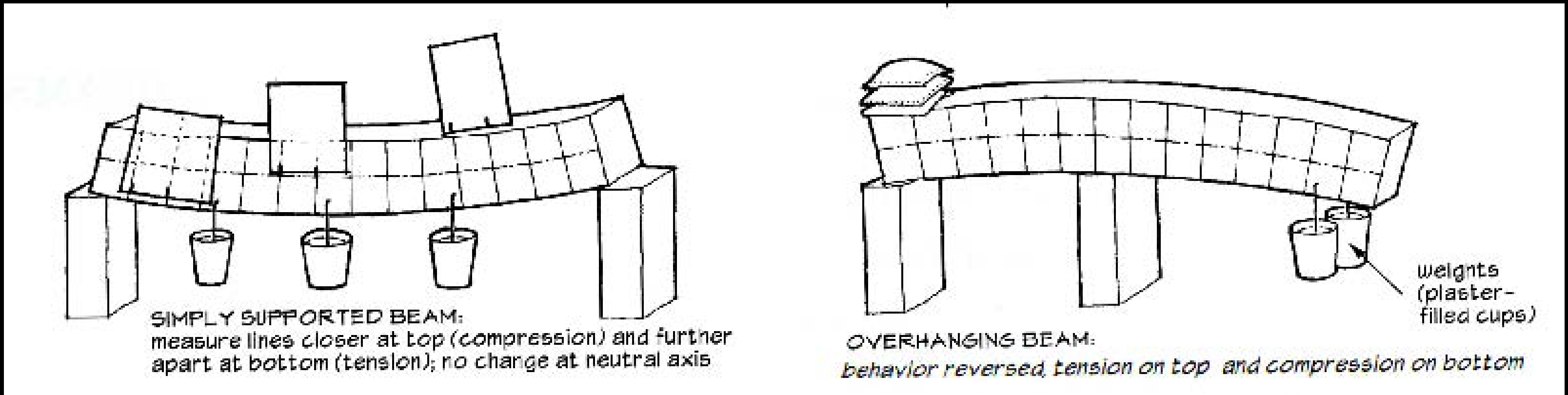
Beams

- *transverse loading*
- *sees:*
  - *bending*
  - *shear*
  - *deflection*
  - *torsion*
  - *bearing*
- *behavior depends on cross section shape*



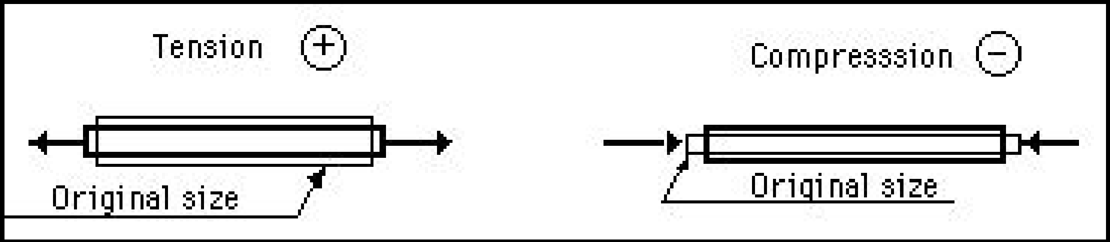
Beams

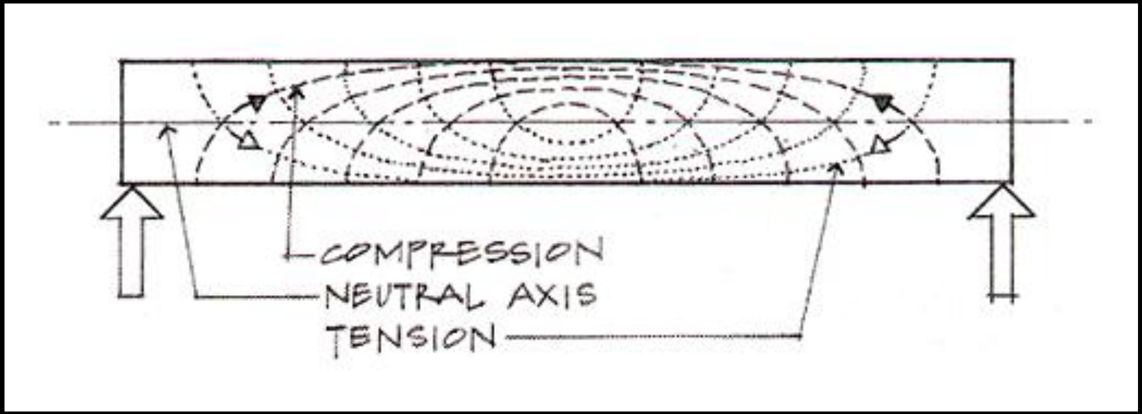
- *bending*
  - *bowing of beam with loads*
  - *one edge surface stretches*
  - *other edge surface squishes*



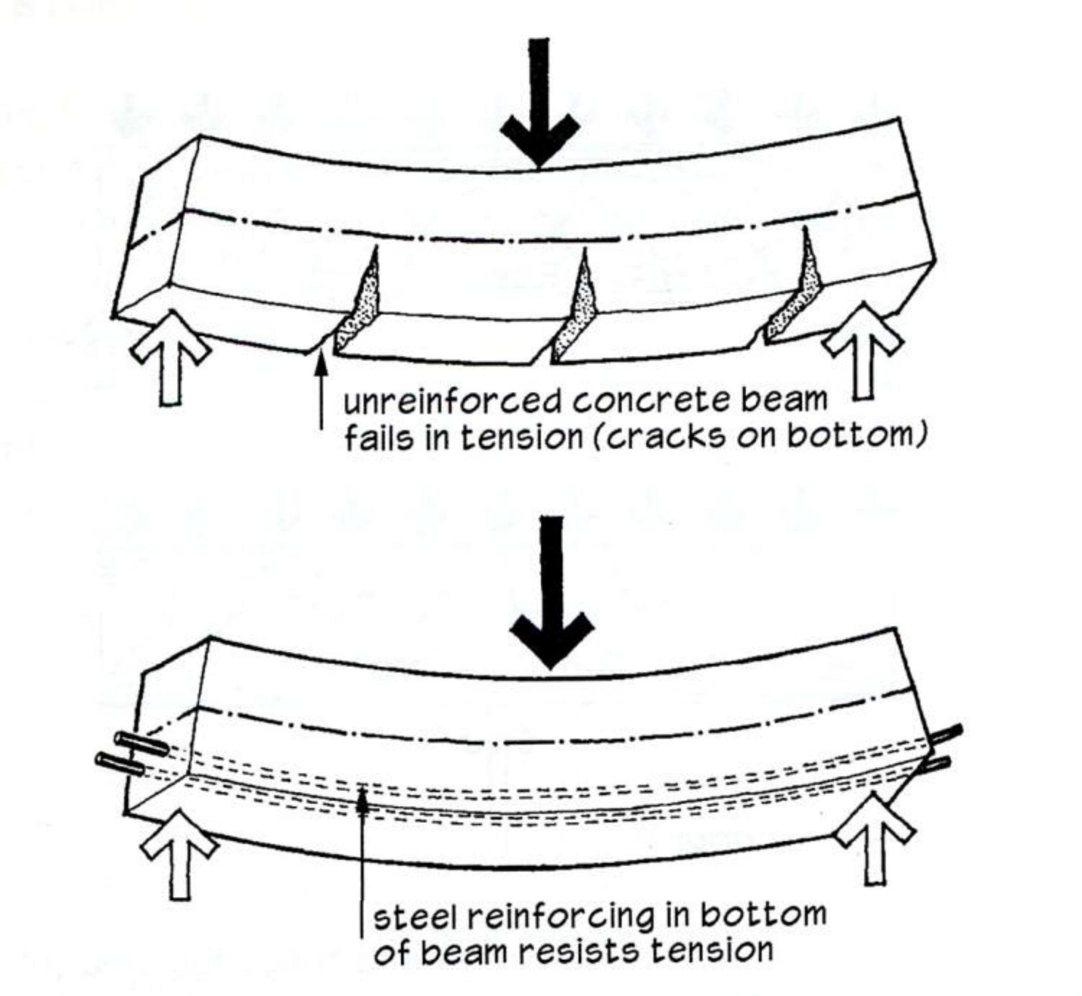
Beam Stresses

- stress = relative force over an area
  - tensile
  - compressive
  - bending
    - tension and compression + ...



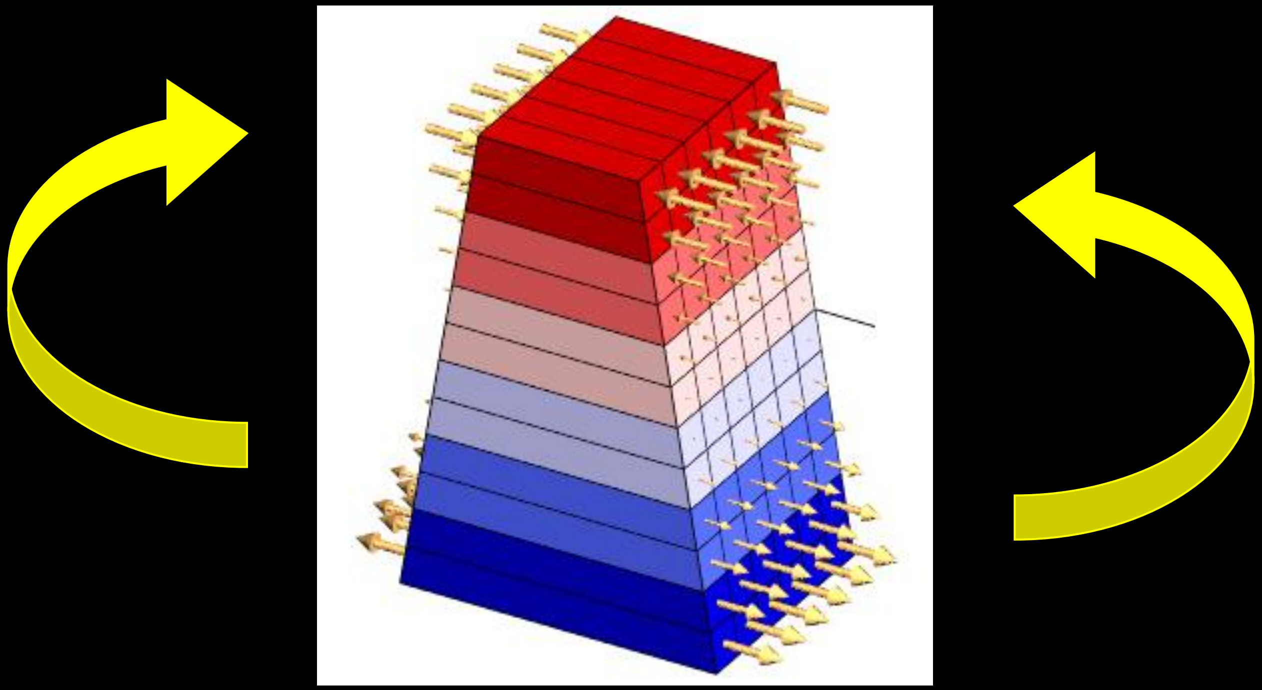


Beam Stresses



Beam Stresses

- *tension and compression*
  - *causes moments*



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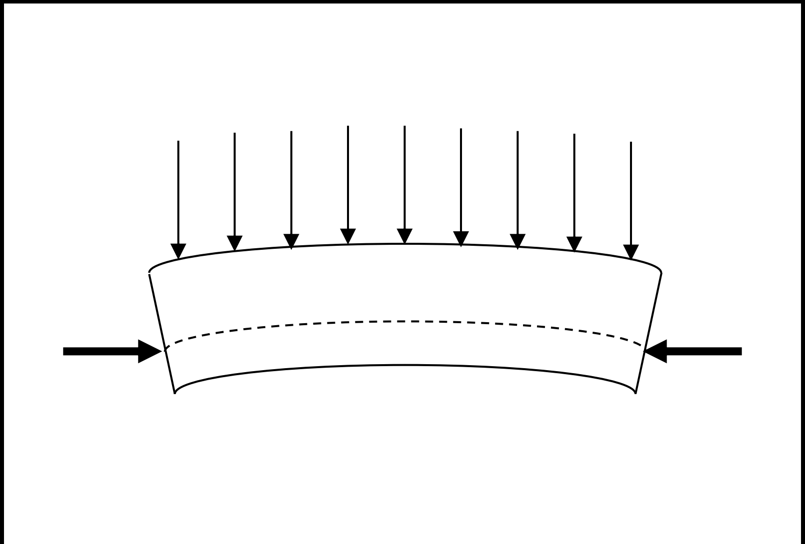
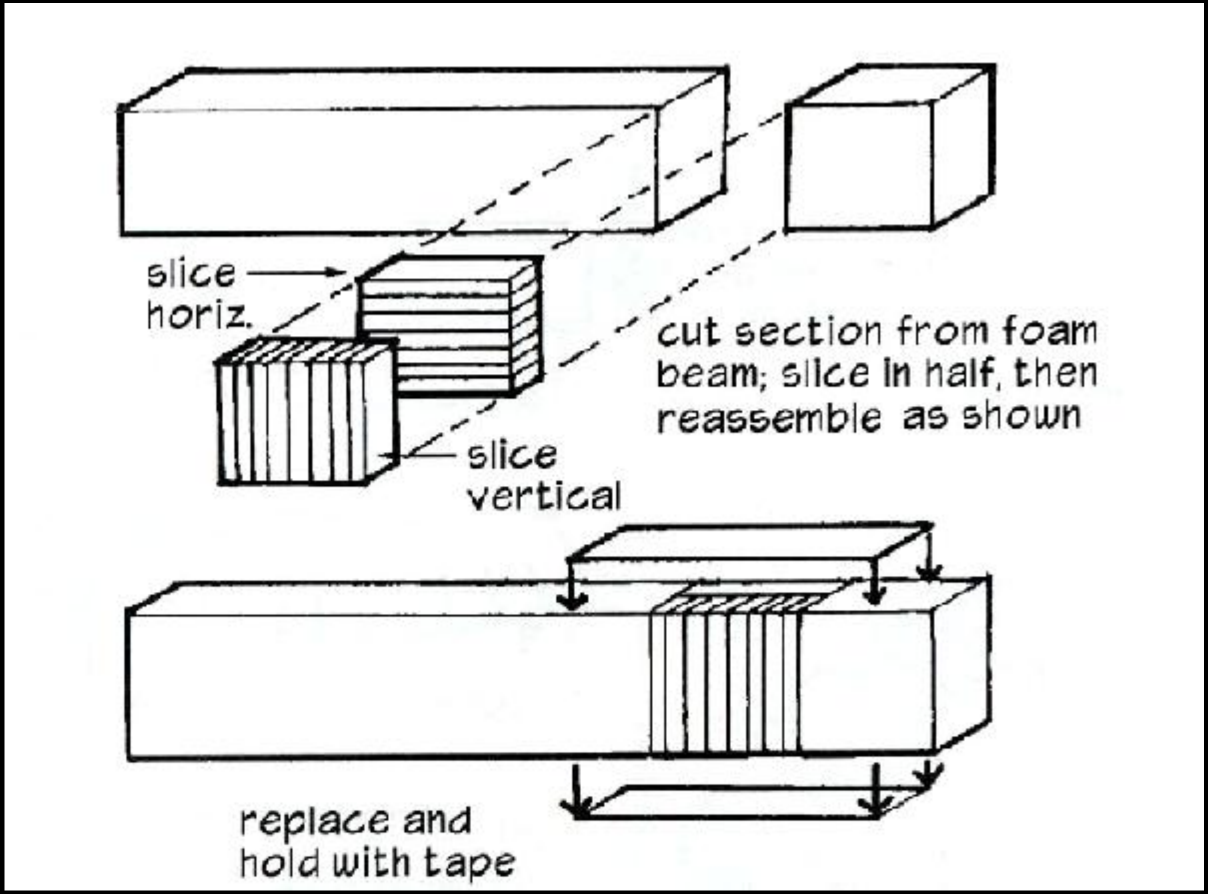
Beam Stresses

- *prestress or post-tensioning*
  - *put stresses in tension area to "pre-compress"*



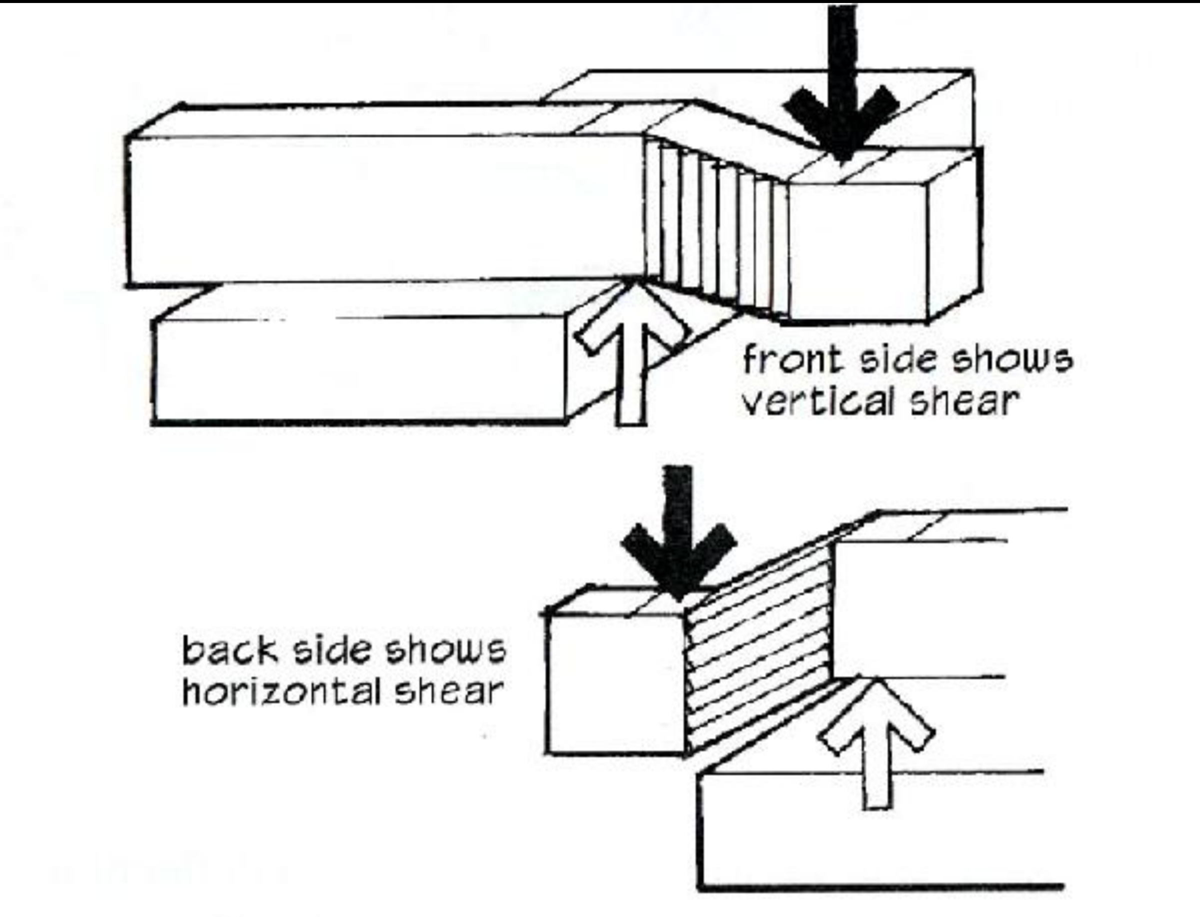
Beam Stresses

- *shear – horizontal & vertical*



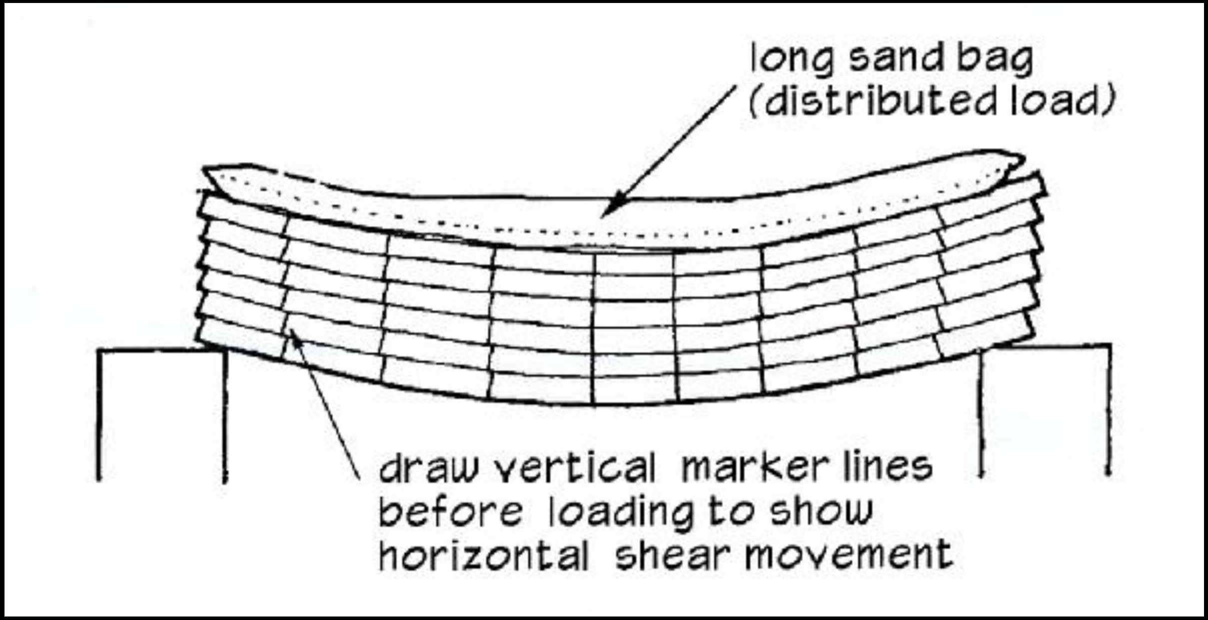
Beam Stresses

- *shear – horizontal & vertical*



Beam Stresses

- *shear – horizontal*



Beam Deflections

- *depends on*
  - *load*
  - *section*
  - *material*

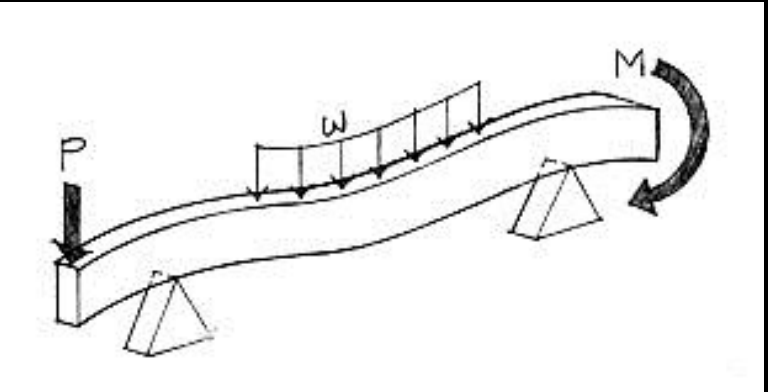
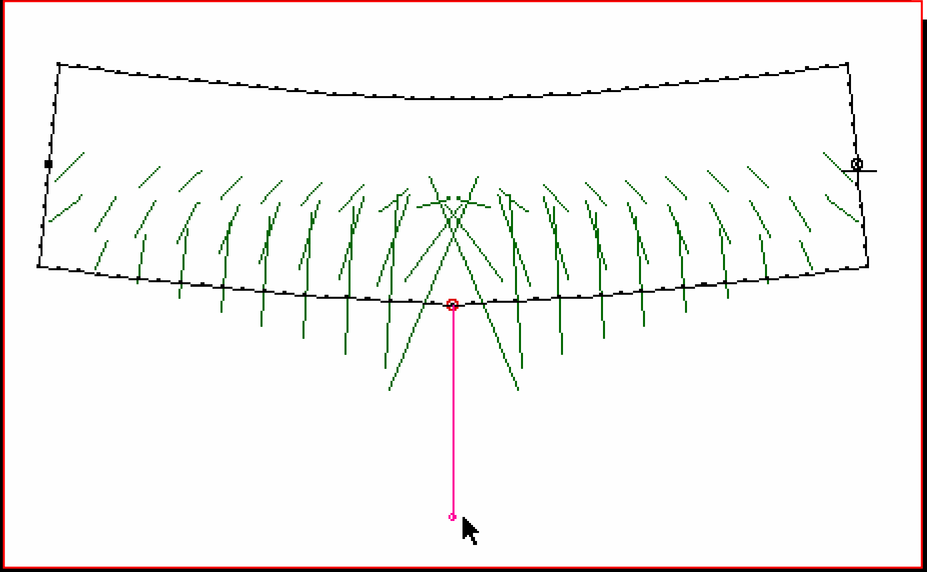
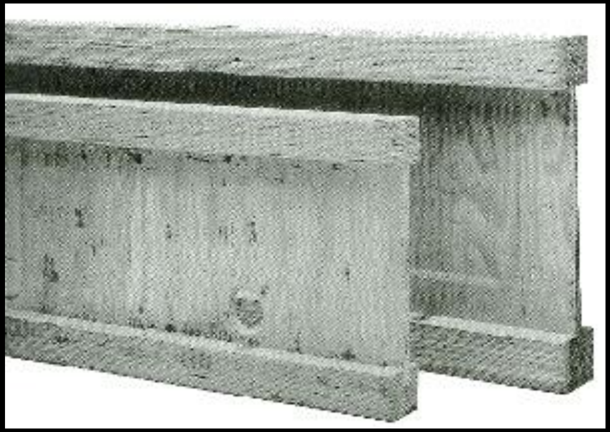


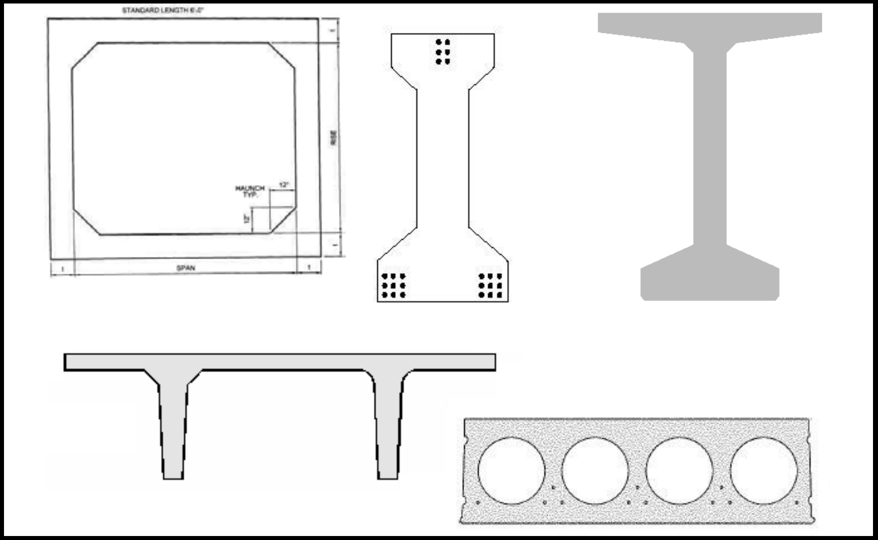
Figure 5.4 Bending (flexural) loads on a beam.

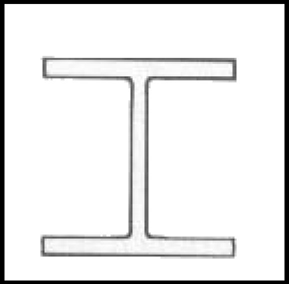


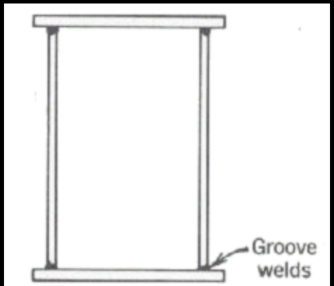
Beam Deflections

- *“moment of inertia”*









(e) Welded box girder.

Beam Styles

- *vierendeel*

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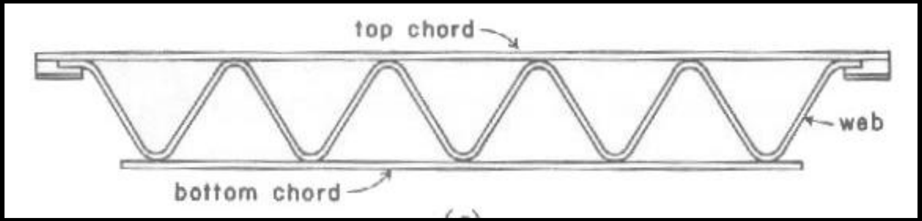
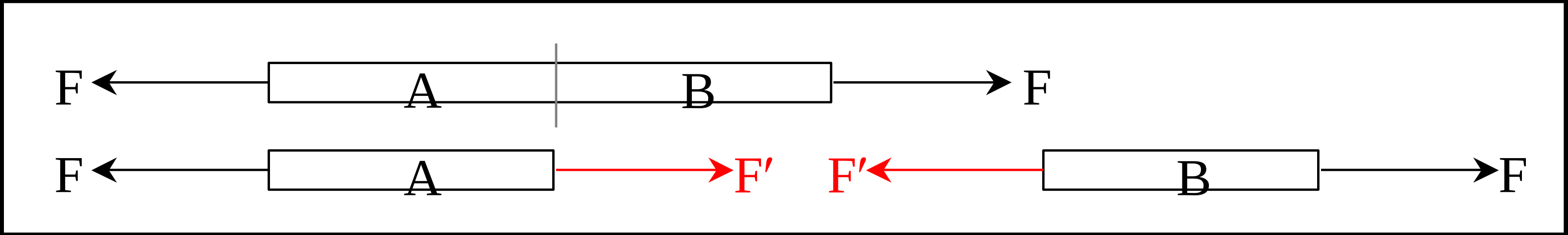
- *open web joists*



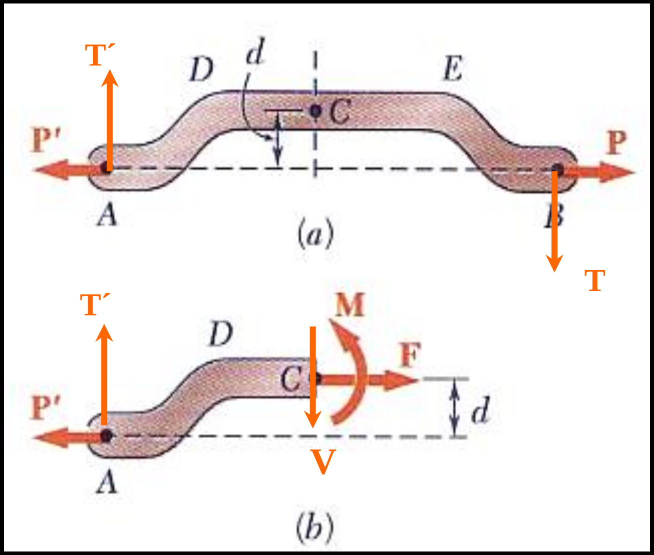
- *manufactured*

Internal Forces

- trusses
  - axial only, (compression & tension)

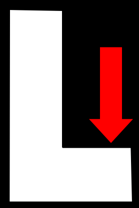


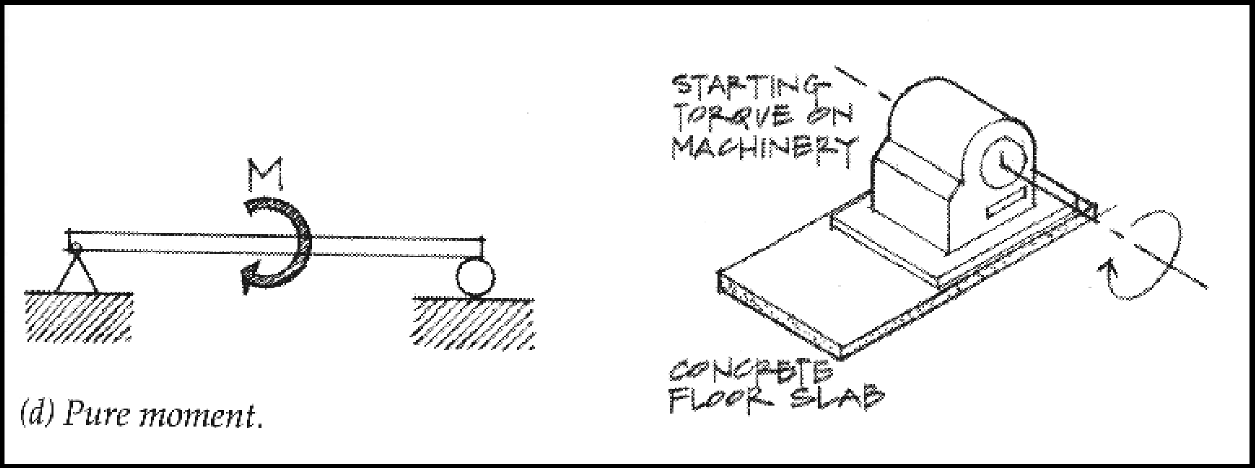
- in general
  - axial force
  - shear force, V
  - bending moment, M



Beam Loading

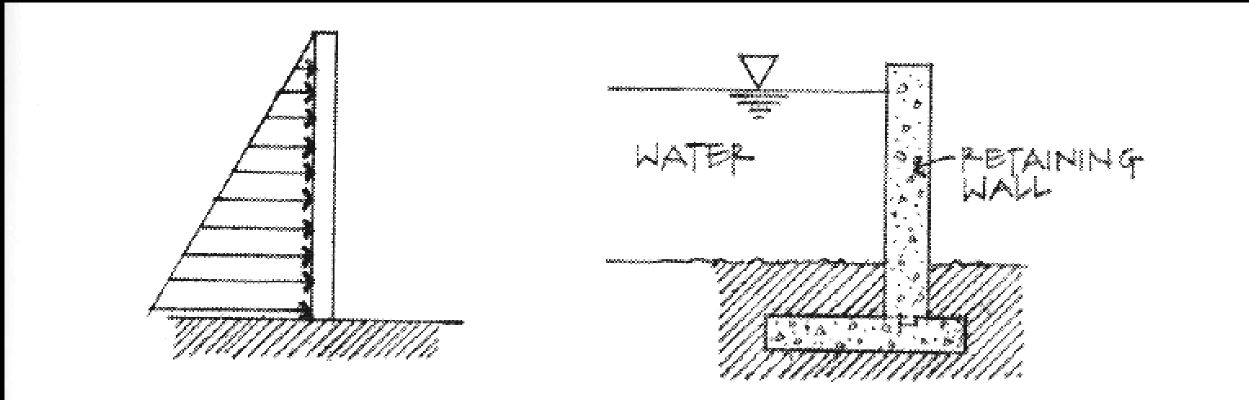
- *concentrated force*
- *concentrated <u>moment</u>*
  - *– spandrel beams*





Beam Loading

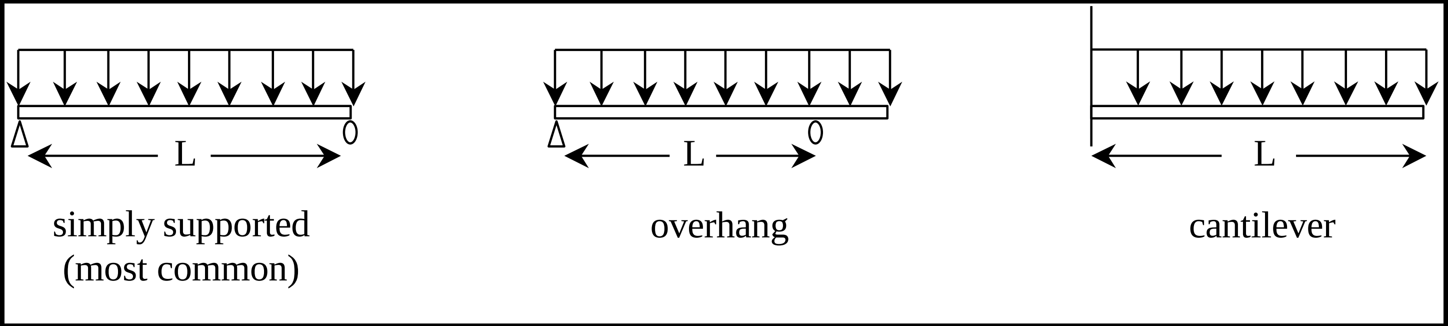
- *uniformly distributed load (line load)*
- *non-uniformly distributed load*
  - *hydrostatic pressure = γh*
  - *wind loads*



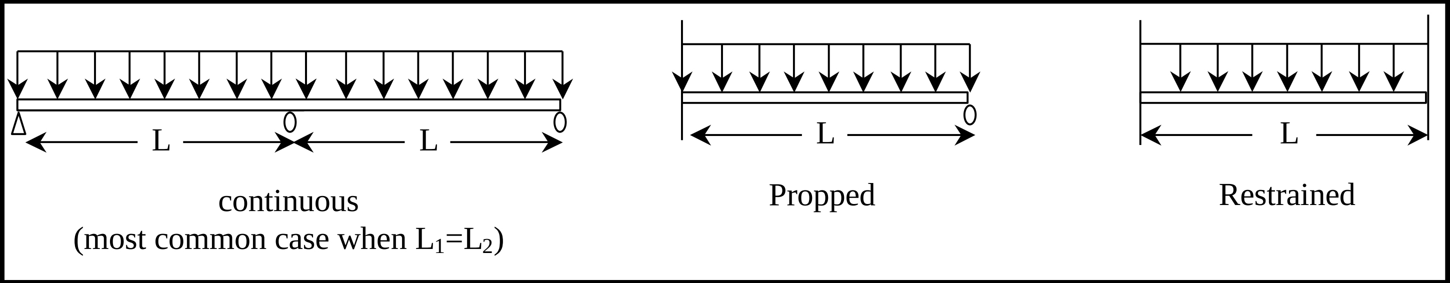
(c) Nonuniformly distributed load.

Beam Supports

- *statically determinate*

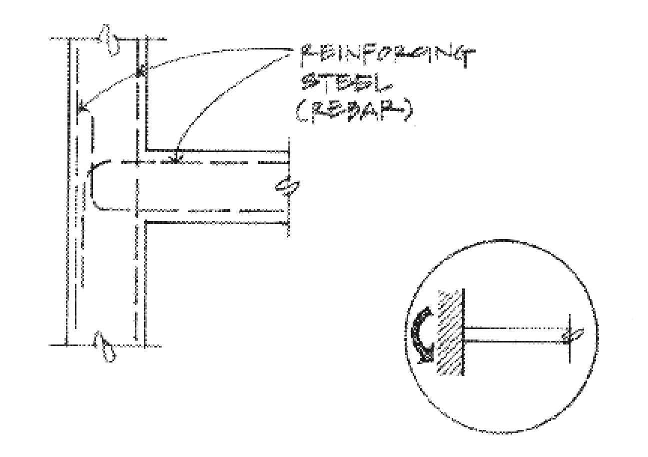


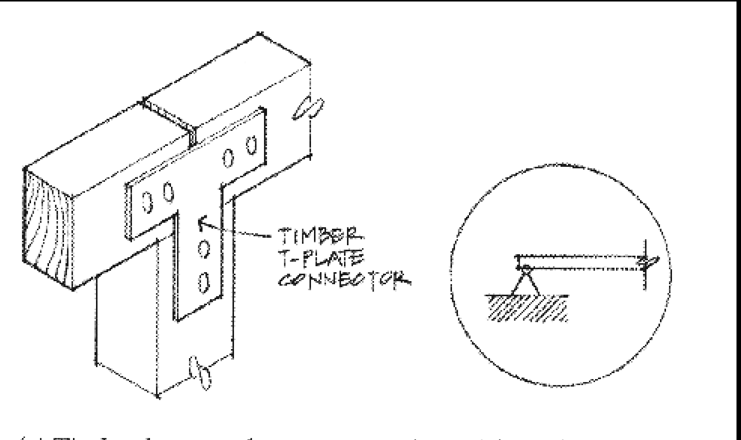
- *statically indeterminate*



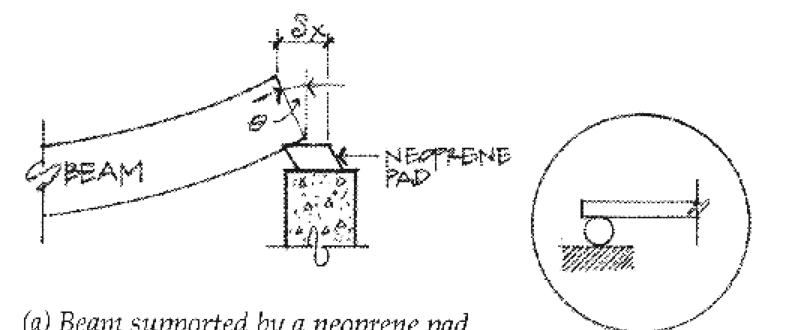
Beam Supports

- *in the real world, modeled type*





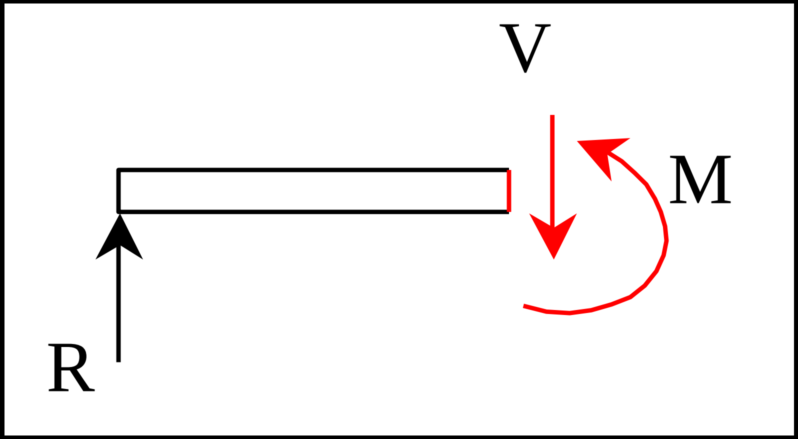
(c) Timber beam–column connection with T-plate.



(a) Beam supported by a neoprene pad.

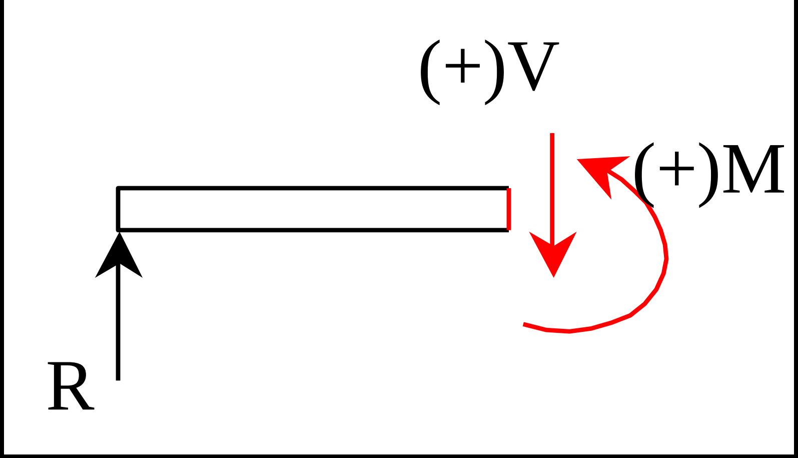
Internal Forces in Beams

- *like method of sections / joints*
  - *no axial forces*
- *section <u>must</u> be in equilibrium*
- *want to know where biggest internal forces and moments are for designing*



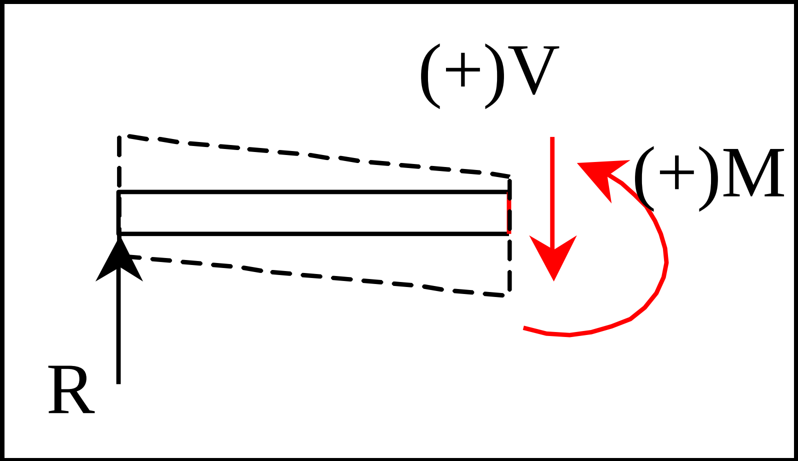
V & M Diagrams

- *tool to locate V_{max} and M_{max} (at V = 0)*
- *<u>necessary</u> for designing*
- *have a <u>different sign convention</u> than external forces, moments, and reactions*

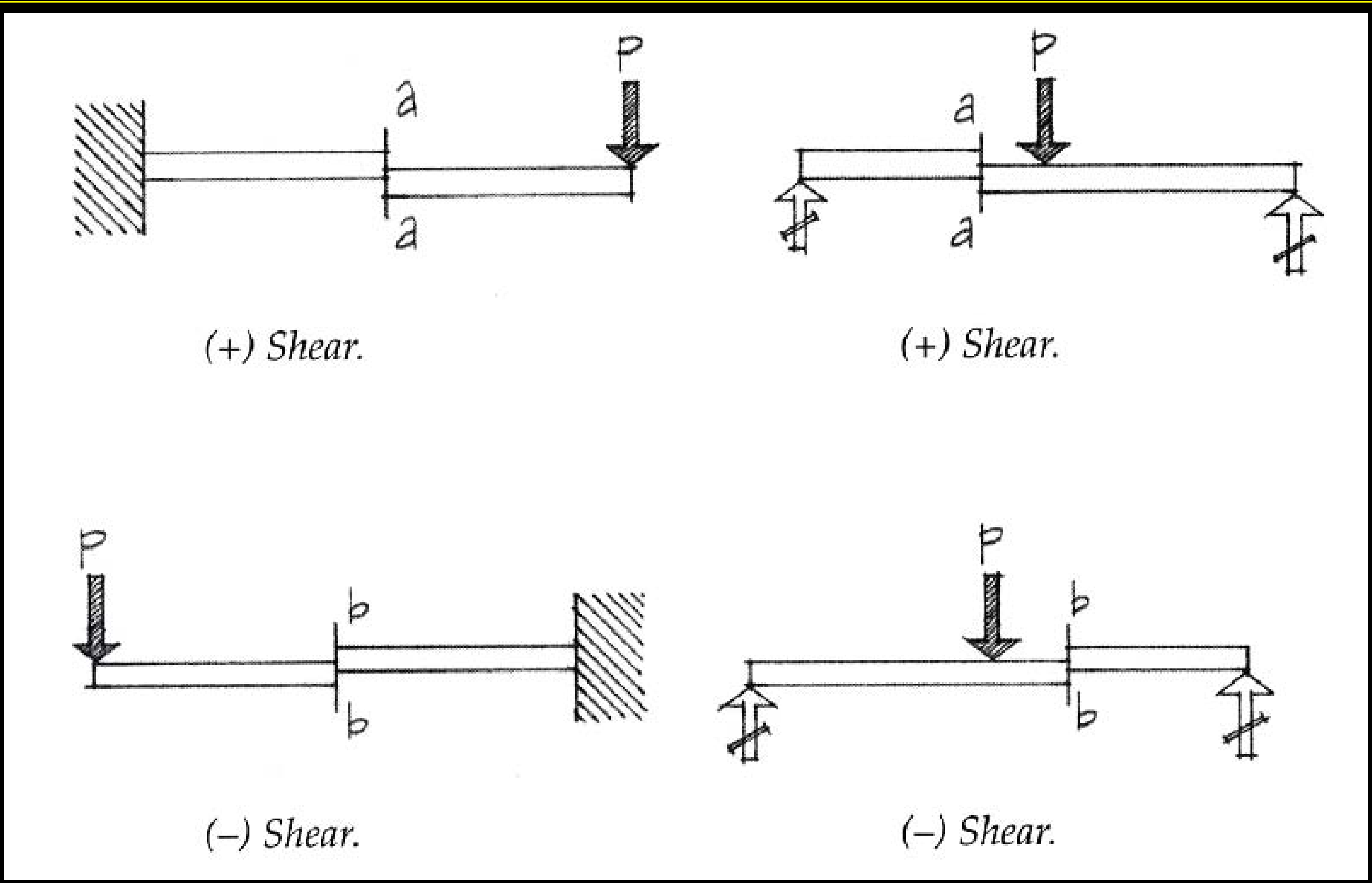


Sign Convention

- *shear force, V:*
  - *cut section to LEFT*
  - *if $\sum F_y$ is positive by statics, V acts down and is POSITIVE*
  - *beam has to resist shearing apart by V*

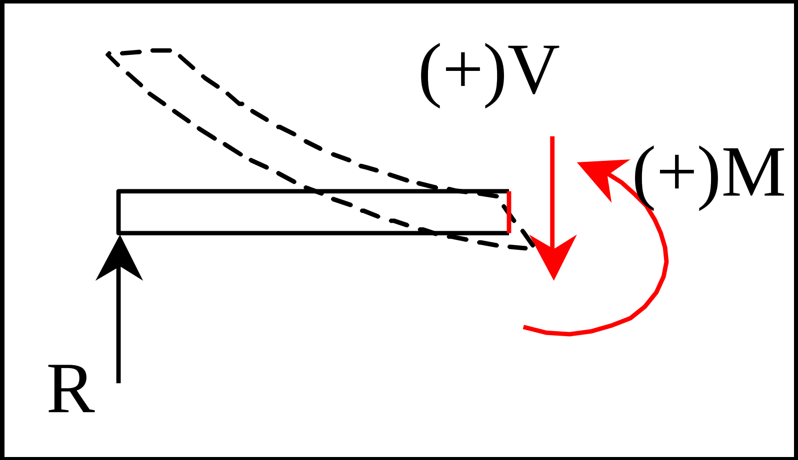


Shear Sign Convention

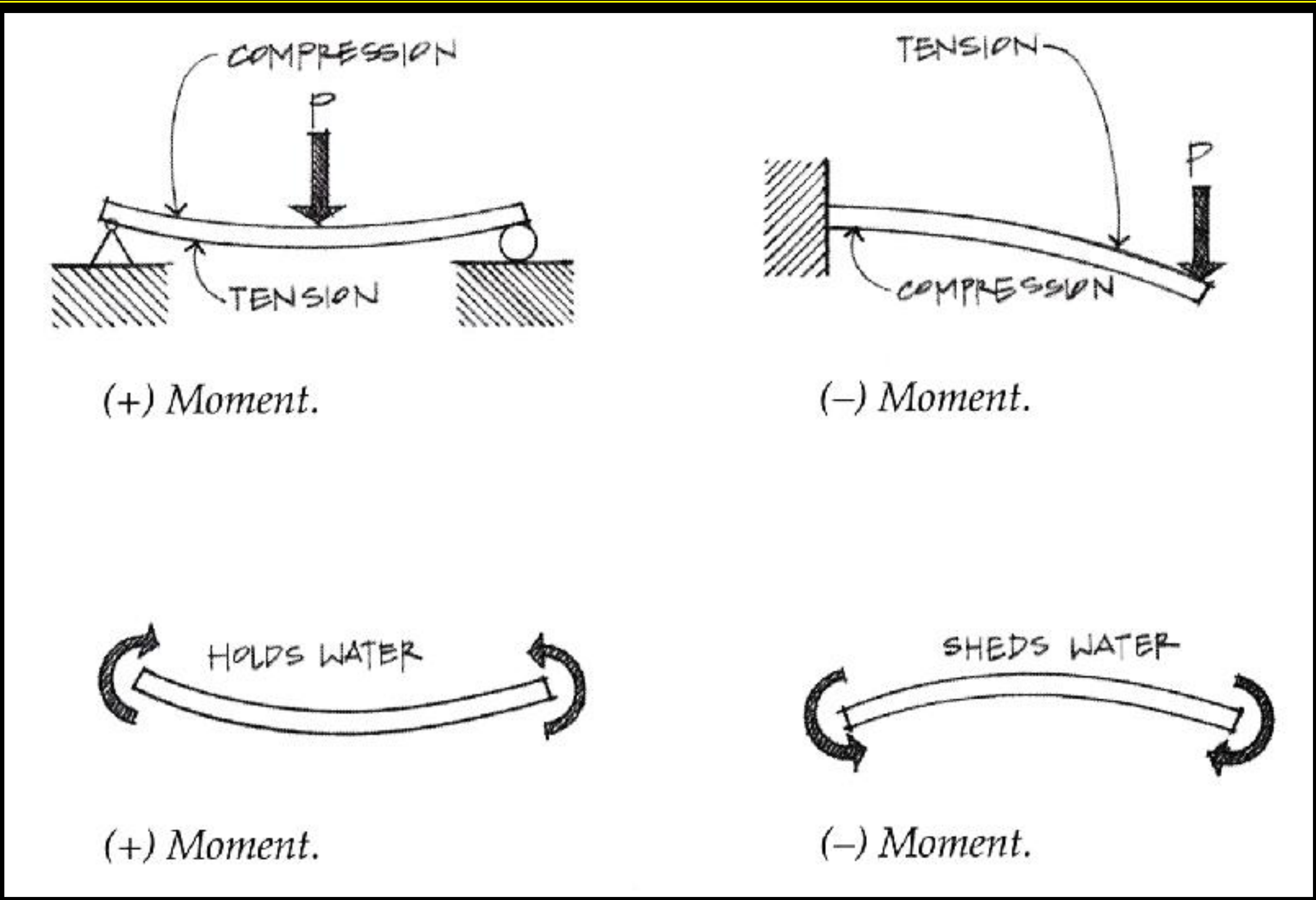


Sign Convention

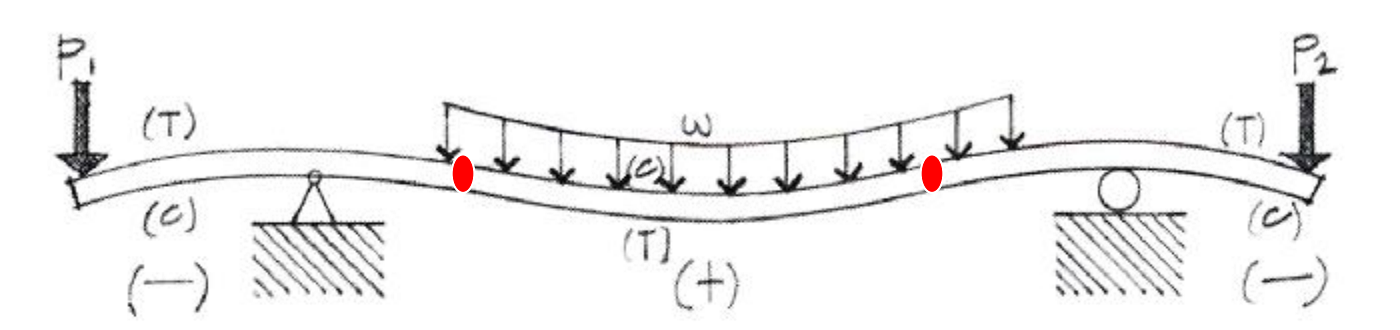
- *bending moment, M:*
  - *cut section to LEFT*
  - *if $\sum M_{cut}$ is clockwise, M acts ccw and is POSITIVE – flexes into a "smiley" beam has to resist bending apart by M*



Bending Moment Sign Convention



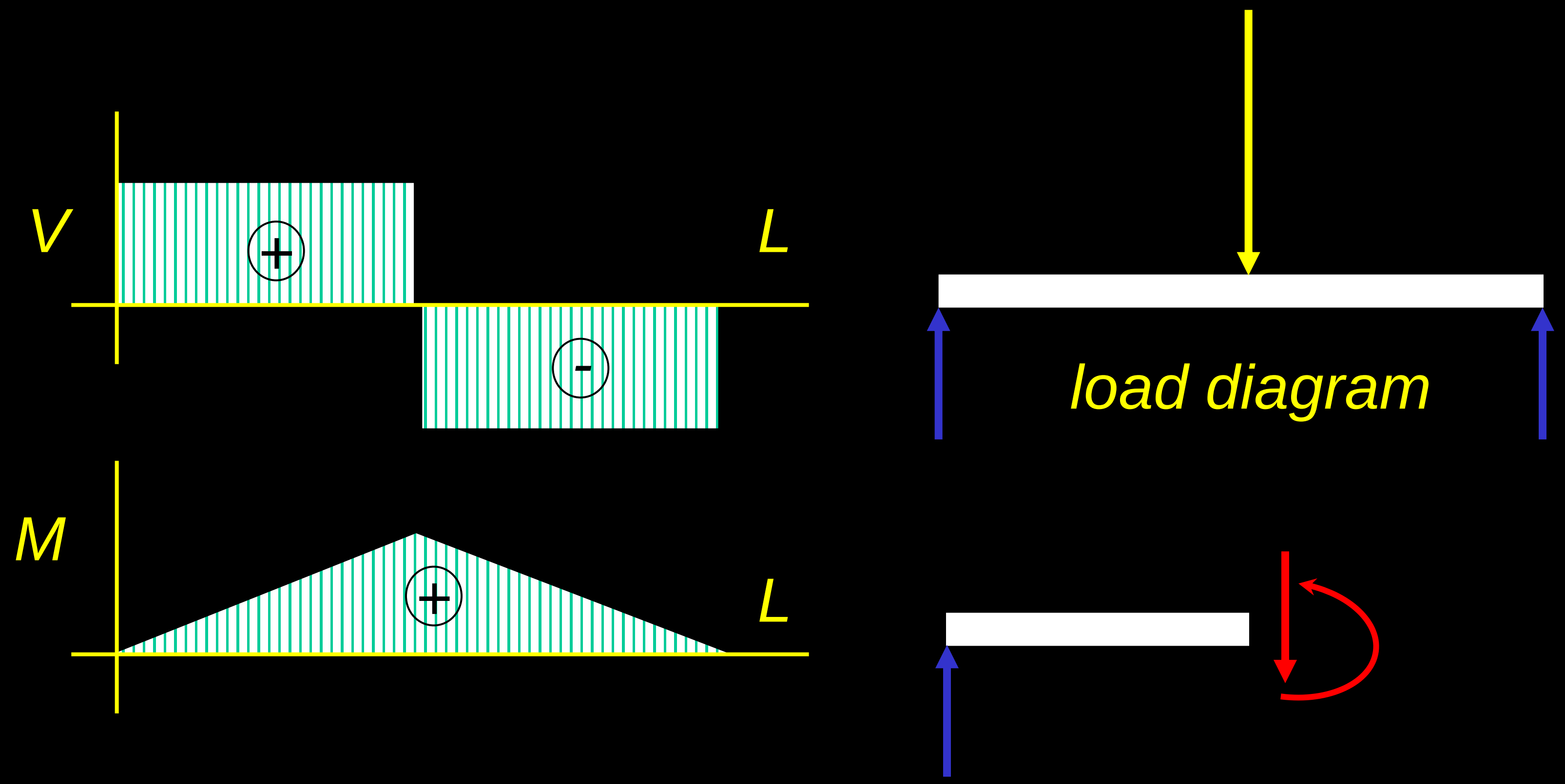
Deflected Shape



- *positive bending moment*
  - *tension in bottom, compression in top*
- *negative bending moment*
  - *tension in top, compression in bottom*
- *zero bending moment*
  - *inflection point*

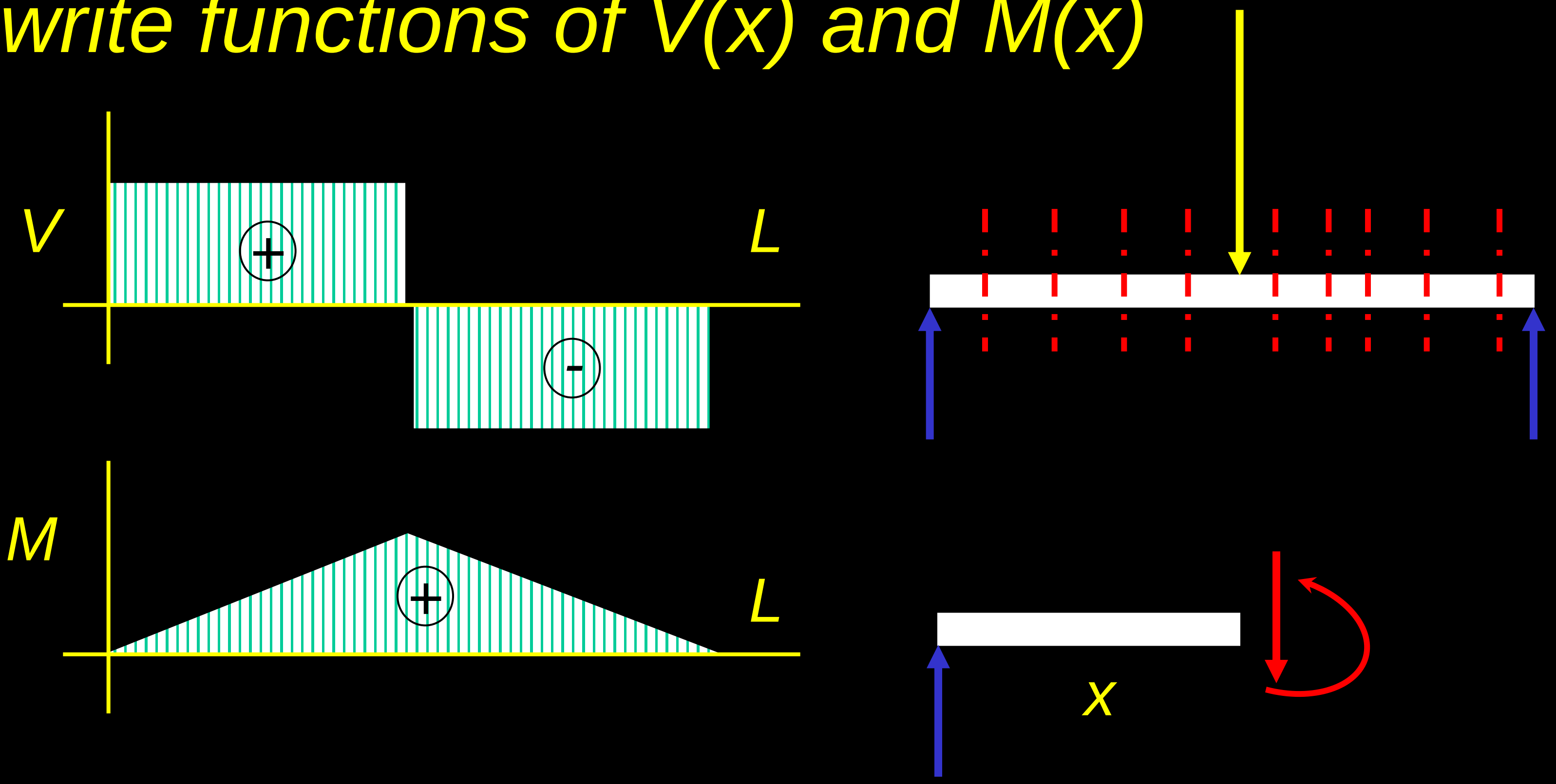
Constructing V & M Diagrams

- *along the beam length, plot V, plot M*



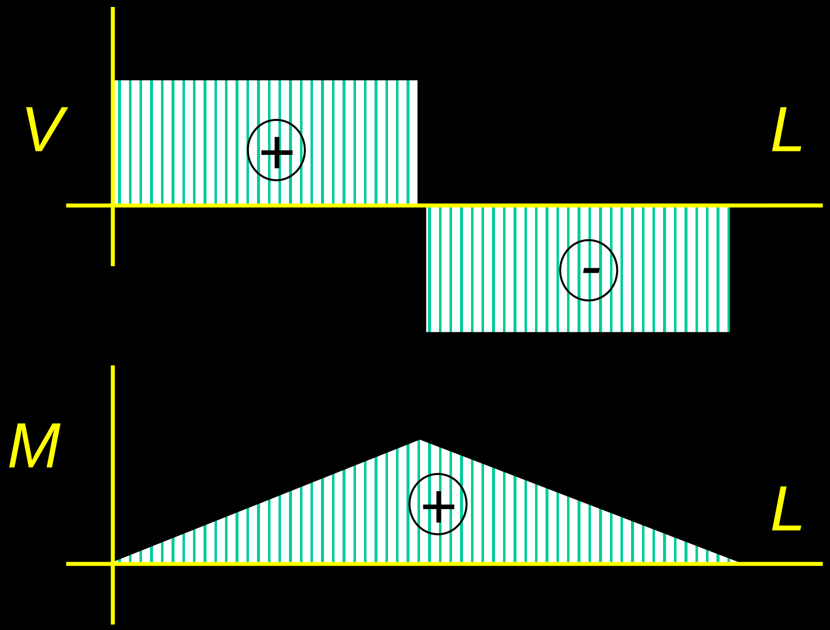
Mathematical Method

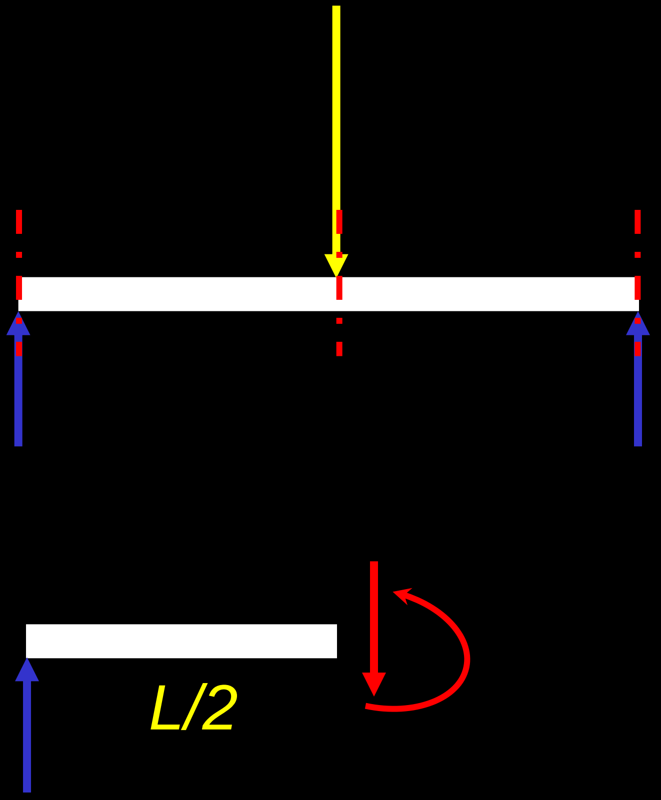
- *cut sections with x as width*
- *write functions of V(x) and M(x)*



Method 1: Equilibrium

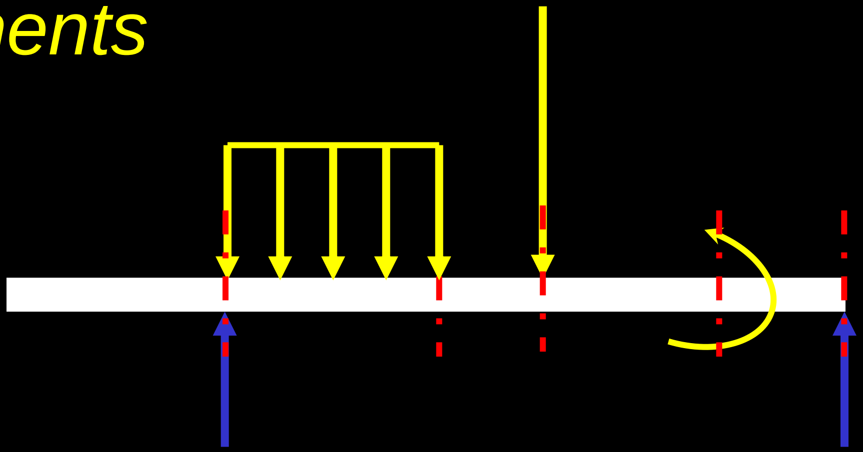
- *cut sections at important places*
- *plot V & M*





Method 1: Equilibrium

- *important places*
  - *supports*
  - *concentrated loads*
  - *start and end of distributed loads*
  - *concentrated moments*
- *free ends*
  - *zero forces*



Method 2: Semigraphical

- *by knowing*
  - *area under loading curve = change in V*
  - *area under shear curve = change in M*
  - *concentrated forces cause “jump” in V*
  - *concentrated moments cause “jump” in M*

$$V_D - V_C = -\int_{x_C}^{x_D} w\,dx \qquad M_D - M_C = \int_{x_C}^{x_D} V\,dx$$

Method 2

- *relationships*

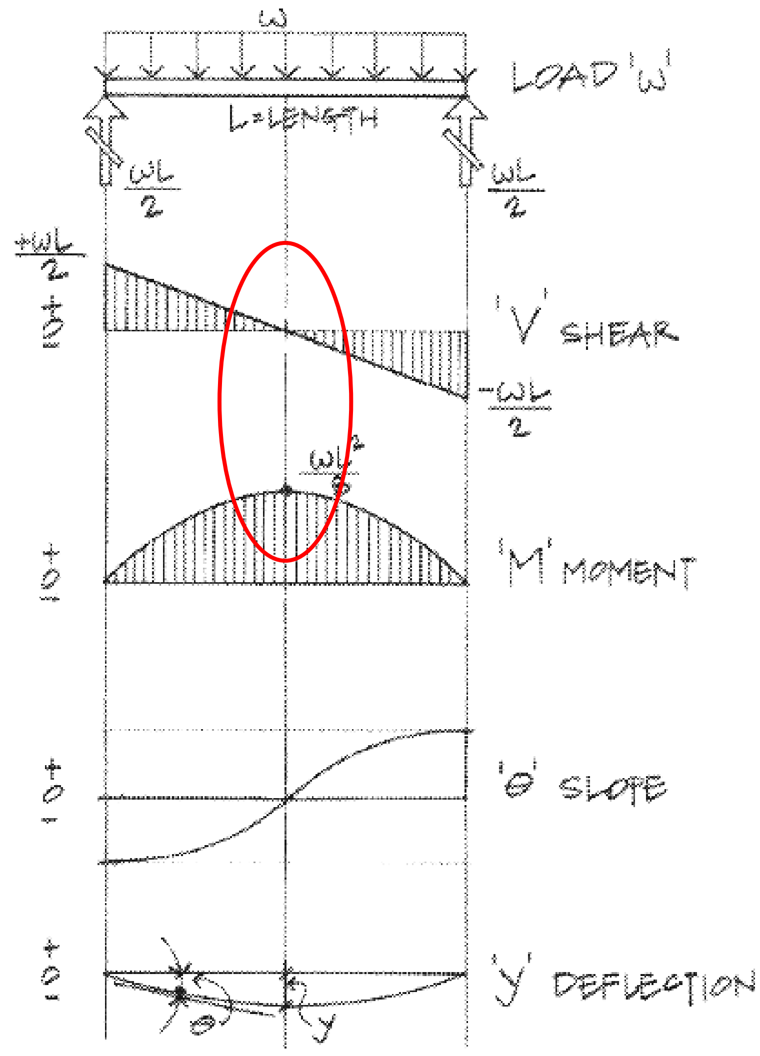
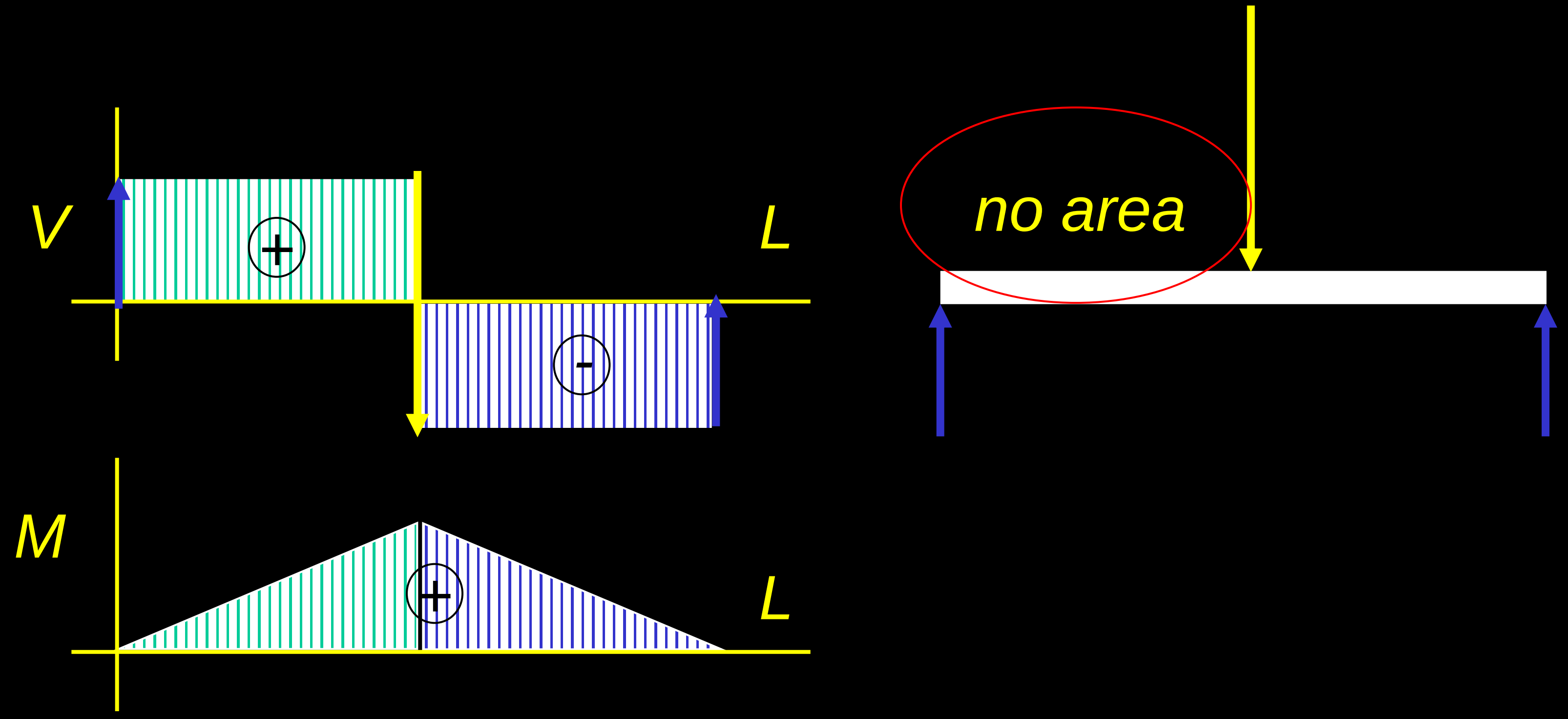


Figure 7.11 Relationship of load, shear, moment, slope, and deflection diagrams.

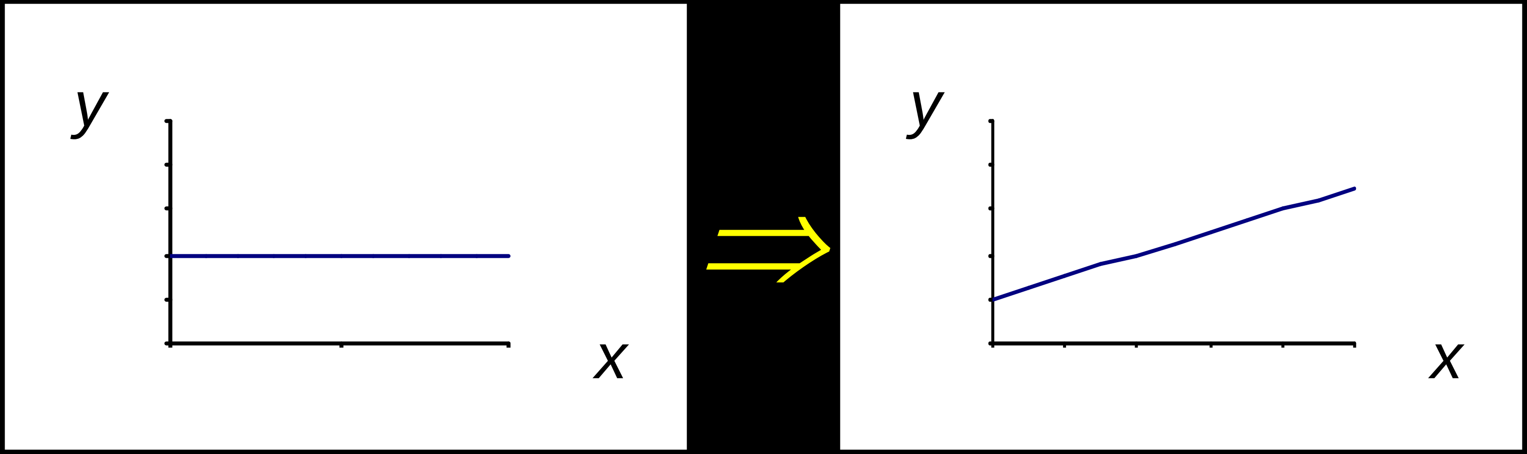
Method 2: Semigraphical

- M_{max} occurs where $V = 0$ (calculus)



Curve Relationships

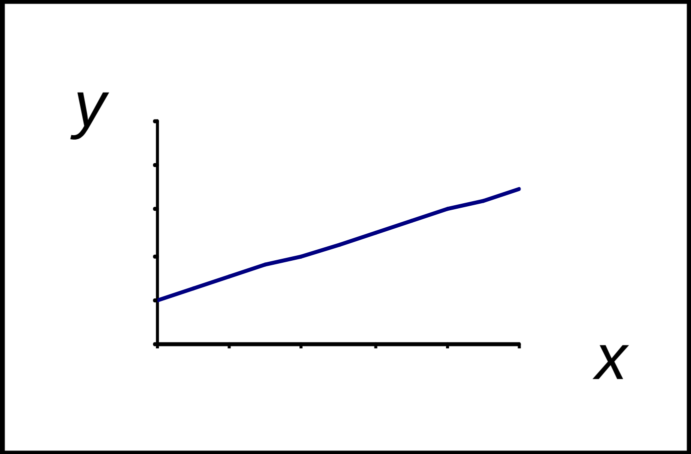
- integration of functions
- line with 0 slope, integrates to sloped



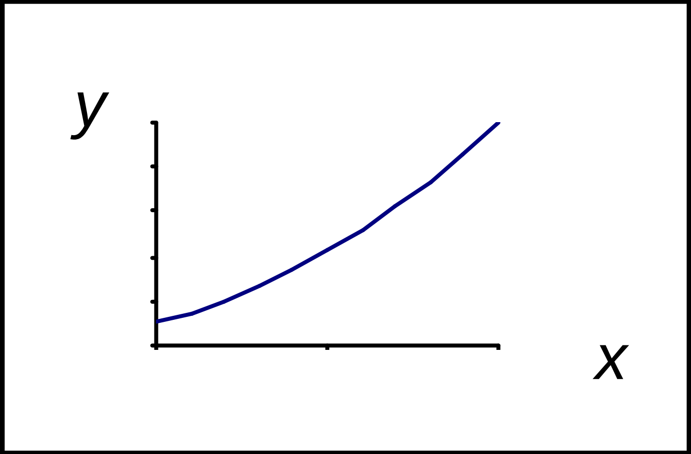
- ex: load to shear, shear to moment

Curve Relationships

- line with slope, integrates to parabola



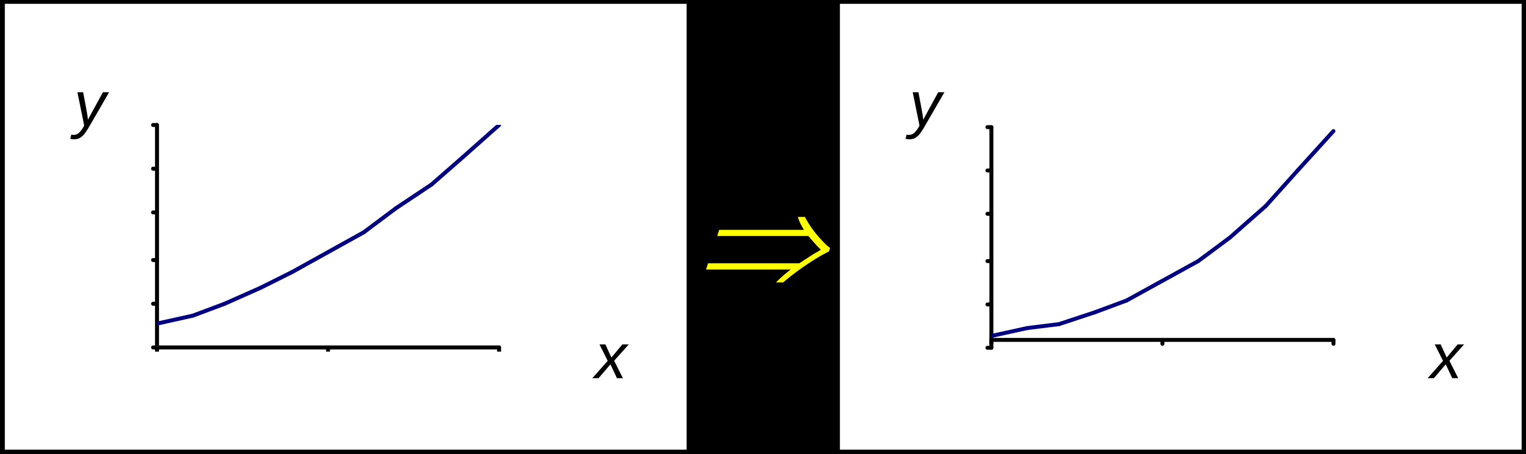
$\Rightarrow$



- ex: load to shear, shear to moment

Curve Relationships

- parabola, integrates to 3rd order curve



- ex: load to shear, shear to moment

Basic Procedure with Sections

1. *Find reaction forces & moments*

   Plot axes, underneath beam load diagram

V:

2. *Starting at left*
3. *Shear is 0 at free ends*
4. *Shear has 2 values at point loads*
5. *<u>Sum vertical forces at each section</u>*

Basic Procedure with Sections

M:

6. Starting at left
7. Moment is 0 at free ends
8. Moment has 2 values at moments
9. <u>Sum moments at each section</u>
10. Maximum moment is where shear = 0!
    (locate where V = 0)

Basic Procedure by Curves

1. *Find reaction forces & moments*

   Plot axes, underneath beam load diagram

V:

2. *Starting at left*
3. *Shear is 0 at free ends*
4. *Shear jumps with concentrated load*
5. *Shear changes with area under load*

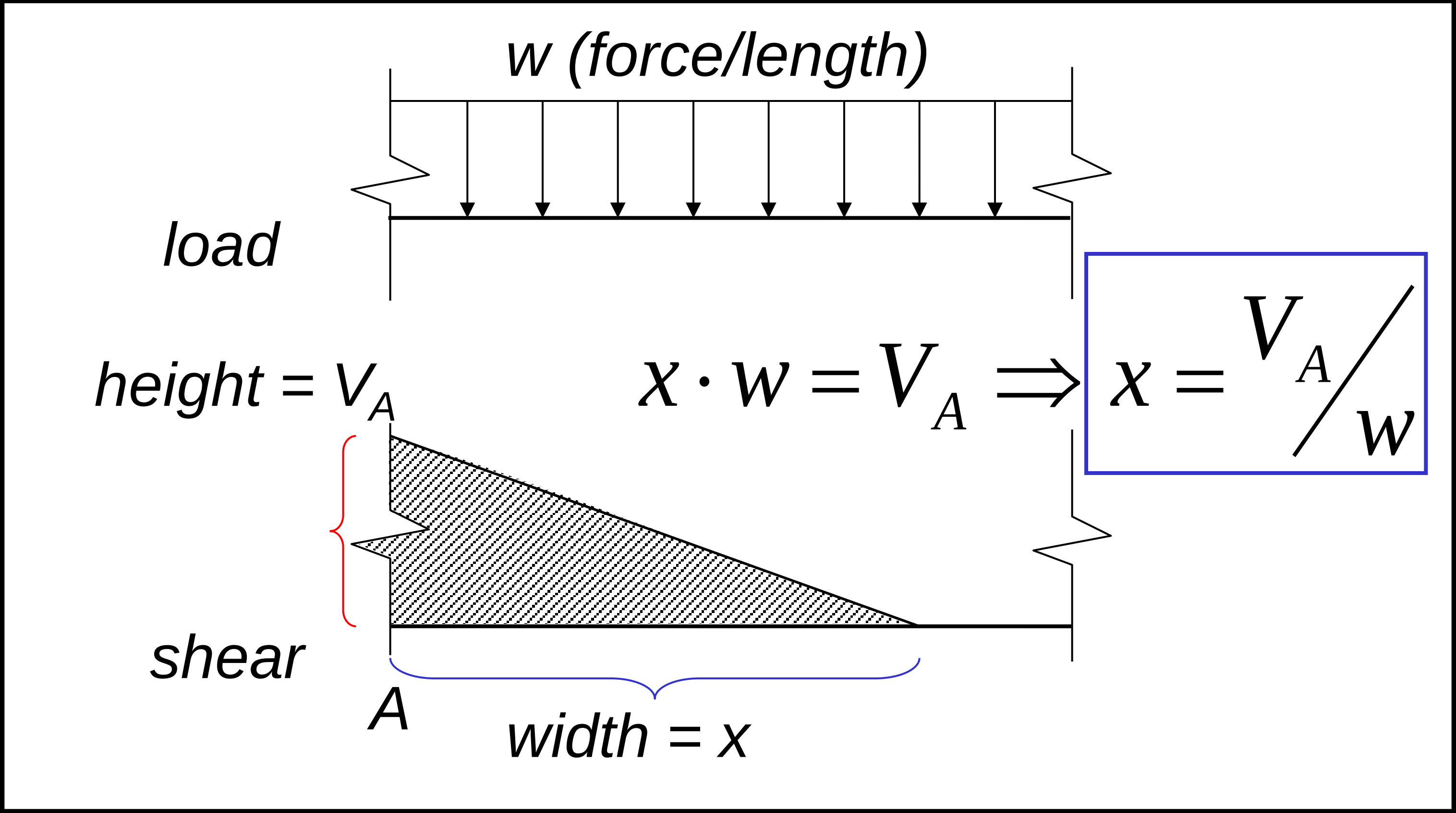
Basic Procedure by Curves

M:

6. Starting at left
7. Moment is 0 at free ends
8. <u>Moment jumps with moment</u>
9. <u>Moment changes with area under V</u>
10. Maximum moment is where shear = 0!
    (locate where V = 0)

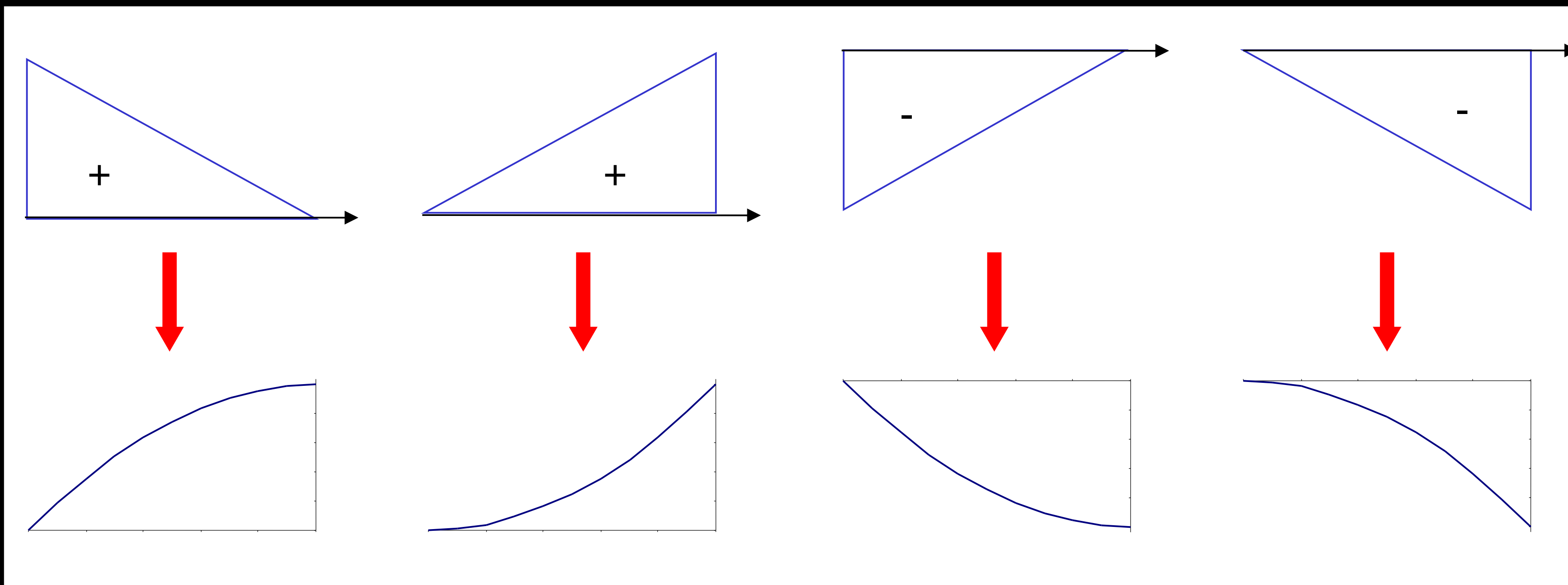
Shear Through Zero

- slope of V is w (-w:1)



Parabolic Shapes

- *cases*



up fast, then slow

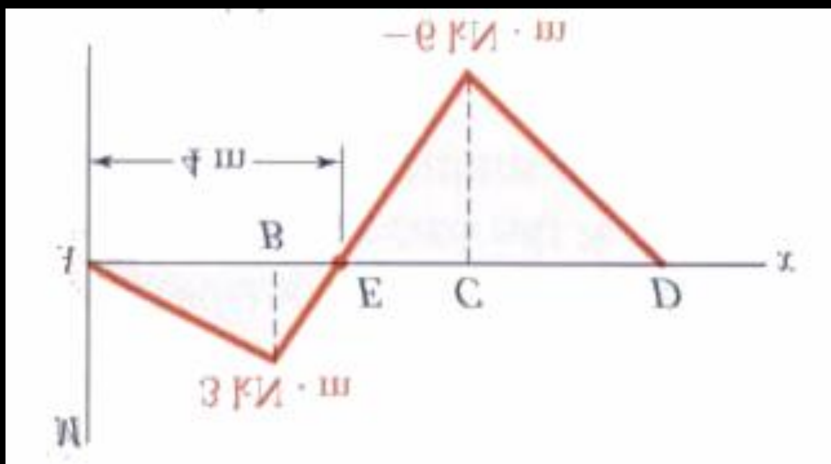
up slow, then fast

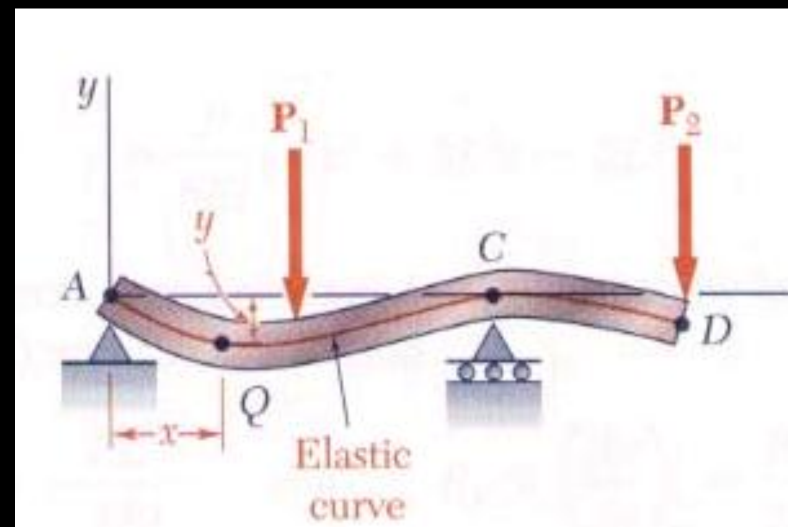
down fast, then slow

down slow, then fast

Deflected Shape & M(x)

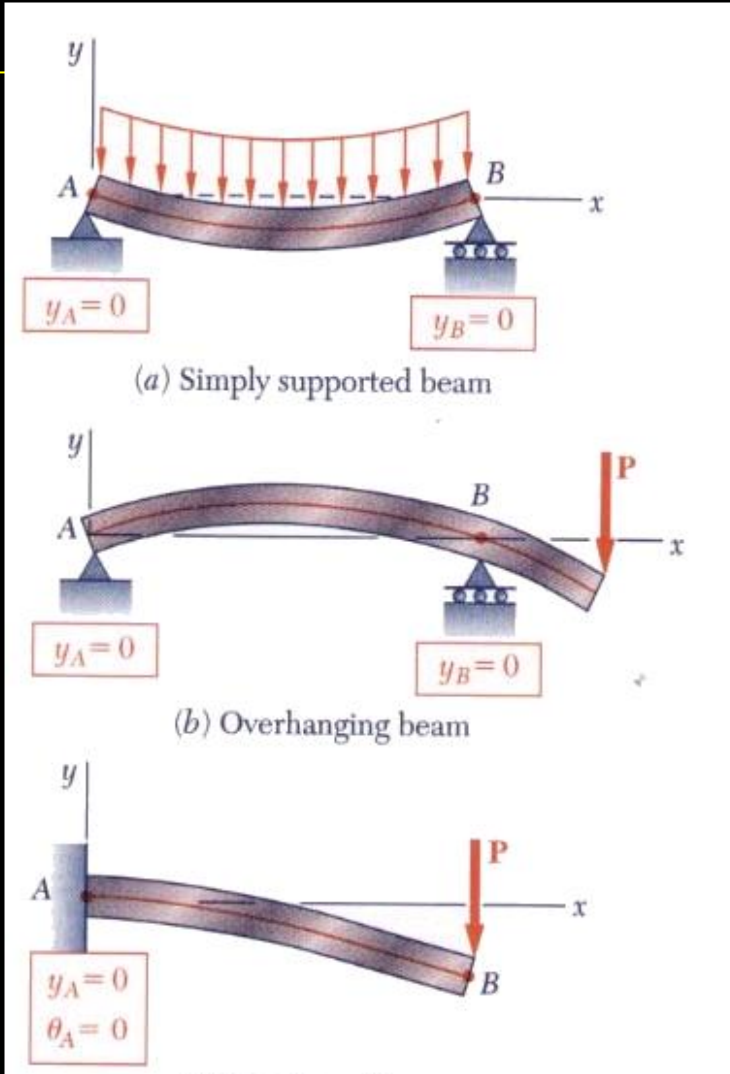
- *-M(x) gives shape indication*
- *boundary conditions must be met*





Boundary Conditions

- *at pins, rollers, fixed supports:* $y = 0$
- *at fixed supports:* $\theta = 0$
- *at inflection points from symmetry:* $\theta = 0$
- y_{max} *at* $\frac{dy}{dx} = 0$

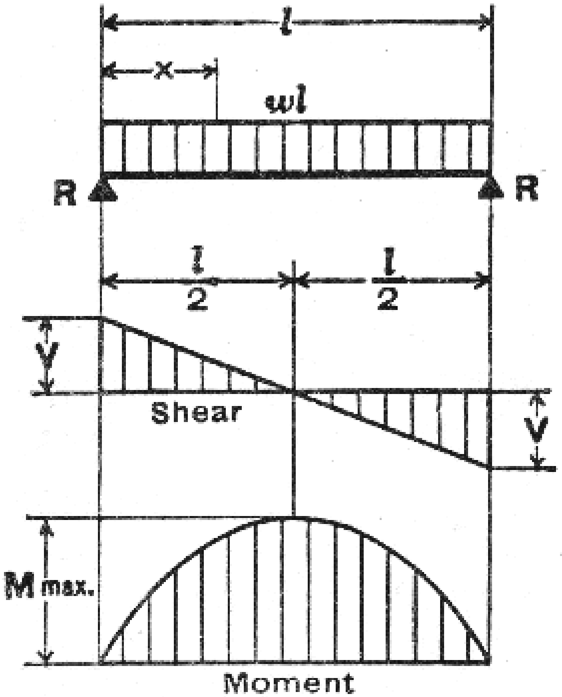


(a) Simply supported beam
(b) Overhanging beam
(c) Cantilever beam

Tabulated Beam Formulas

- *how to read charts*

1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load . . . . $= wl$

$R = V$ . . . . $= \frac{wl}{2}$

V_x . . . . $= w\left(\frac{l}{2} - x\right)$

M max. (at center) . . . . $= \frac{wl^2}{8}$

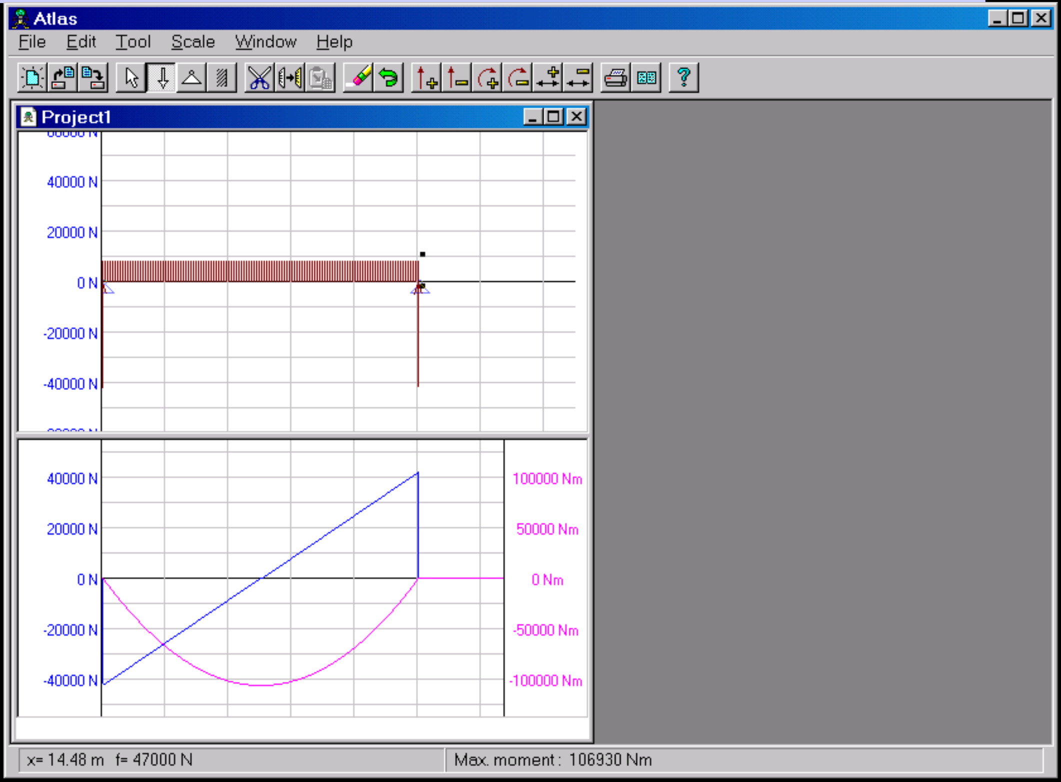
M_x . . . . $= \frac{wx}{2}(l - x)$

Δmax. (at center) . . . . $= \frac{5wl^4}{384EI}$

Δx . . . . $= \frac{wx}{24EI}(l^3 - 2lx^2 + x^3)$

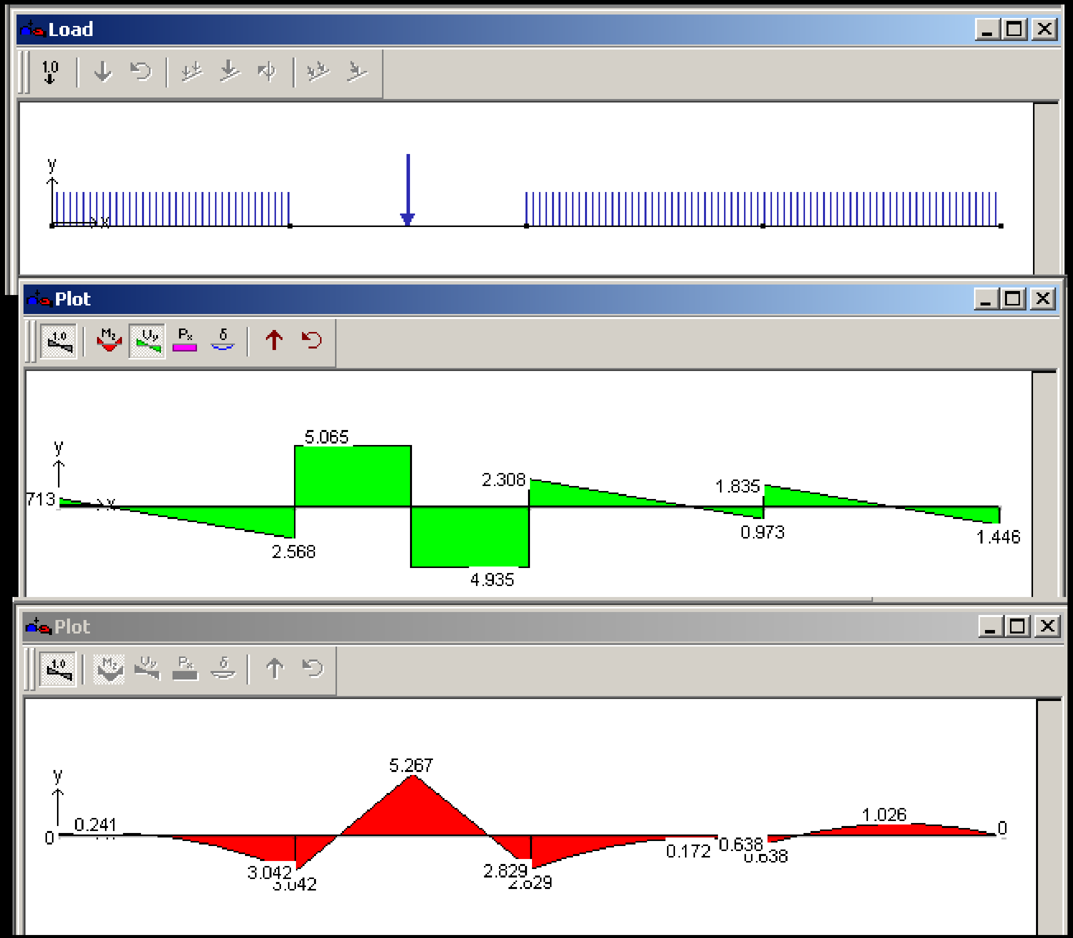
Tools

- *software & spreadsheets help*
- *http://www.rekenwonder.com/atlas.htm*

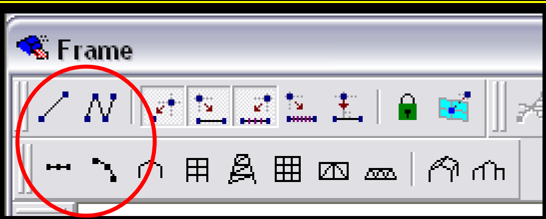


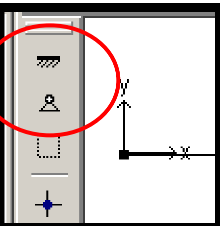
Tools – Multiframe

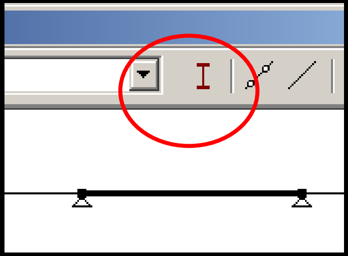
- *in computer lab*



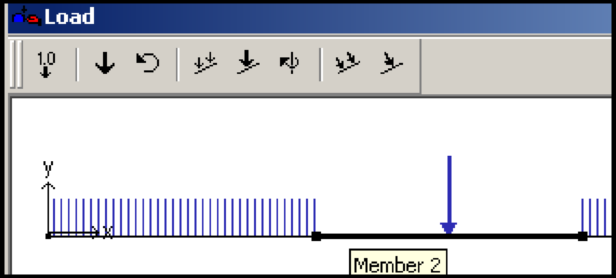
Tools – Multiframe





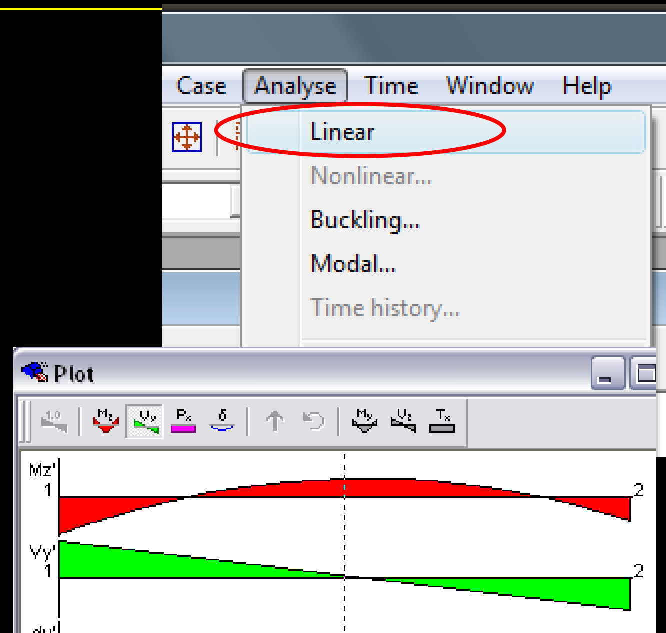


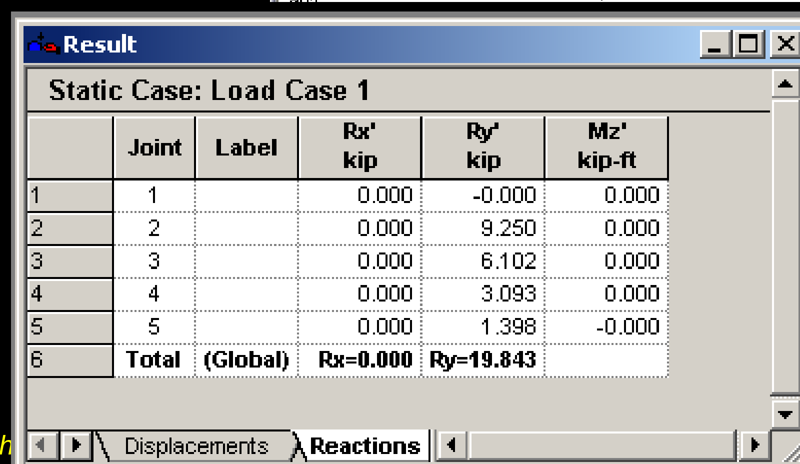
- *frame window*
  - *define beam members*
  - *select points, assign supports*
  - *select members, assign section*
- *load window*
  - *select point or member, add point or distributed loads*



Tools – Multiframe

- to run analysis choose
  - Analyze menu
    - Linear
- plot
  - choose options
  - double click (all)
- results
  - choose options




Result

Static Case: Load Case 1

	Joint	Label	Rx' kip	Ry' kip	Mz' kip-ft
1	1		0.000	-0.000	0.000
2	2		0.000	9.250	0.000
3	3		0.000	6.102	0.000
4	4		0.000	3.093	0.000
5	5		0.000	1.398	-0.000
6	Total	(Global)	Rx=0.000	Ry=19.843	

Displacements | Reactions