

**ARCHITECTURAL STRUCTURES:
FORM, BEHAVIOR, AND DESIGN**

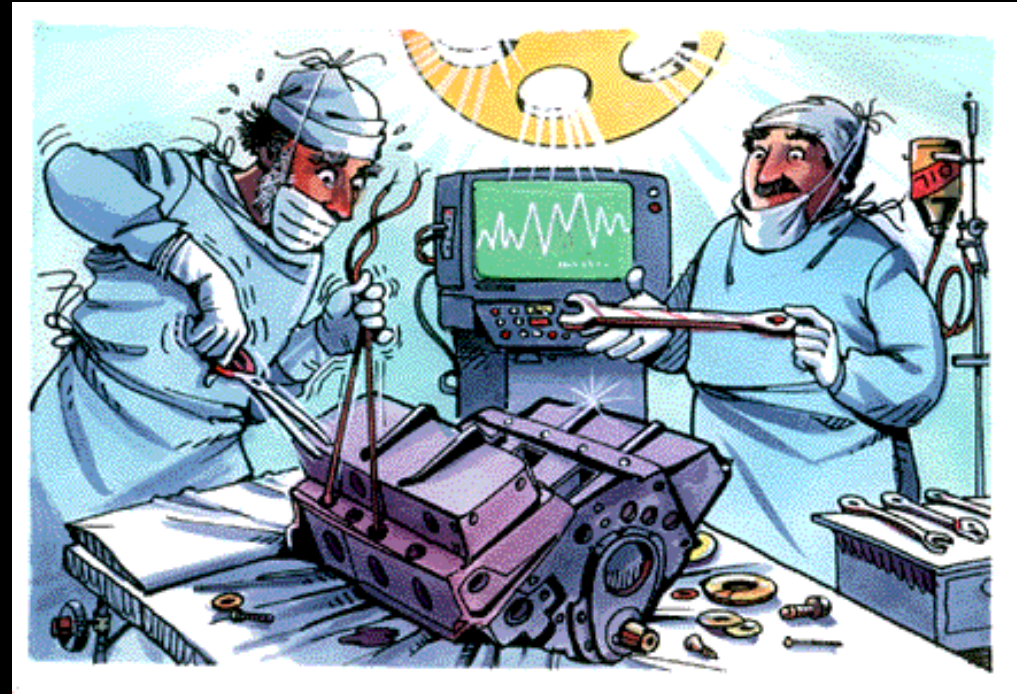
ARCH 331

DR. ANNE NICHOLS

SUMMER 2013

**lecture
five**

**mechanics
of materials**



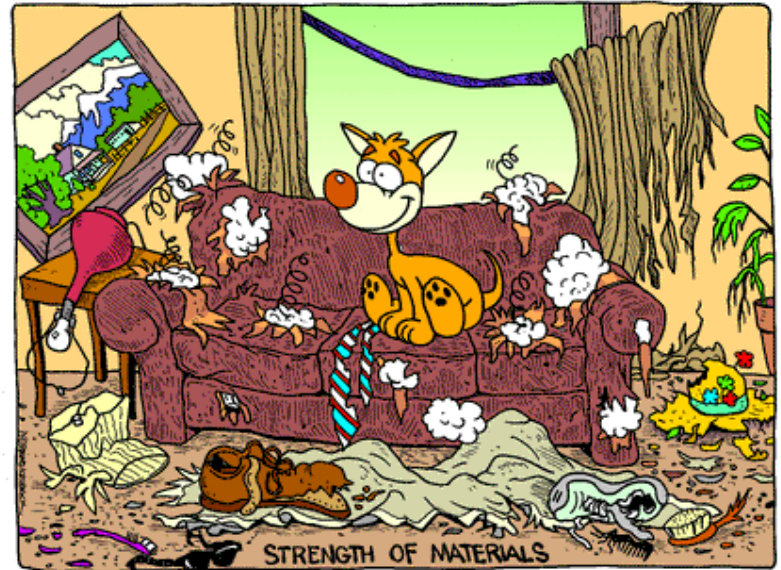
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Mechanics of Materials

- **MECHANICS**



- **MATERIALS**



Mechanics of Materials

- *external loads and their effect on deformable bodies*
- *use it to answer question if structure meets requirements of*
 - *stability and equilibrium*
 - *strength and stiffness*
- *other principle building requirements*
 - *economy, functionality and aesthetics*

Knowledge Required

- *material properties*
- *member cross sections*
- *ability of a material to resist breaking*
- *structural elements that resist excessive*
 - *deflection*
 - *deformation*

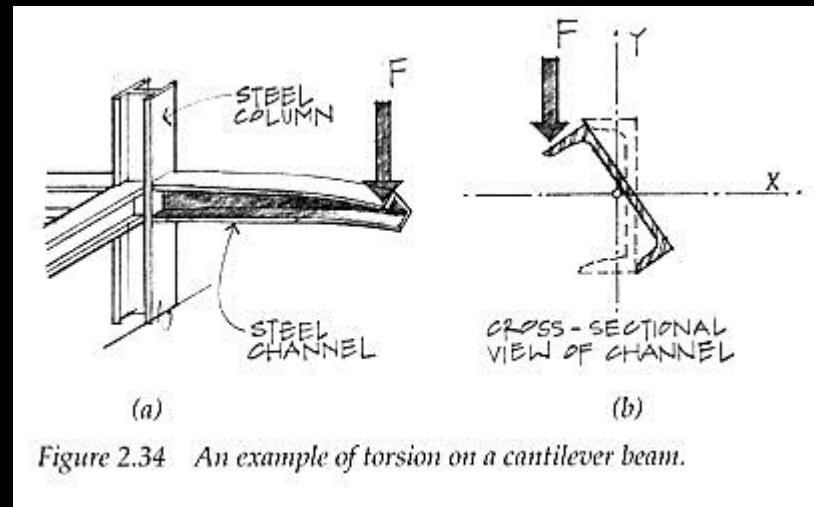


Figure 2.34 An example of torsion on a cantilever beam.

Problem Solving

1. STATICS:

*equilibrium of external forces,
internal forces, stresses*



2. GEOMETRY:

*cross section properties, deformations and
conditions of geometric fit, strains*

3. MATERIAL PROPERTIES:

*stress-strain relationship for each material
obtained from testing*

Stress

- *stress is a term for the intensity of a force, like a pressure*
- *internal or applied*
- *force per unit area*

$$\text{stress} = f = \frac{P}{A}$$



Design

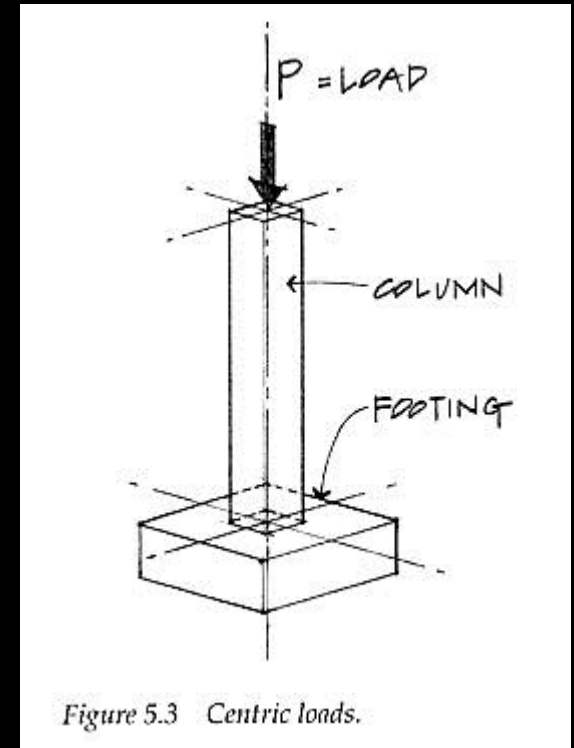
- *materials have a critical stress value where they could break or yield*
 - *ultimate stress*
 - *yield stress*
 - *compressive stress*
 - *fatigue strength*
 - *(creep & temperature)*
- acceptance
vs. failure*

Design (cont)

- we'd like

$$f_{actual} \ll F_{allowable}$$

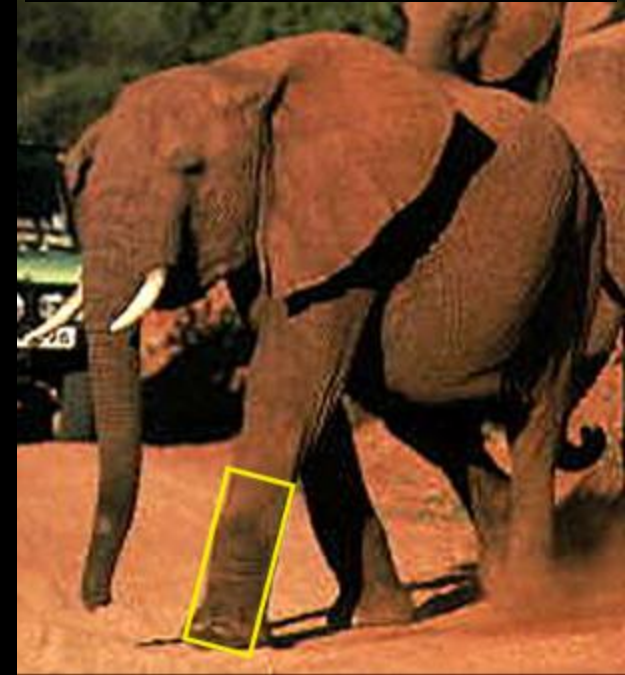
- stress distribution may vary: average
- uniform distribution exists IF the member is loaded axially (concentric)



Scale Effect

- *model scale*
 - *material weights by volume, small section areas*
- *structural scale*
 - *much more material weight, bigger section areas*
- *scale for strength is not proportional:*

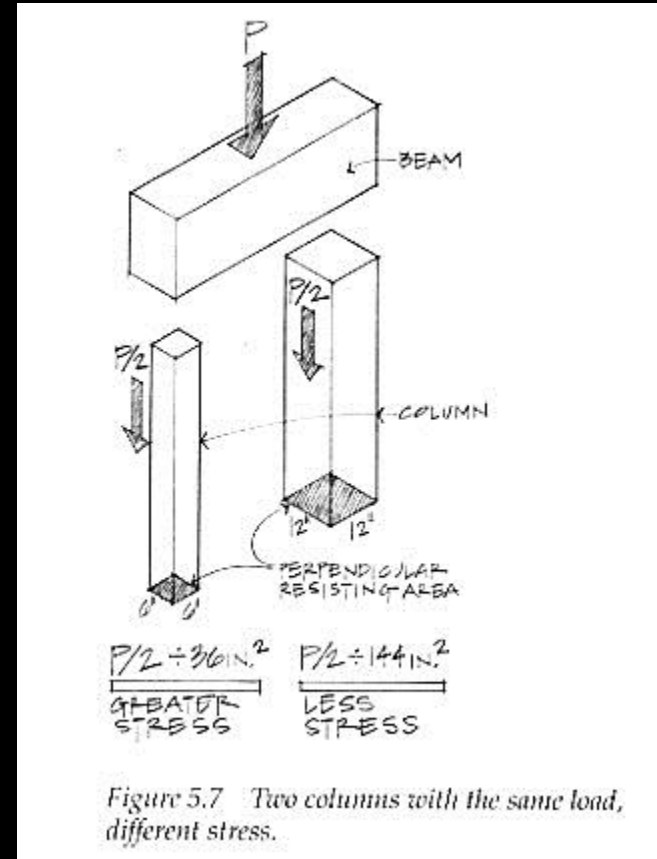
$$\frac{\gamma L^3}{L^2} = \gamma L$$



Normal Stress (direct)

- normal stress is normal to the cross section
 - stressed area is perpendicular to the load

$$f_{t \text{ or } c} \left(\sigma \right) = \frac{P}{A}$$



Shear Stress

- *stress parallel to a surface*

$$f_v = \frac{P}{A} = \frac{P}{td}$$

(τ_{ave})

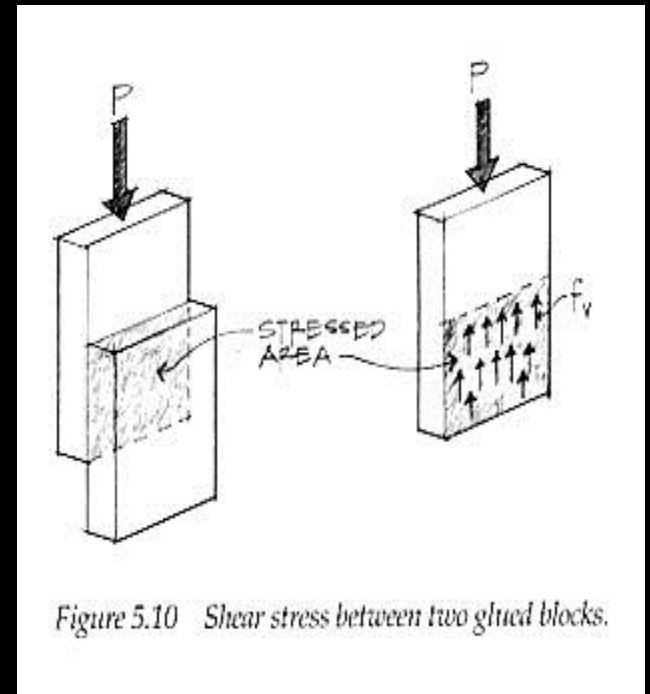


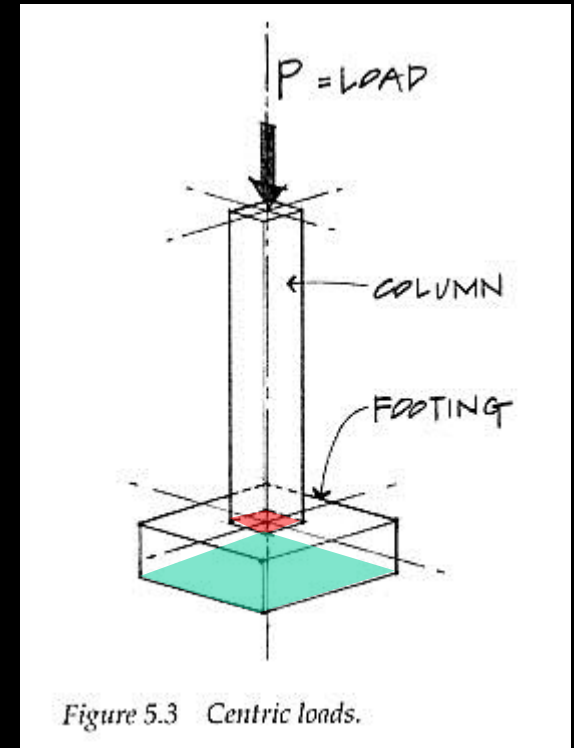
Figure 5.10 Shear stress between two glued blocks.

Bearing Stress

- *stress on a surface by contact in compression*

$$f_p = \frac{P}{A} = \frac{P}{td}$$

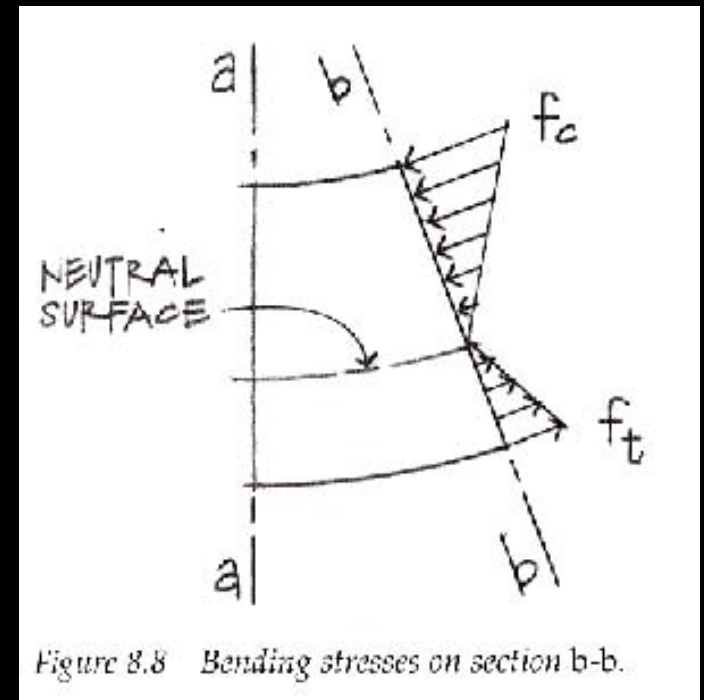
(σ)



Bending Stress

- *normal stress caused by bending*

$$f_b \left(\sigma \right) = \frac{Mc}{I} = \frac{M}{S}$$

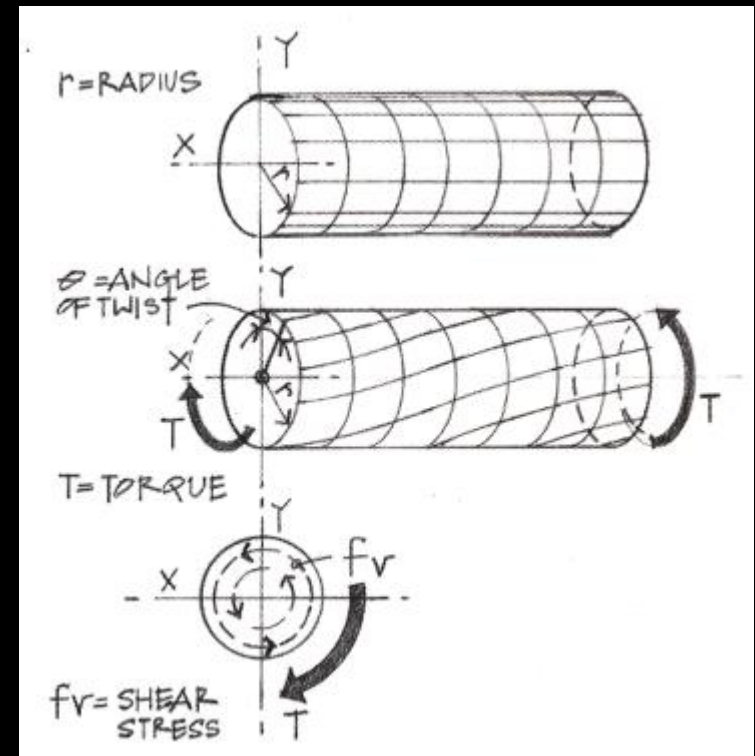


Torsional Stress

- *shear stress caused by twisting*

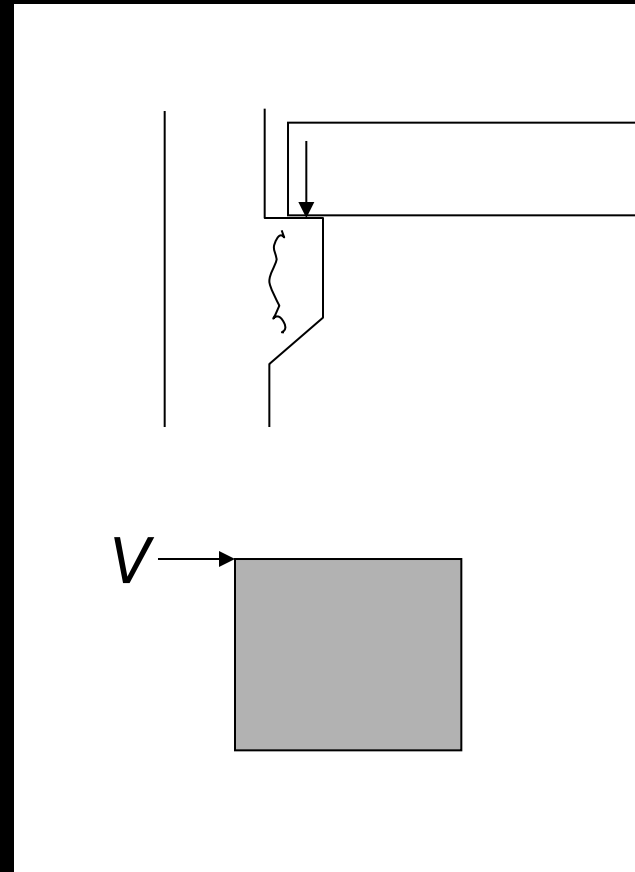
$$f_v = \frac{T\rho}{J}$$

(τ)



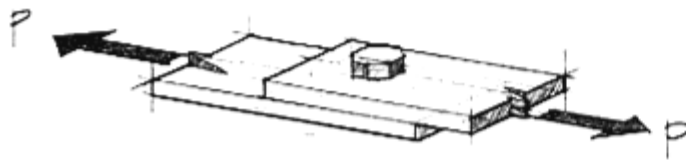
Structures and Shear

- *what structural elements see shear?*
 - *beams*
 - *bolts*
 - *splices*
 - *slabs*
 - *footings*
 - *walls*
 - *wind*
 - *seismic loads*

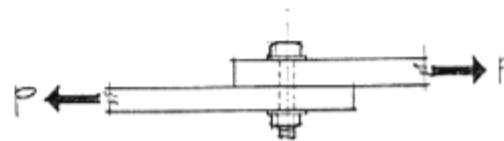


Bolts

- *connected members in tension cause shear stress*

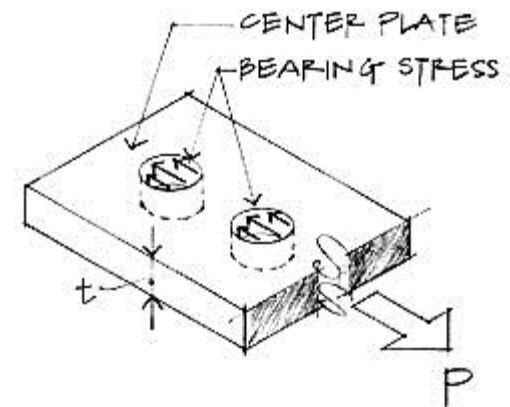


(a) Two steel plates bolted using one bolt.



(b) Elevation showing the bolt in

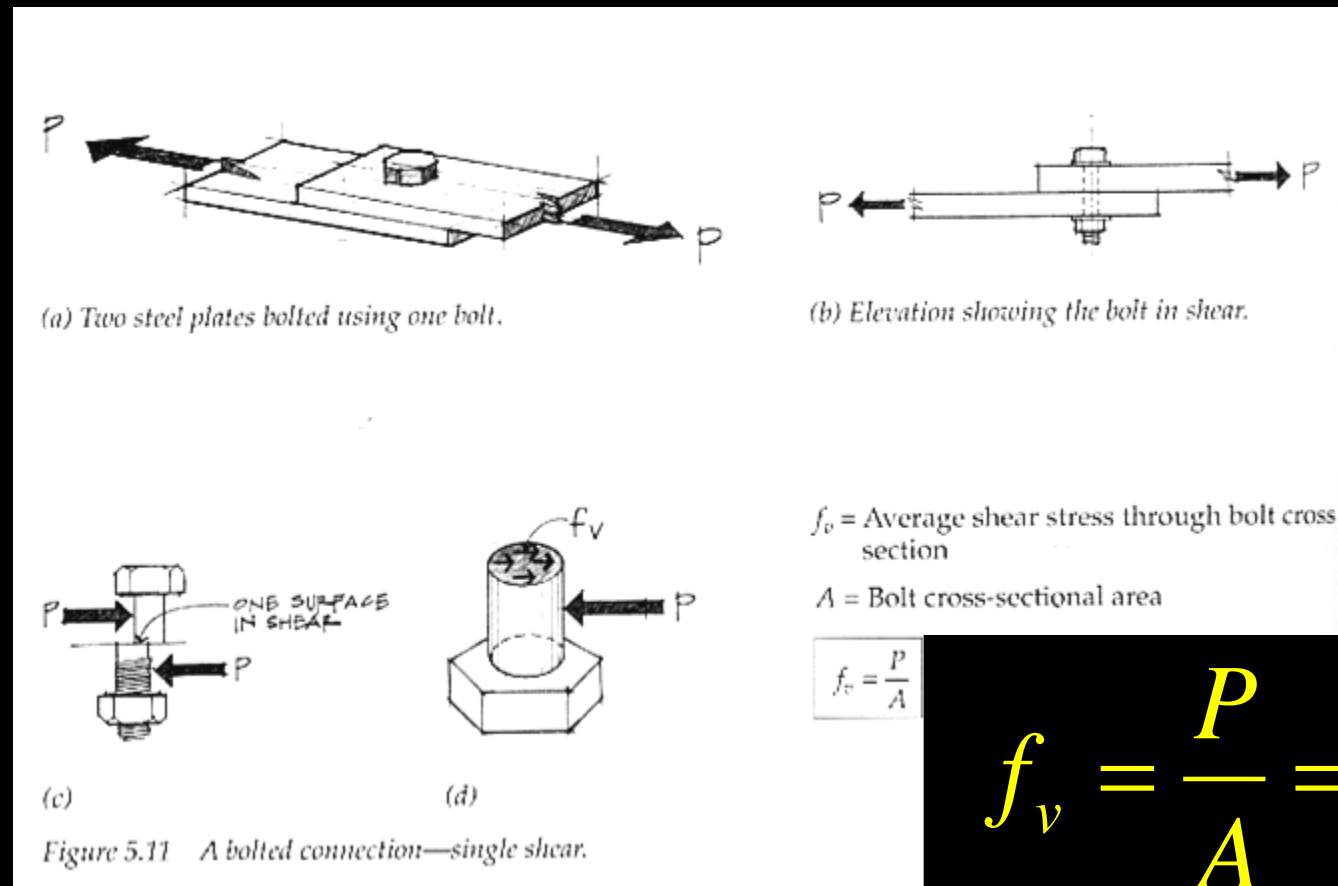
- *connected members in compression cause bearing stress*



Bearing stress on plate.

Single Shear

- seen when 2 members are connected



$$f_v = \frac{P}{A} = \frac{P}{\pi d^2 / 4}$$

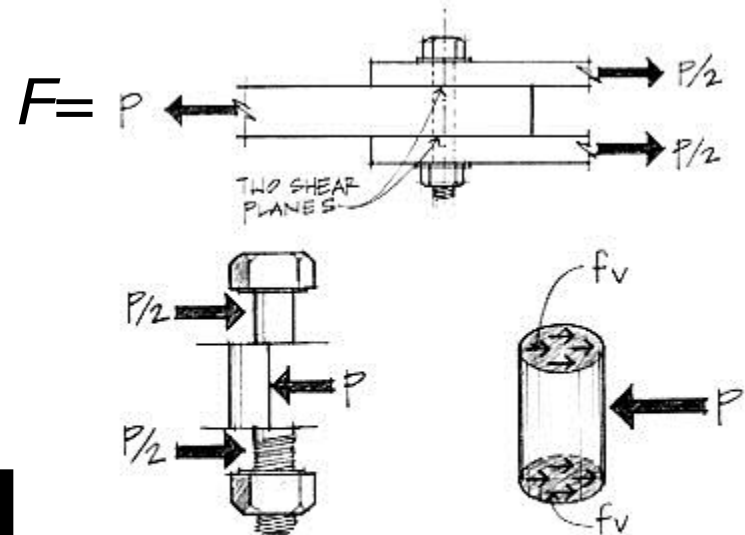
Double Shear

- seen when 3 members are connected
- two areas

$$f_v = \frac{P}{2A}$$

(two shear planes)

$$f_v = \frac{P}{2A} = \frac{P/2}{A} = \frac{P/2}{\pi d^2/4}$$

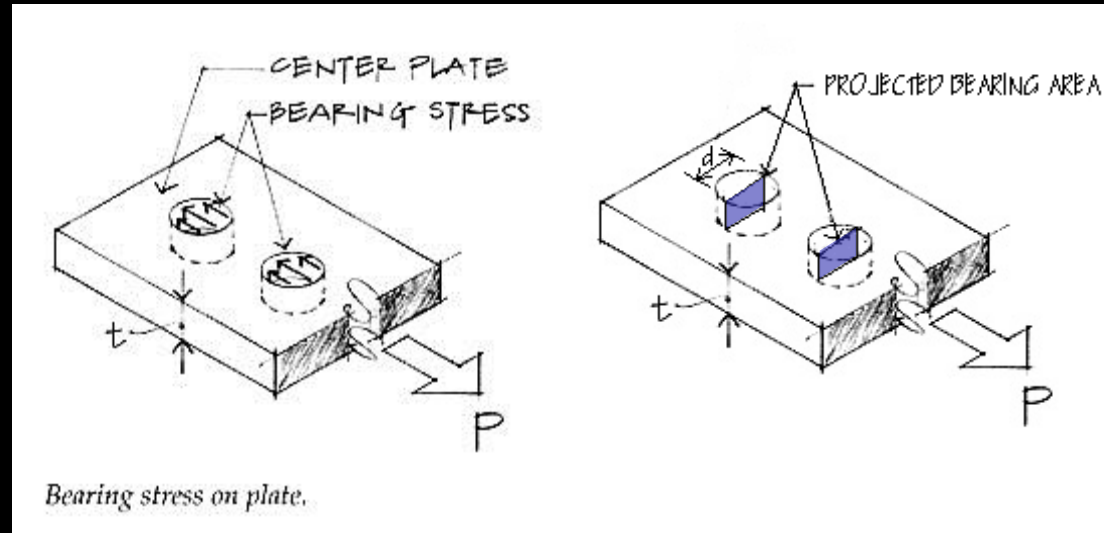


Free-body diagram of middle section of the bolt in shear.

Figure 5.12 A bolted connection in double shear.

Bolt Bearing Stress

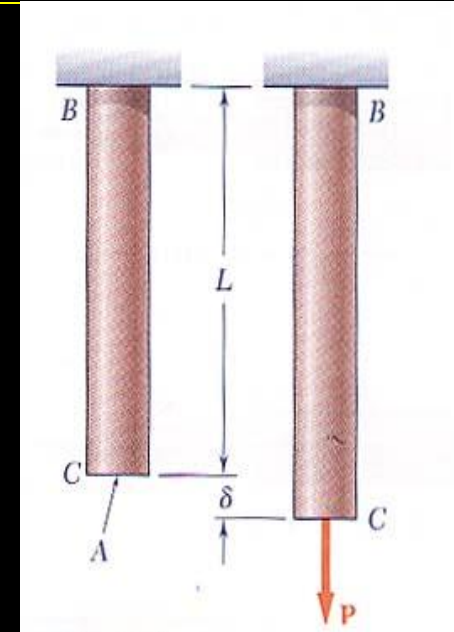
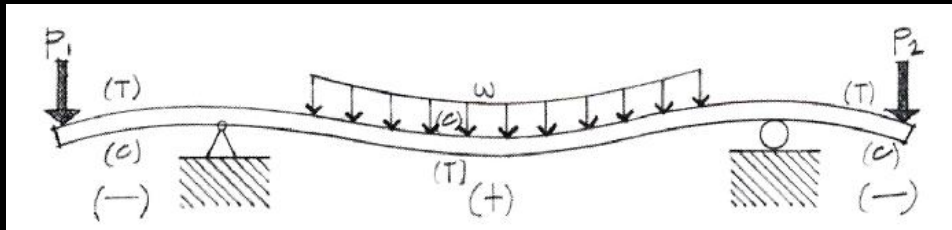
- *compression & contact*
- *projected area*



$$f_p = \frac{P}{A_{\text{projected}}} = \frac{P}{td}$$

Strain

- *materials deform*
- *axially loaded materials change length*
- *bending materials deflect*



- **STRAIN:**

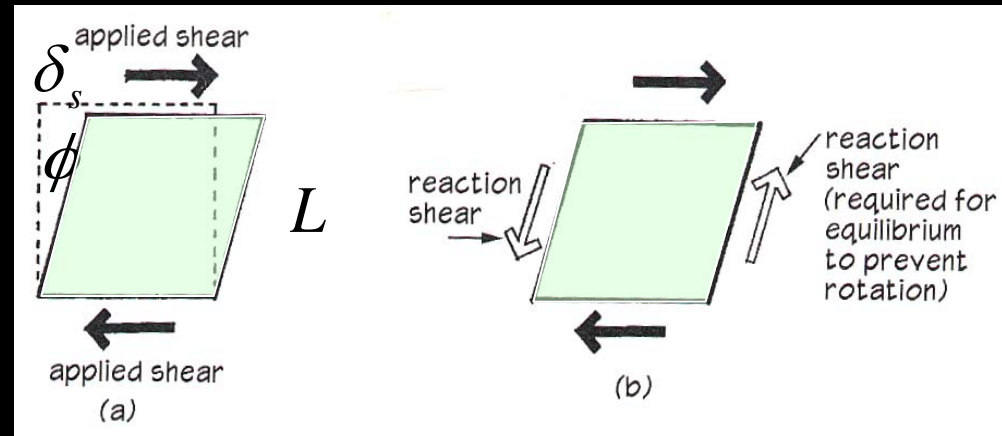
– change in length

over length + UNITLESS

$$\text{strain} = \varepsilon = \frac{\Delta L}{L}$$

Shearing Strain

- *deformations with shear*
- *parallelogram*
- *change in angles*
- *stress: τ*
- *strain: γ*
 - *unitless (radians)*

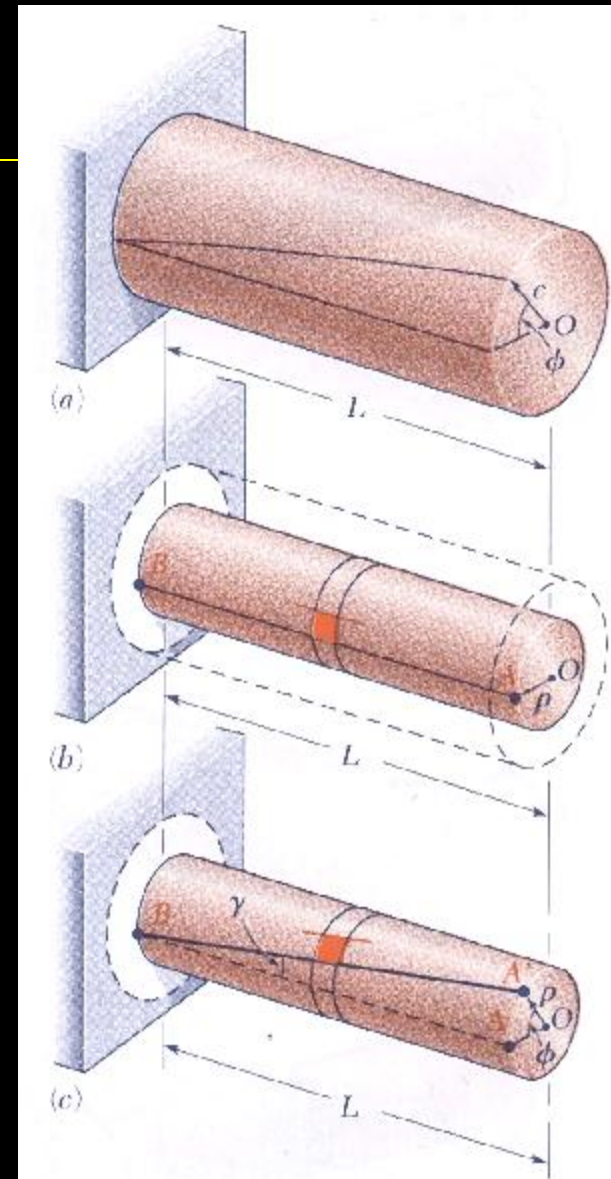


$$\gamma = \frac{\delta_s}{L} = \tan \phi \cong \phi$$

Shearing Strain

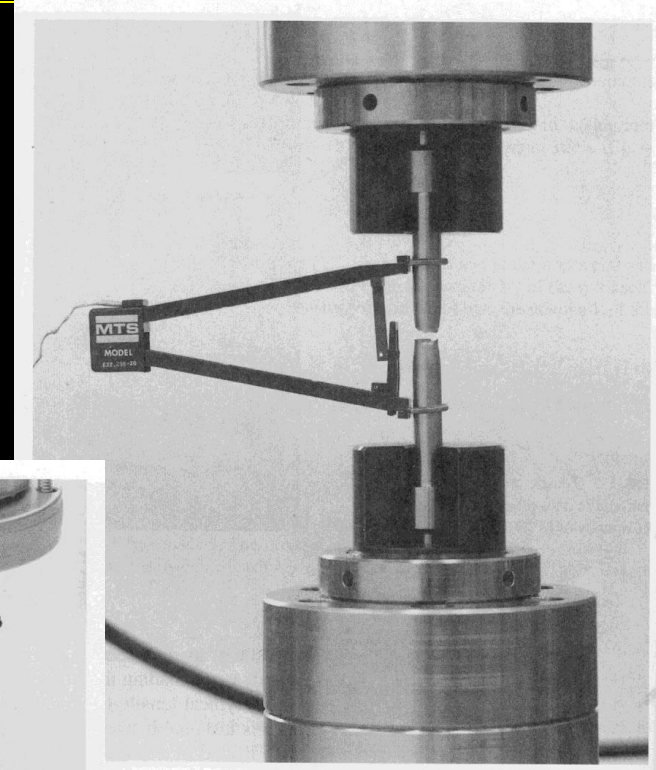
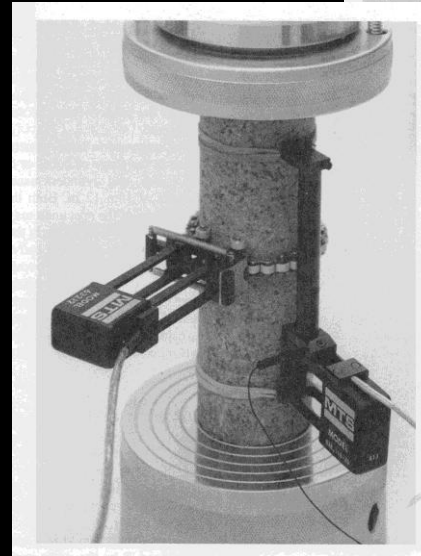
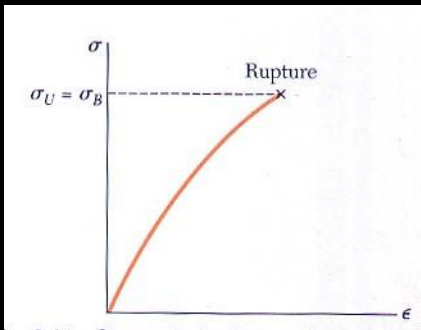
- *deformations with torsion*
- *twist*
- *change in angle of line*
- *stress:* τ
- *strain:* γ
– *unitless (radians)*

$$\gamma = \frac{\rho\phi}{L}$$



Load and Deformation

- for stress, need P & A
- for strain, need δ & L
 - how?
 - TEST with load and measure
 - plot P/A vs. ϵ



Material Behavior

- every material has its own response
 - 10,000 psi
 - $L = 10$ in
 - Douglas Fir vs. steel?

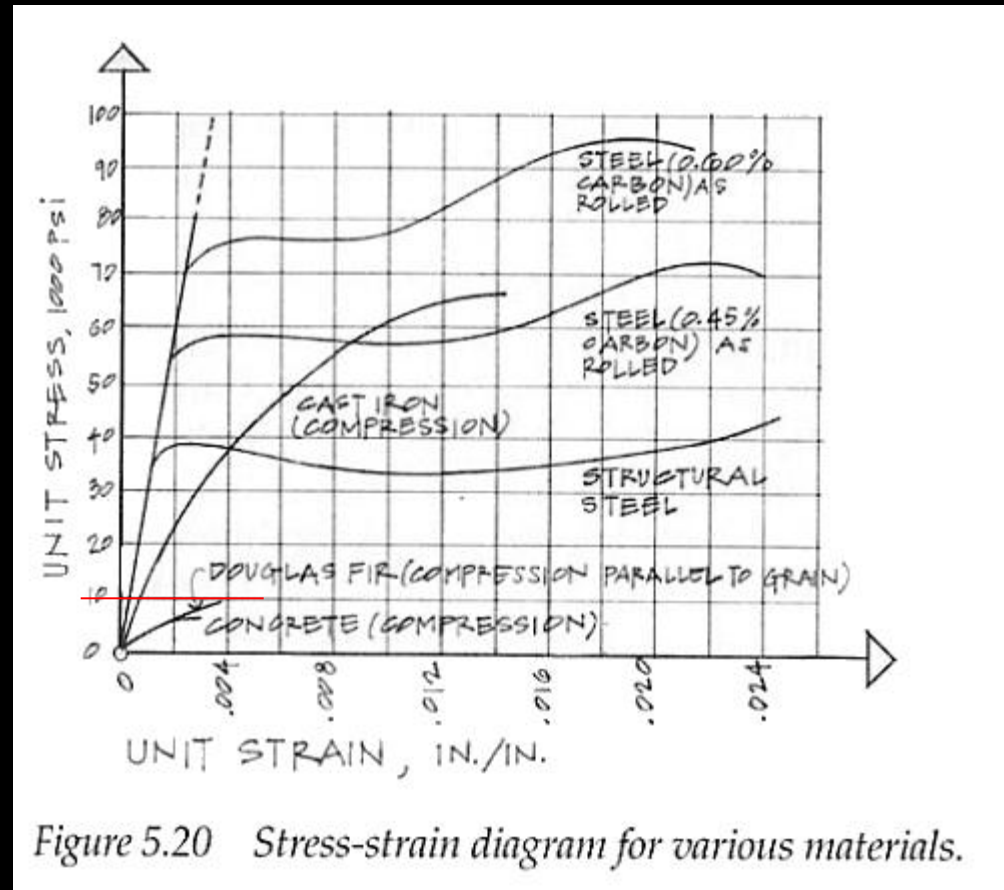


Figure 5.20 Stress-strain diagram for various materials.

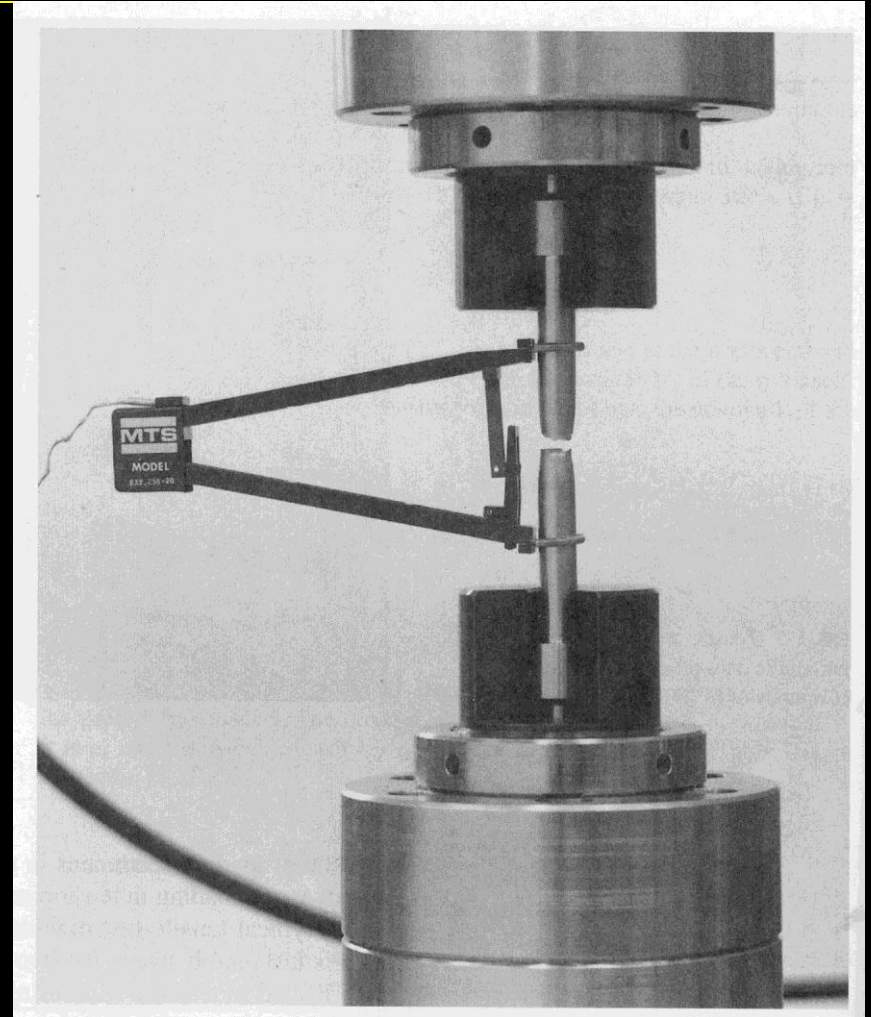
Behavior Types

- ductile - “necking”
- true stress

$$f = \frac{P}{A}$$

- engineering stress
– (simplified)

$$f = \frac{P}{A_o}$$



Behavior Types

- *brittle*

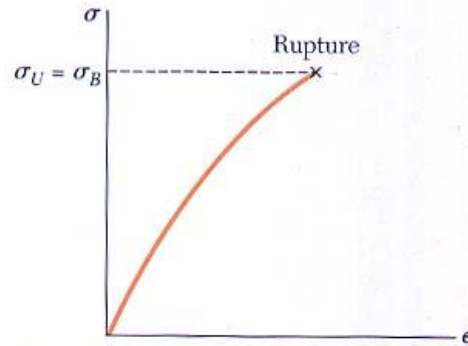


Fig. 2.11 Stress-strain diagram for a typical brittle material.

- *semi-brittle*

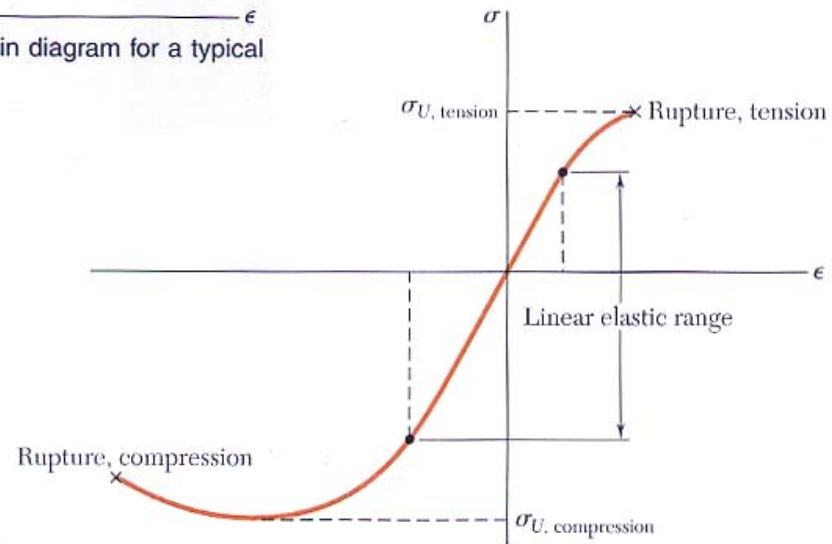


Fig. 2.14 Stress-strain diagram for concrete.

Stress to Strain

- important to us in f - ϵ diagrams:
 - straight section
 - **LINEAR-ELASTIC**
 - recovers shape (no permanent deformation)

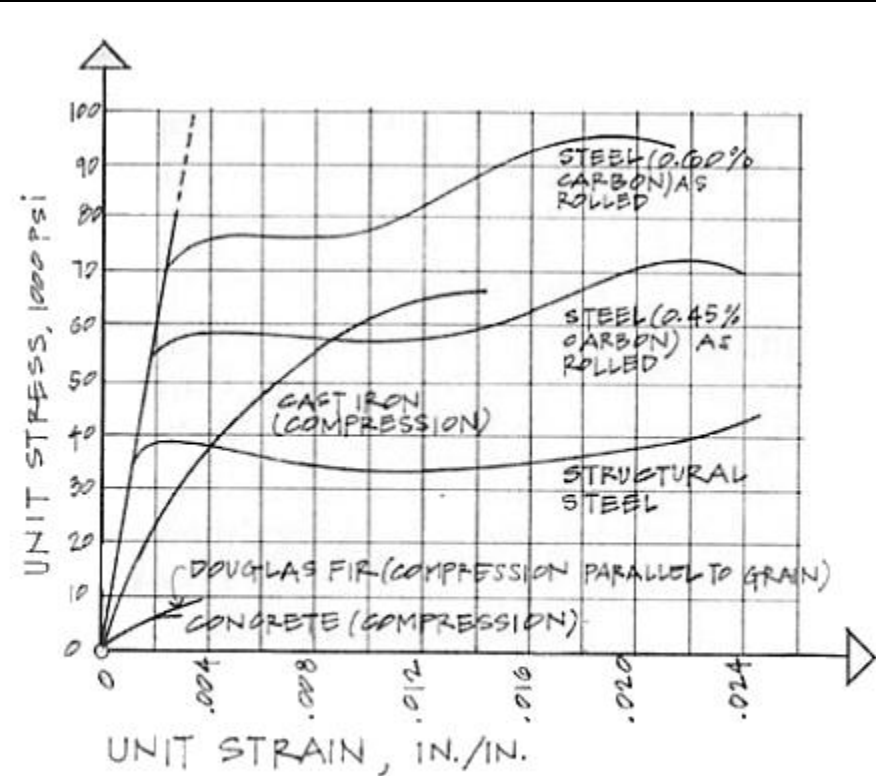


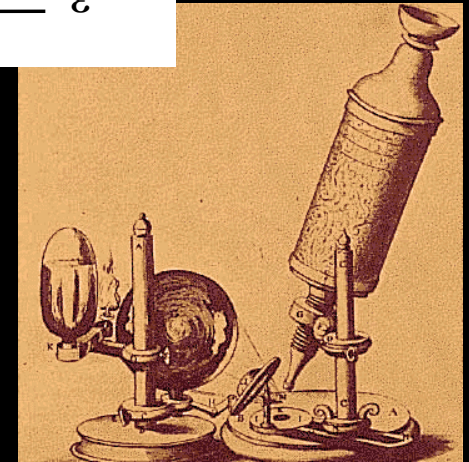
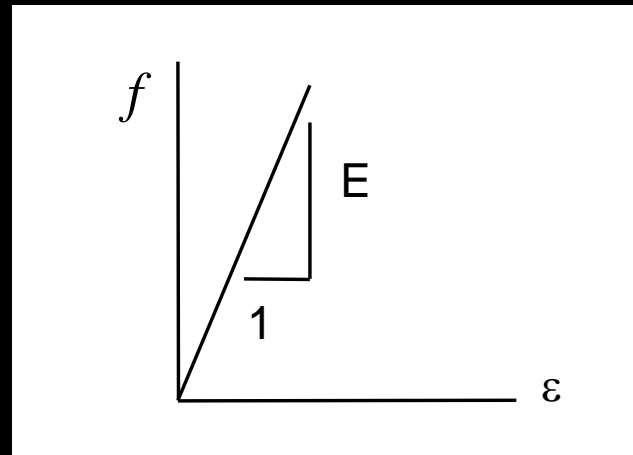
Figure 5.20 Stress-strain diagram for various materials.

Hooke's Law

- *straight line has constant slope*
- *Hooke's Law*

$$f = E \cdot \varepsilon$$

- *E*
 - *Modulus of elasticity*
 - *Young's modulus*
 - *units just like stress*



Stiffness

- *ability to resist strain*

- *steels*

- *same E*
- *different yield points*
- *different ultimate strength*

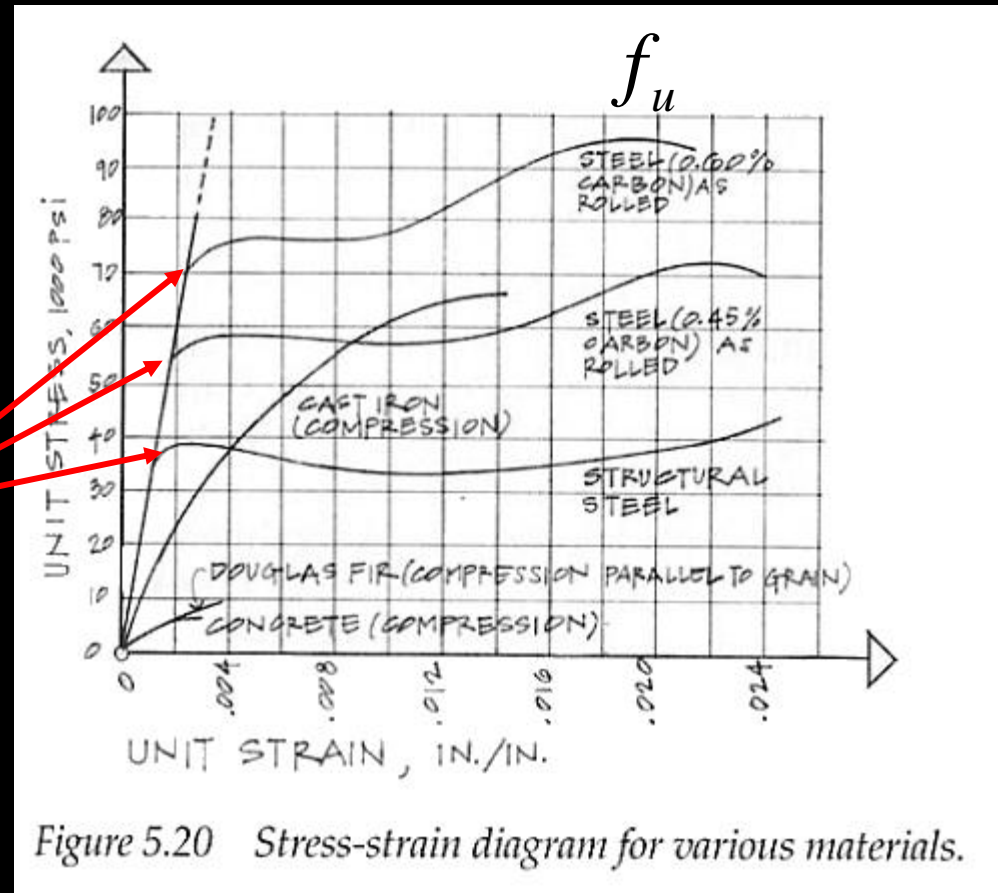
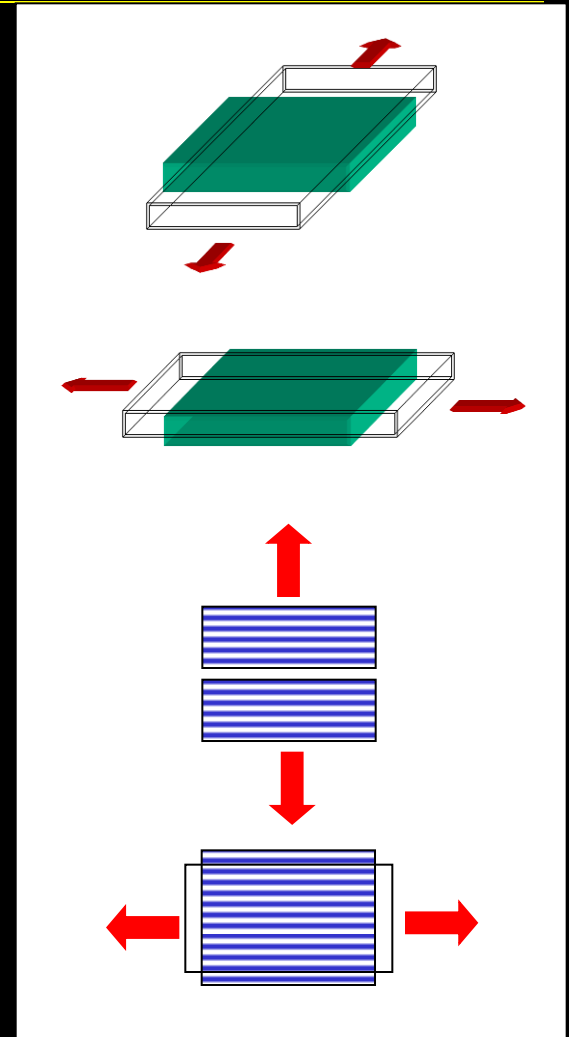


Figure 5.20 Stress-strain diagram for various materials.

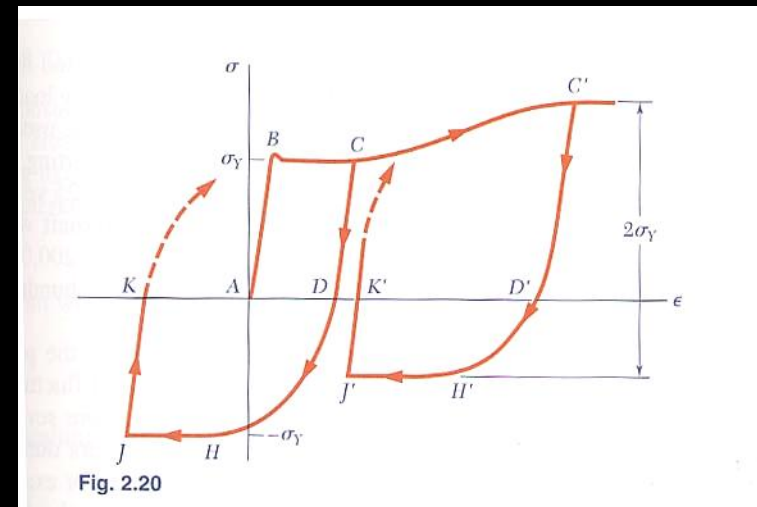
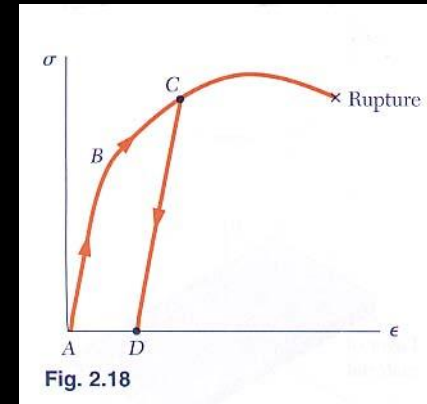
Isotropy & Anisotropy

- **ISOTROPIC**
 - materials with E same at any direction of loading
 - ex. steel
- **ANISOTROPIC**
 - materials with different E at any direction of loading
 - ex. wood is orthotropic



Elastic, Plastic, Fatigue

- *elastic springs back*
- *plastic has permanent deformation*
- *fatigue caused by reversed loading cycles*



Plastic Behavior

- ductile

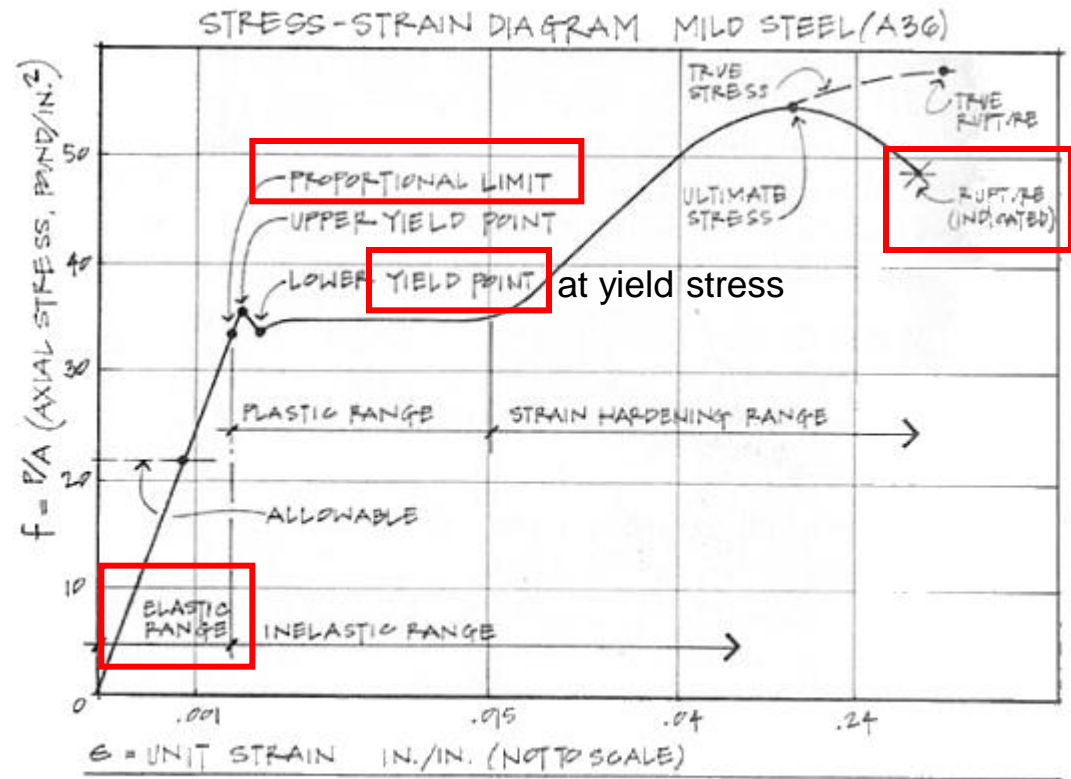


Figure 5.22 Stress-strain diagram for mild steel (A36) with key points highlighted.

Lateral Strain

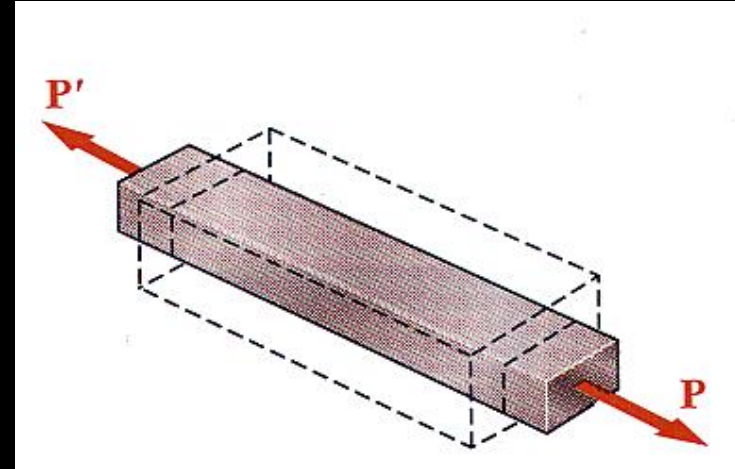
- or “what happens to the cross section with axial stress”

$$\varepsilon_x = \frac{f_x}{E}$$

$$f_y = f_z = 0$$

- strain in lateral direction
 - negative
 - equal for isometric materials

$$\varepsilon_y = \varepsilon_z$$

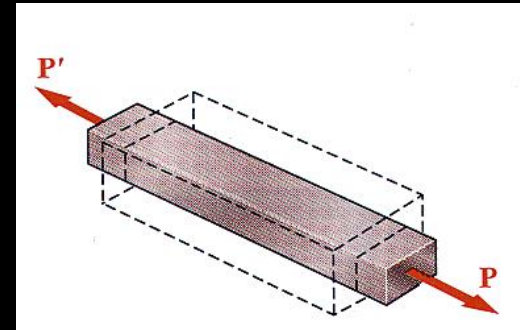


Poisson's Ratio

- constant relationship between longitudinal strain and lateral strain

$$\mu = -\frac{\text{lateral strain}}{\text{axial strain}} = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}$$
$$\varepsilon_y = \varepsilon_z = -\frac{\mu f_x}{E}$$

- sign! $0 < \mu < 0.5$



Calculating Strain

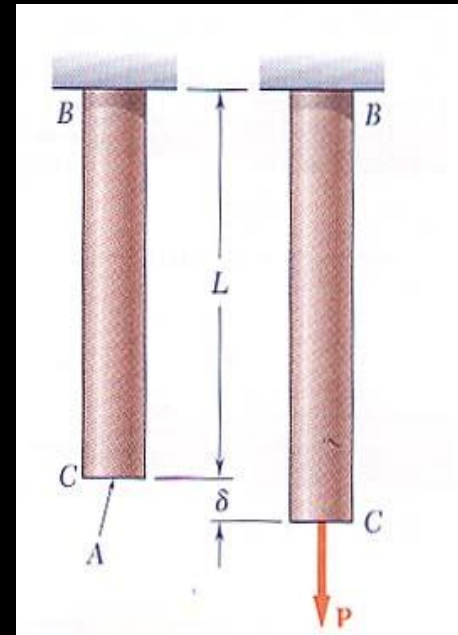
- from Hooke's law

$$f = E \cdot \varepsilon$$

- substitute

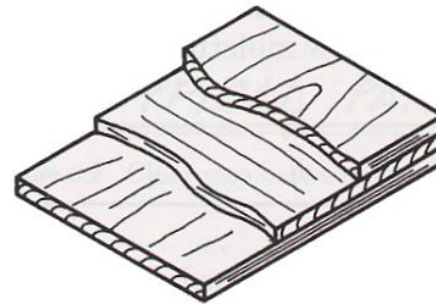
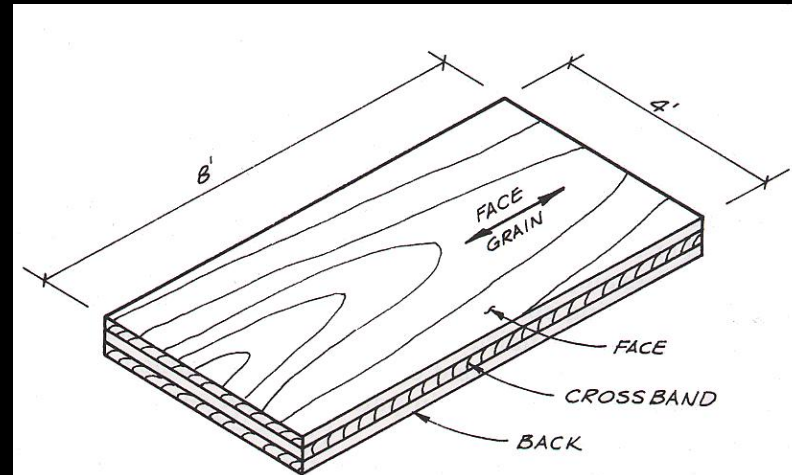
$$\frac{P}{A} = E \cdot \frac{\delta}{L}$$

- get \Rightarrow
$$\delta = \frac{PL}{AE}$$

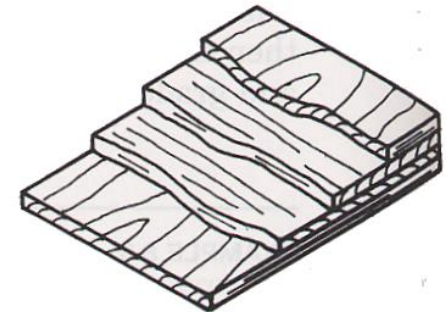


Orthotropic Materials

- *non-isometric*
- *directional values of E and μ*
- *ex:*
 - *plywood*
 - *laminates*
 - *polymer composites*



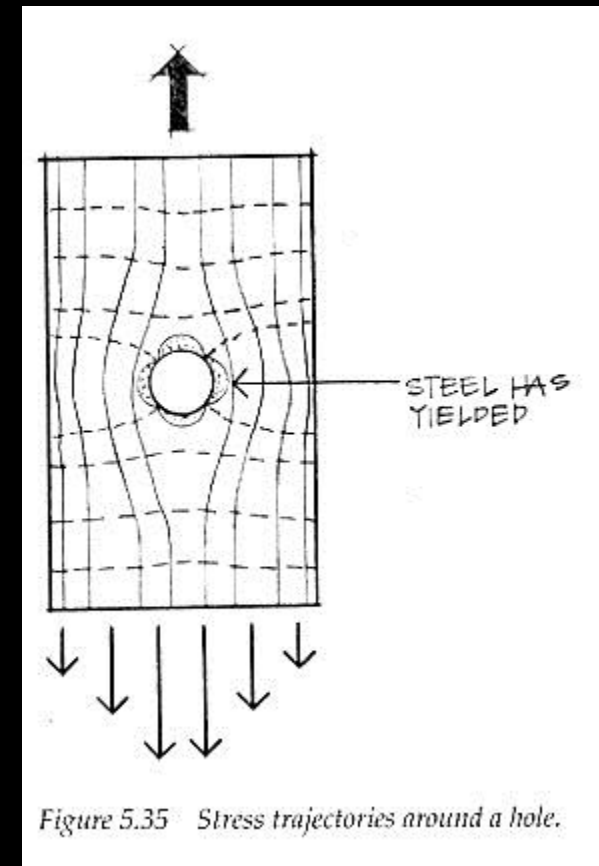
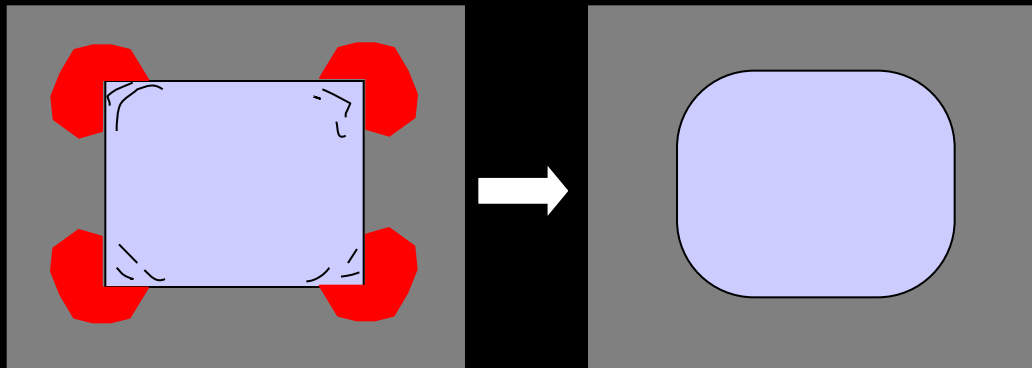
3 LAYER
3 PLY CONSTRUCTION



3 LAYER
4 PLY CONSTRUCTION

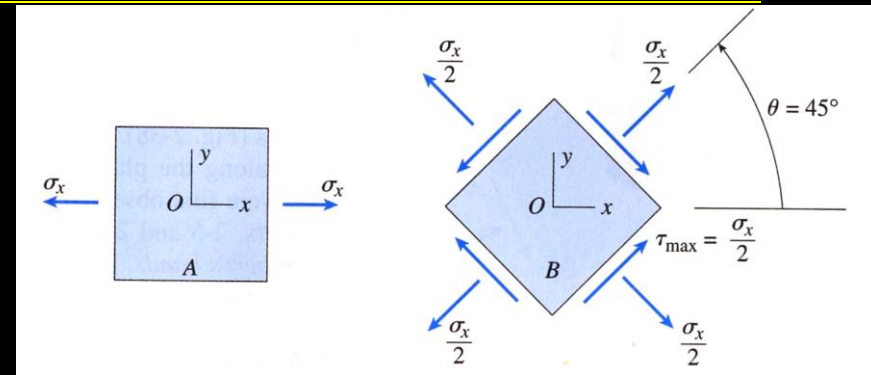
Stress Concentrations

- why we use f_{ave}
- increase in stress at changes in geometry
 - sharp notches
 - holes
 - corners



Maximum Stresses

- if we need to know where max f and f_v happen:



$$\theta = 0^\circ \rightarrow \cos \theta = 1 \quad f_{\max} = \frac{P}{A_o}$$

$$\theta = 45^\circ \rightarrow \cos \theta = \sin \theta = \sqrt{0.5}$$

$$f_{v-\max} = \frac{P}{2A_o} = \frac{f_{\max}}{2}$$

Maximum Stresses



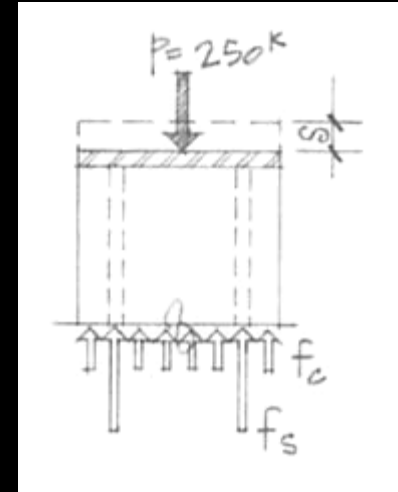
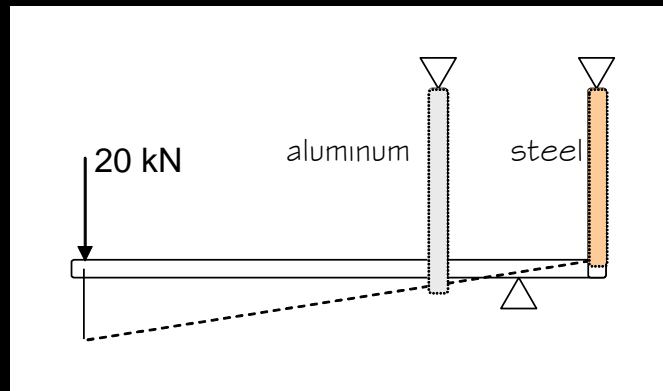
FIG. 2-37 Shear failure along a 45° plane of a wood block loaded in compression



FIG. 2-38 Slip bands (or Lüders' bands) in a polished steel specimen loaded in tension

Deformation Relationships

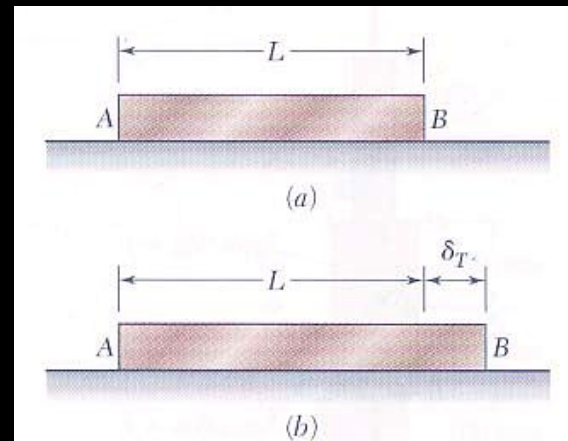
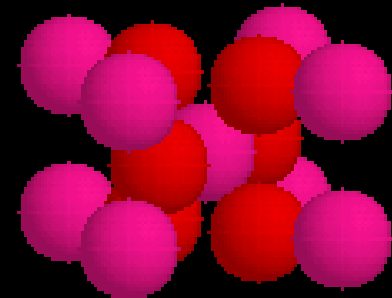
- *physical movement*
 - axially (same or zero)
 - rotations from axial changes



- $\delta = \frac{PL}{AE}$ relates δ to P

Deformations from Temperature

- *atomic chemistry reacts to changes in energy*
- *solid materials*
 - *can contract with decrease in temperature*
 - *can expand with increase in temperature*
- *linear change can be measured per degree*



Thermal Deformation

- α - the rate of strain per degree

- UNITS : $/^{\circ}\text{F}$, $/^{\circ}\text{C}$

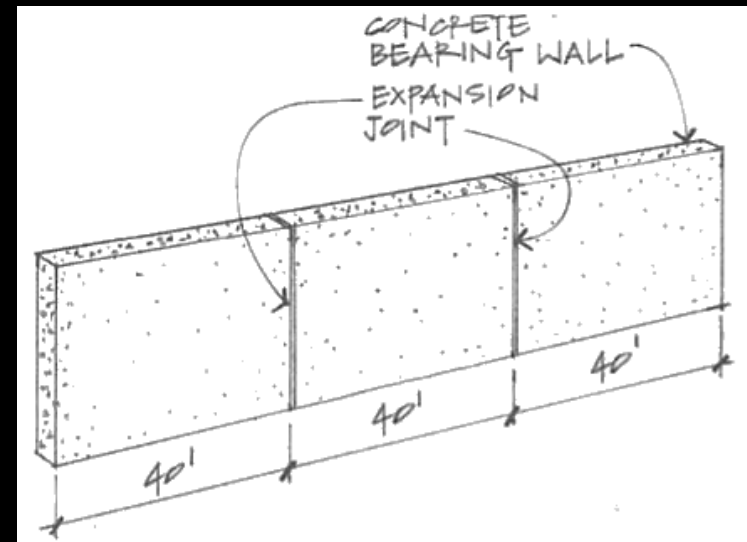
- length change: $\delta_T = \alpha(\Delta T)L$

- thermal strain: $\varepsilon_T = \alpha(\Delta T)$

– no stress when movement allowed

Coefficients of Thermal Expansion

Material	Coefficients (α) [in./in./°F]
Wood	3.0×10^{-6}
Glass	4.4×10^{-6}
Concrete	5.5×10^{-6}
Cast Iron	5.9×10^{-6}
Steel	6.5×10^{-6}
Wrought Iron	6.7×10^{-6}
Copper	9.3×10^{-6}
Bronze	10.1×10^{-6}
Brass	10.4×10^{-6}
Aluminum	12.8×10^{-6}



Stresses and Thermal Strains

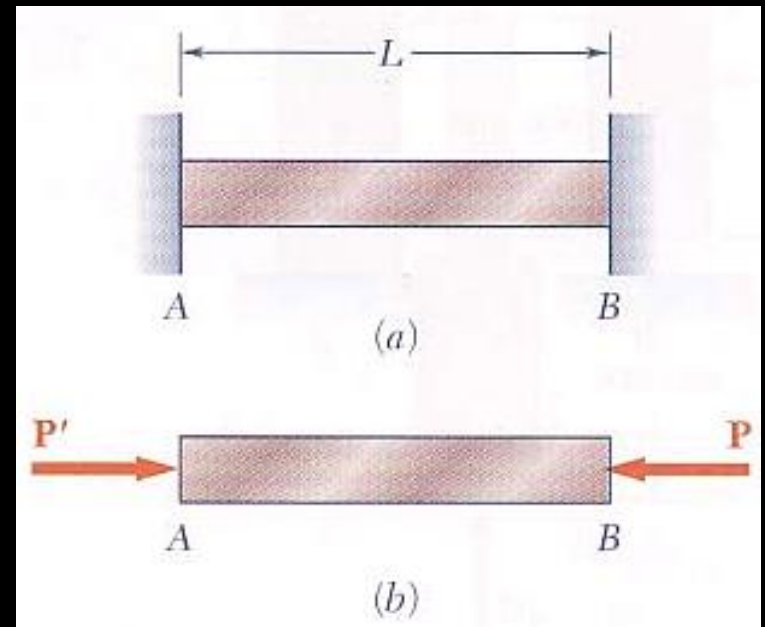
- if thermal movement is restrained stresses are induced

1. bar pushes on supports

2. support pushes back

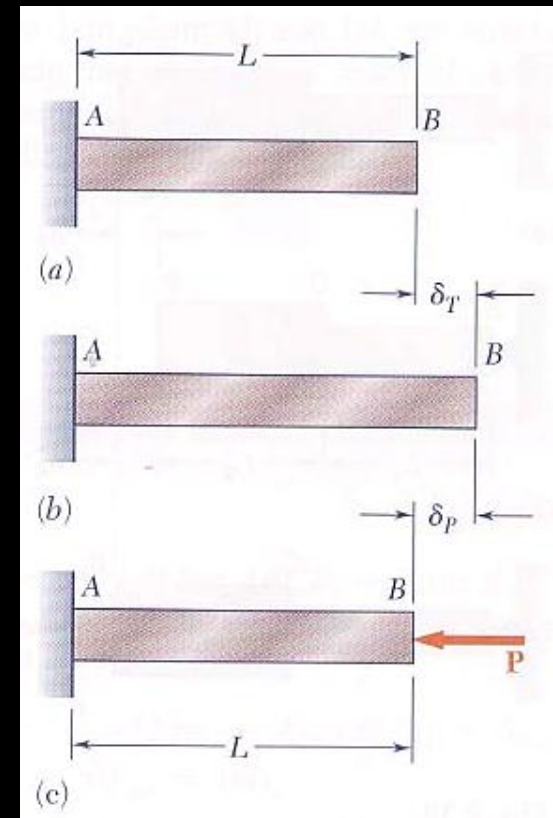
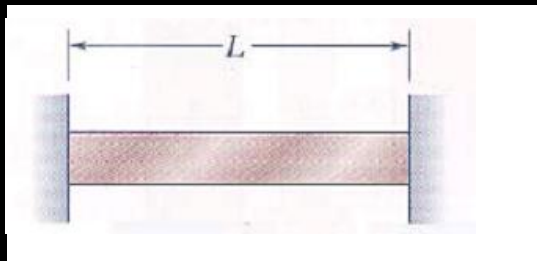
3. reaction causes internal stress

$$f = \frac{P}{A} = \frac{\delta}{L} E$$



Superposition Method

- can remove a support to make it look determinate
- replace the support with a reaction
- enforce the geometry constraint



Superposition Method

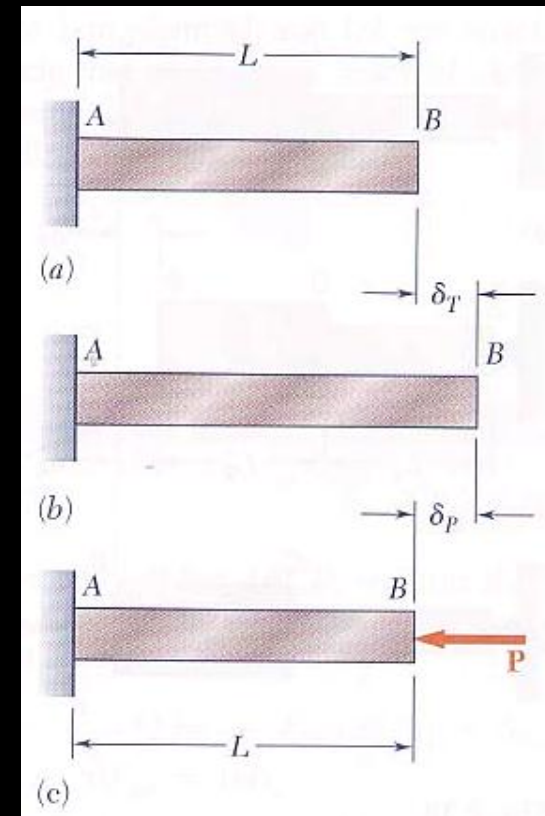
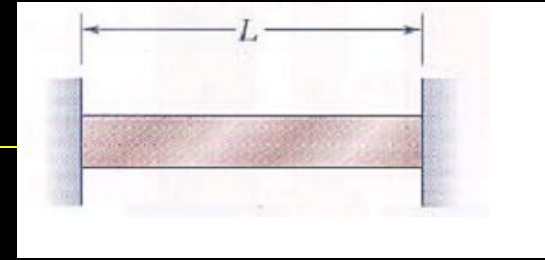
- total length change restrained to zero

$$\text{constraint: } \delta_P + \delta_T = 0$$

$$\delta_p = -\frac{PL}{AE} \quad \delta_T = \alpha(\Delta T)L$$

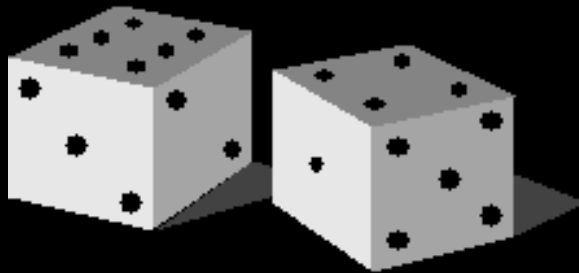
$$\text{sub: } -\frac{PL}{AE} + \alpha(\Delta T)L = 0$$

$$f = -\frac{P}{A} = -\alpha(\Delta T)E$$



Design of Members

- *beyond allowable stress...*
- *materials aren't uniform 100% of the time*
 - *ultimate strength or capacity to failure may be different and some strengths hard to test for*
- **RISK & UNCERTAINTY**



$$f_u = \frac{P_u}{A}$$

Factor of Safety

- *accommodate uncertainty with a safety factor:*

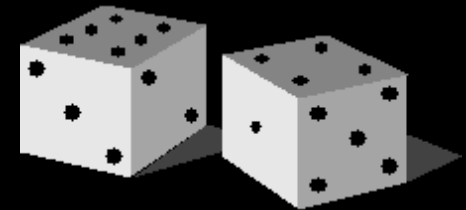
$$\text{allowable load} = \frac{\text{ultimate load}}{F.S}$$

- *with linear relation between load and stress:*

$$F.S = \frac{\text{ultimate load}}{\text{allowable load}} = \frac{\text{ultimate stress}}{\text{allowable stress}}$$

Load and Resistance Factor Design

- *loads on structures are*
 - *not constant*
 - *can be more influential on failure*
 - *happen more or less often*
 - **UNCERTAINTY**



$$R_u = \gamma_D R_D + \gamma_L R_L \leq \phi R_n$$

ϕ - *resistance factor*

γ - *load factor for (D)ead & (L)ive load*