ARCHITECTURAL STRUCTURES:
FORM, BEHAVIOR, AND DESIGN

ARCH 331

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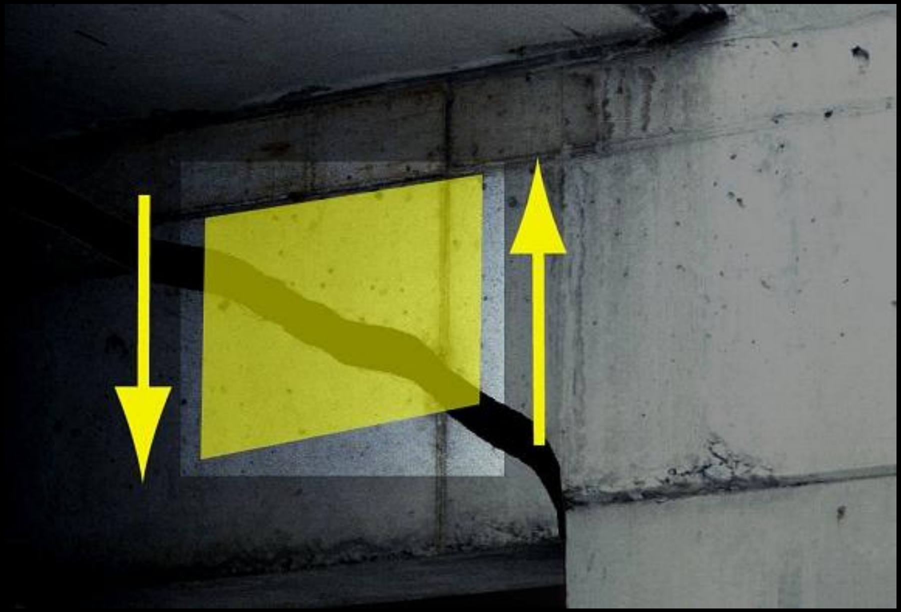
SUMMER 2013

lecture
# twenty one

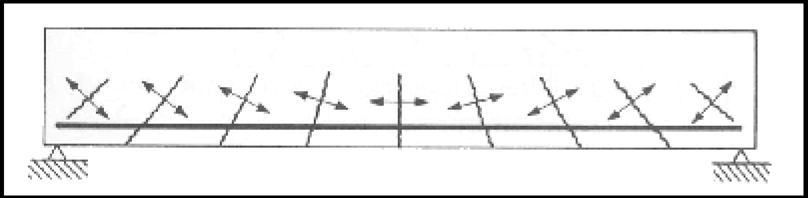




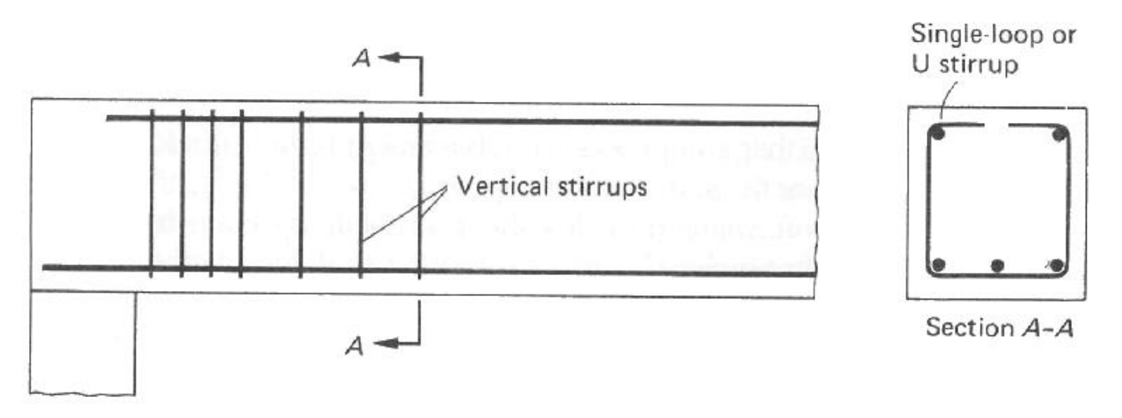
# concrete construction: shear & deflection

# *Shear in Concrete Beams*

- *flexure combines with shear to form diagonal cracks*



- *horizontal reinforcement doesn't help*
- *stirrups = vertical reinforcement*



# ACI Shear Values

- $V_u$ is at distance d from face of support
- shear capacity: $$V_c = \upsilon_c \times b_w d$$
  - where $b_w$ means thickness of web at n.a.

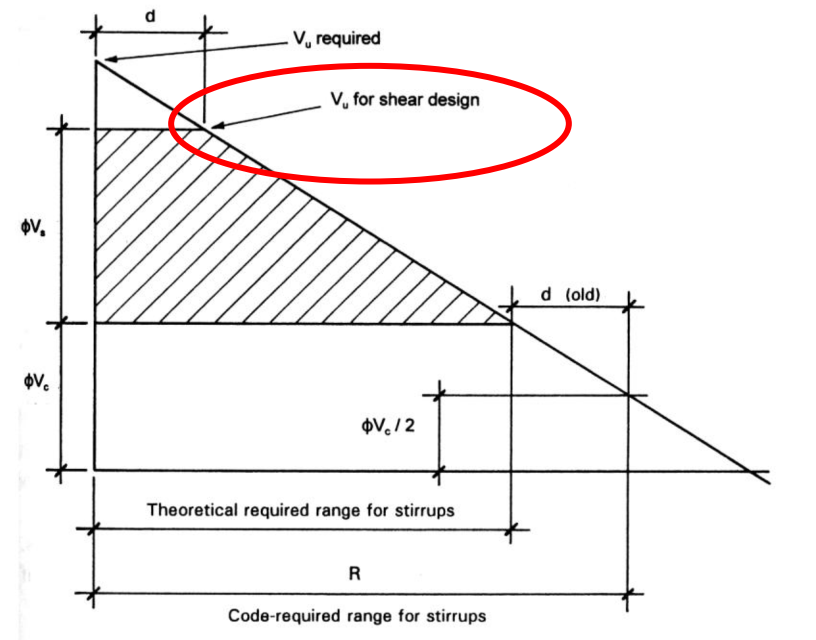


**Figure 13.16** Layout for shear stress analysis: ACI Code requirements.

# ACI Shear Values

- *shear stress (beams)*
  - $\upsilon_c = 2\sqrt{f_c'}$

$$\phi V_c = \phi 2\sqrt{f_c'}\, b_w d$$

$\phi$ *= 0.75 for shear*

*f'c is in psi*

- *shear strength:*

$$V_u \le \phi V_c + \phi V_s$$

  - *$V_s$ is strength from stirrup reinforcement*

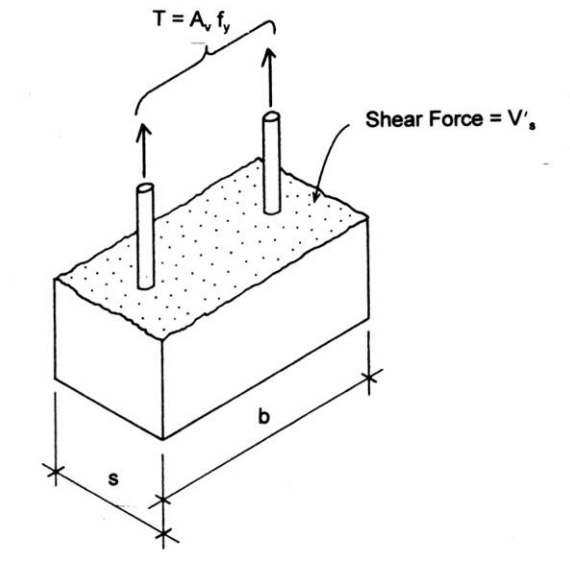


**Figure 13.17** Consideration for spacing of a single stirrup.

# Stirrup Reinforcement

- *shear capacity:*

$$V_s = \frac{A_v f_y d}{s}$$

  – $A_v$ = area in all legs of stirrups
  – $s$ = spacing of stirrup

- *may need stirrups when concrete has enough strength!*

# *Required Stirrup Reinforcement*

- *spacing limits*

Table 3-8 ACI Provisions for Shear Design*

| | | $V_u \le \frac{\phi V_c}{2}$ | $\phi V_c \ge V_u > \frac{\phi V_c}{2}$ | $V_u > \phi V_c$ |
|---|---|---|---|---|
| Required area of stirrups, $A_v$** | | none | $\frac{50 b_w s}{f_y}$ | $\frac{(V_u - \phi V_c)s}{\phi f_y d}$ |
| Stirrup spacing, s | Required | — | $\frac{A_v f_y}{50 b_w}$ | $\frac{\phi A_v f_y d}{V_u - \phi V_c}$ |
| | Recommended Minimum† | — | — | 4 in. |
| | Maximum†† (ACI 11.5.4) | — | $\frac{d}{2}$ or 24 in. | $\frac{d}{2}$ or 24 in. for $(V_u - \phi V_c) \le \phi 4\sqrt{f'_c}\, b_w d$ |
| | | | | $\frac{d}{4}$ or 12 in. for $(V_u - \phi V_c) > \phi 4\sqrt{f'_c}\, b_w d$ |

*Members subjected to shear and flexure only; $\phi V_c = \phi 2\sqrt{f'_c}\, b_w d$, $\phi = 0.75$ (ACI 11.3.1.1)

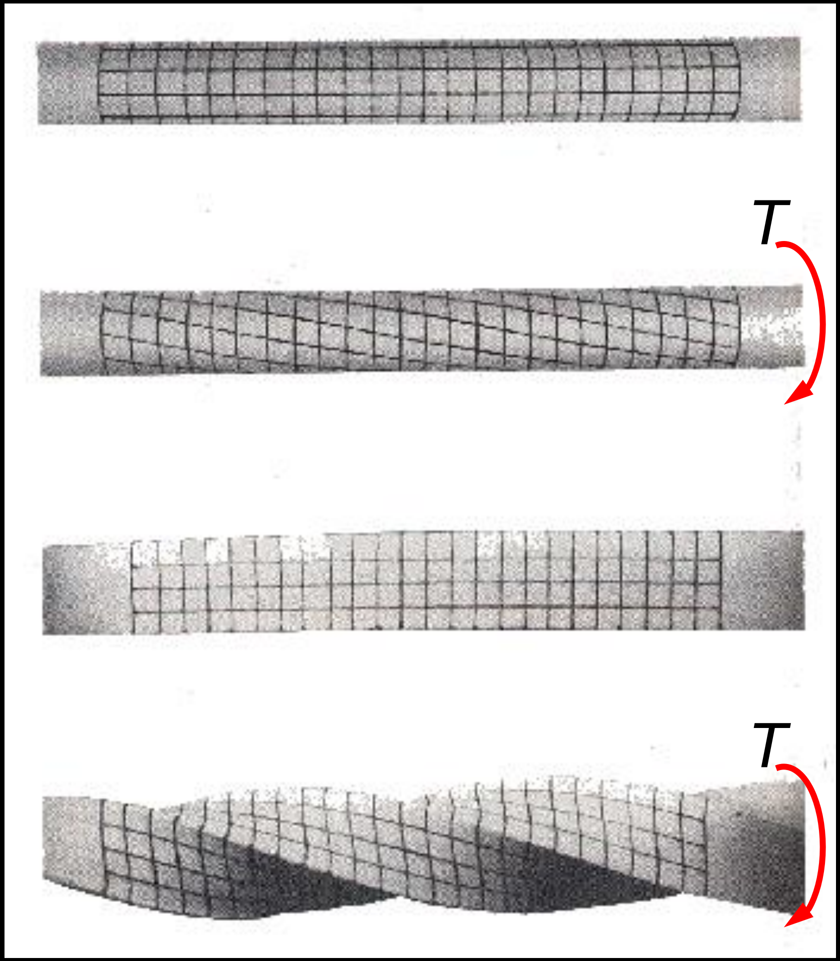
**$A_v = 2 \times A_b$ for U stirrups; $f_y \le 60$ ksi (ACI 11.5.2)

†A practical limit for minimum spacing is d/4

††Maximum spacing based on minimum shear reinforcement ($= A_v f_y / 50 b_w$) must also be considered (ACI 11.5.5.3).

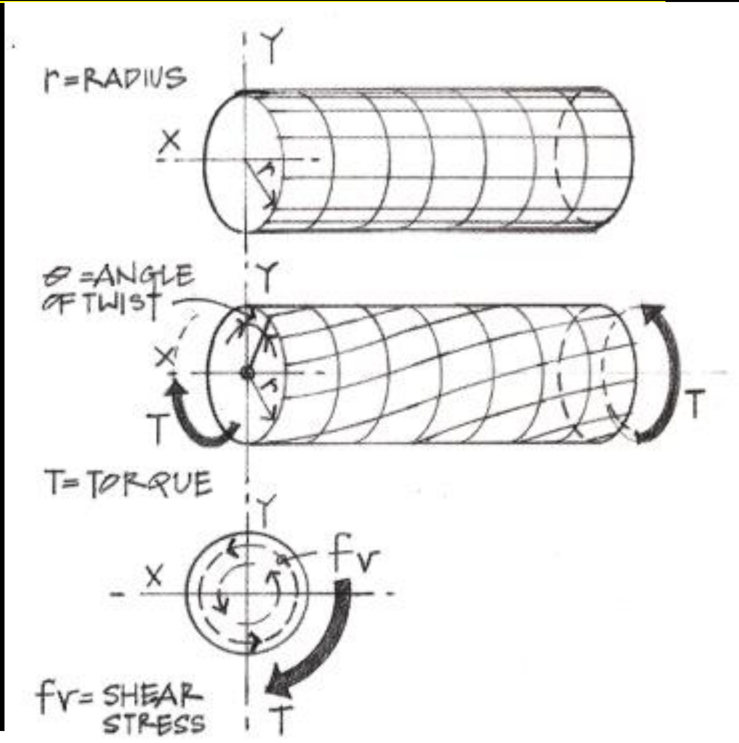
# Torsional Stress & Strain

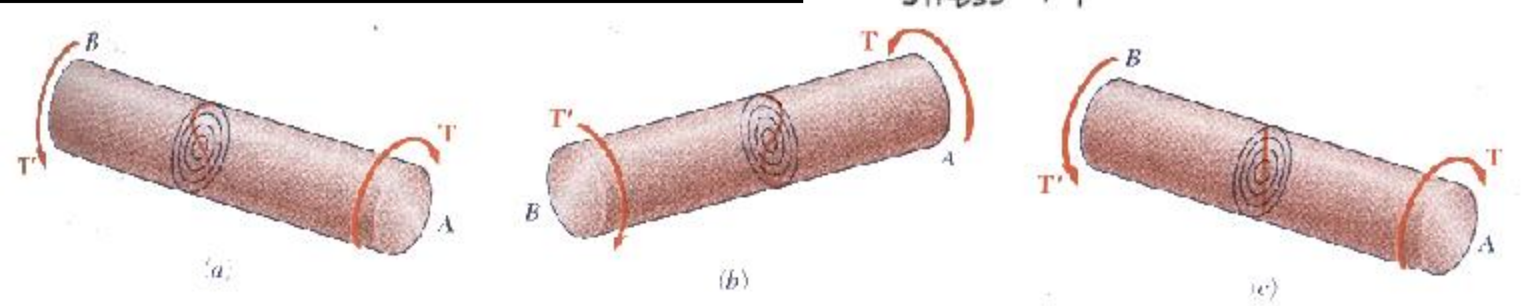
- can see torsional stresses & twisting of axi-symmetrical cross sections
  - torque
  - remain plane
  - undistorted
  - rotates
- not true for square sections….



# Shear Stress Distribution

- *depend on the deformation*
- *$\phi$ = angle of twist*
  - *measure*
- *can prove planar section doesn't distort*



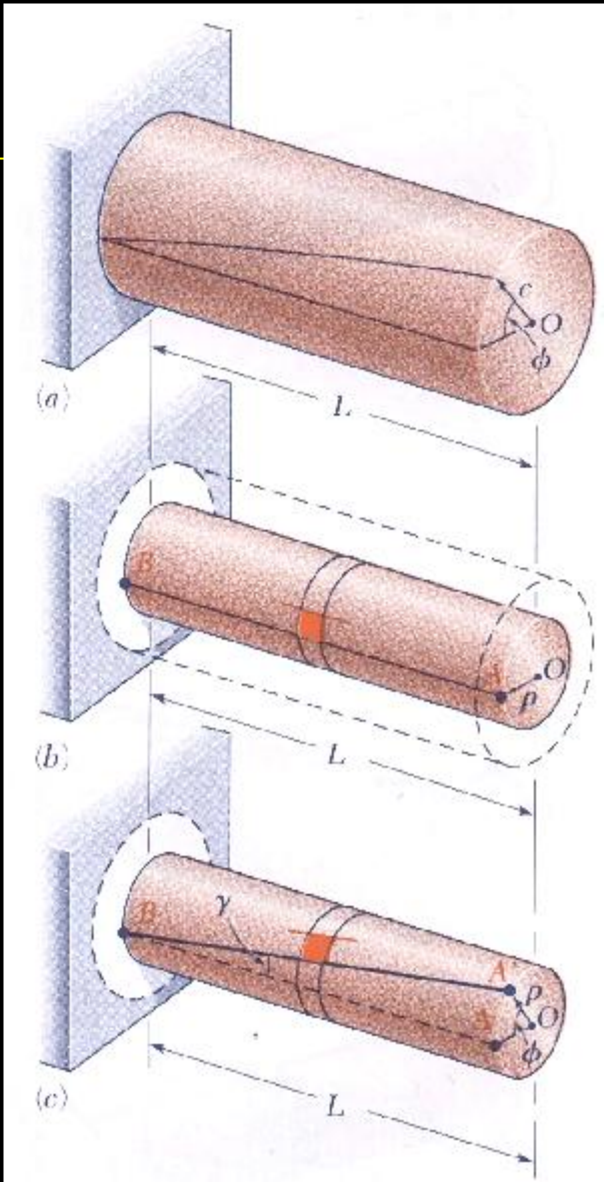



# Shearing Strain

- *related to* $\phi$

$$\gamma = \frac{\rho\phi}{L}$$

- $\rho$ *is the radial distance from the centroid to the point under strain*
- *shear strain varies linearly along the radius:* $\gamma_{max}$ *is at outer diameter*

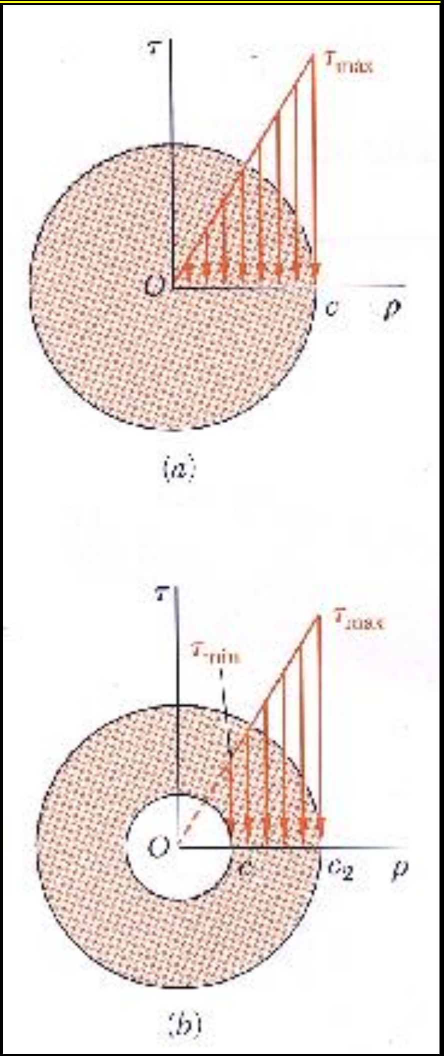


# Torsional Stress - Strain

- know $f_v = \tau = G \cdot \gamma$ and $\gamma = \dfrac{\rho\phi}{L}$
- so $\tau = G \cdot \dfrac{\rho\phi}{L}$
- where G is the <u>Shear Modulus</u>

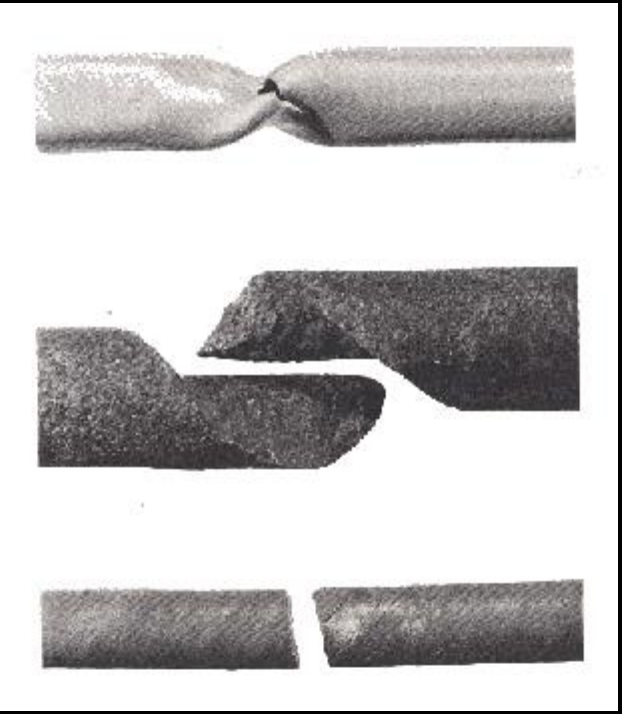
# Torsional Stress - Strain

- from $$T = \Sigma \tau(\rho) \Delta A$$
- can derive $$T = \frac{\tau J}{\rho}$$
  - where J is the polar moment of inertia
  - elastic range $$\tau = \frac{T\rho}{J}$$



# *Shear Stress*

- *$\tau_{max}$ happens at outer diameter*



- *combined shear and axial stresses*
  - *maximum shear stress at 45° "twisted" plane*

# Shear Strain

- *knowing* $\tau = G \cdot \dfrac{\rho\phi}{L}$ *and* $\tau = \dfrac{T\rho}{J}$
- *solve:* $\phi = \dfrac{TL}{JG}$
- *composite shafts:* $\phi = \sum_i \dfrac{T_i L_i}{J_i G_i}$

# Noncircular Shapes

- *torsion depends on J*
- *plane sections don't remain plane*
- $\tau_{max}$ *is still at outer diameter*

$$\tau_{\max} = \frac{T}{c_1 ab^2} \quad \phi = \frac{TL}{c_2 ab^3 G}$$

– *where a is longer side (> b)*

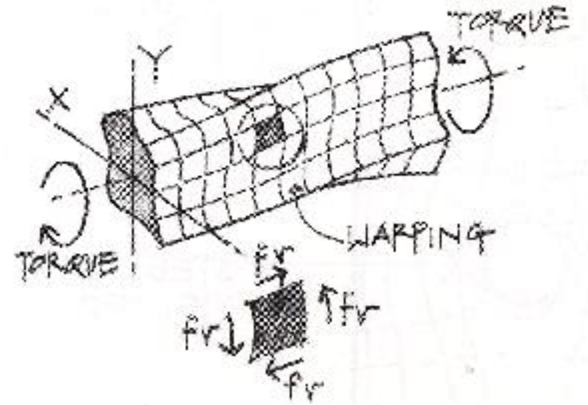
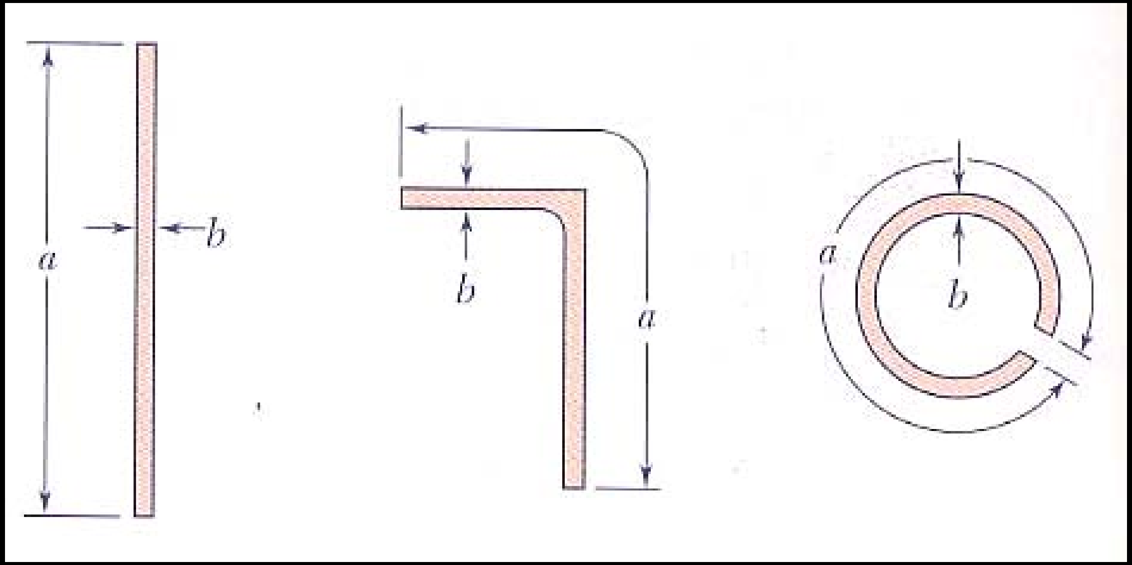



**TABLE 3.1. Coefficients for Rectangular Bars in Torsion**

| $a/b$ | $c_1$ | $c_2$ |
|---|---|---|
| 1.0 | 0.208 | 0.1406 |
| 1.2 | 0.219 | 0.1661 |
| 1.5 | 0.231 | 0.1958 |
| 2.0 | 0.246 | 0.229 |
| 2.5 | 0.258 | 0.249 |
| 3.0 | 0.267 | 0.263 |
| 4.0 | 0.282 | 0.281 |
| 5.0 | 0.291 | 0.291 |
| 10.0 | 0.312 | 0.312 |
| ∞ | 0.333 | 0.333 |

# Open Thin-Walled Sections

- *with very large a/b ratios:*



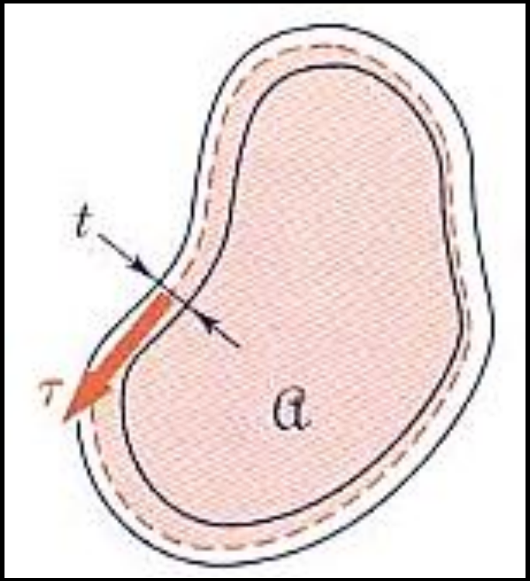

$$\tau_{\max} = \frac{T}{\frac{1}{3}ab^2} \qquad \phi = \frac{TL}{\frac{1}{3}ab^3G}$$

# Shear Flow in Closed Sections

- q is the internal shear force/unit length

$$\tau = \frac{T}{2t\mathcal{a}}$$

$$\phi = \frac{TL}{4t\mathcal{a}^2}\sum_i \frac{s_i}{t_i}$$

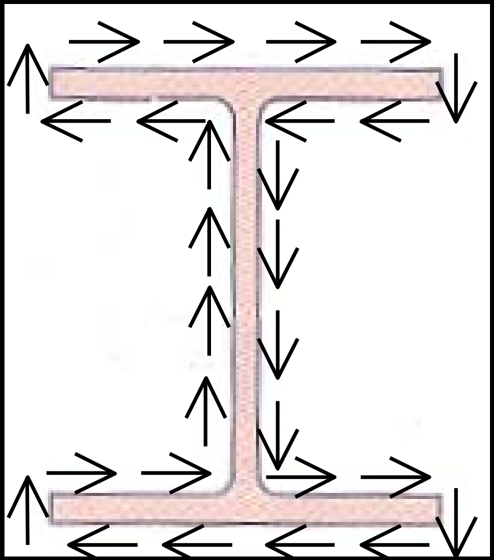



- $\mathcal{a}$ is the area bounded by the centerline
- $s_i$ is the length segment, $t_i$ is the thickness

# Shear Flow in Open Sections

- *each segment has proportion of T with respect to torsional rigidity,*

$$\tau_{max} = \frac{Tt_{max}}{\frac{1}{3}\Sigma b_i t_i^3}$$



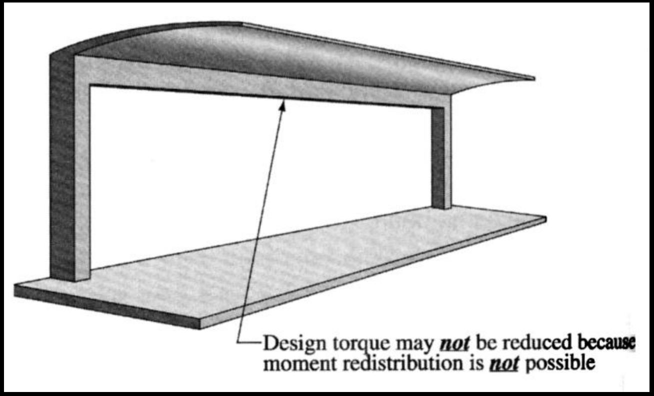
- *total angle of twist:*

$$\phi = \frac{TL}{\frac{1}{3}G\Sigma b_i t_i^3}$$

- *I beams - web is thicker, so $\tau_{max}$ is in <u>web</u>*

# Torsional Shear Stress

- *twisting moment*
- *and beam shear*



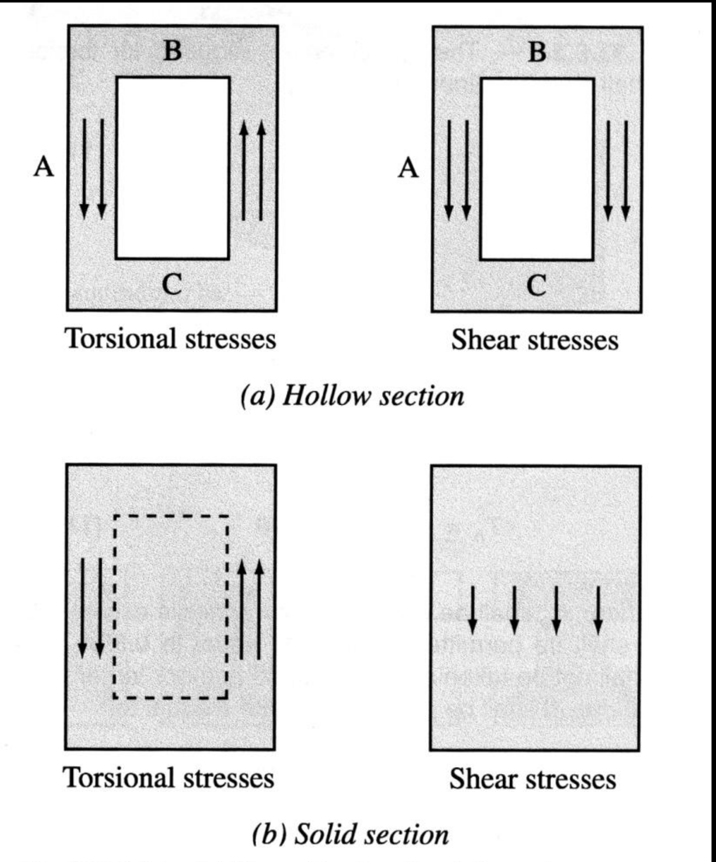




Fig. R11.6.3.1—Addition of torsional and shear stresses

# Torsional Shear Reinforcement

- closed stirrups
- more longitudinal reinforcement

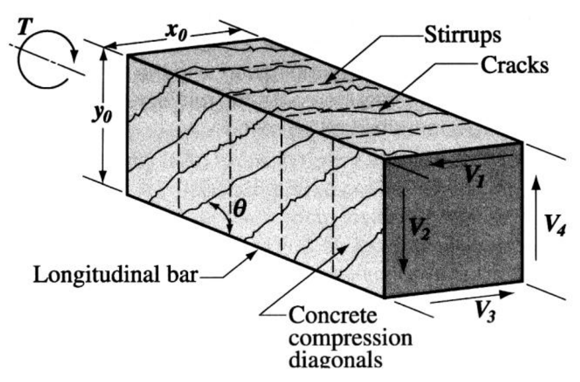


Fig. R11.6.3.6(a)—Space truss analogy

- area enclosed by shear flow

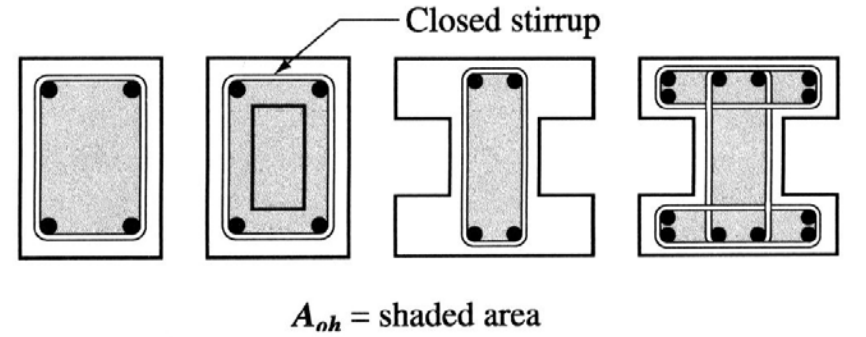
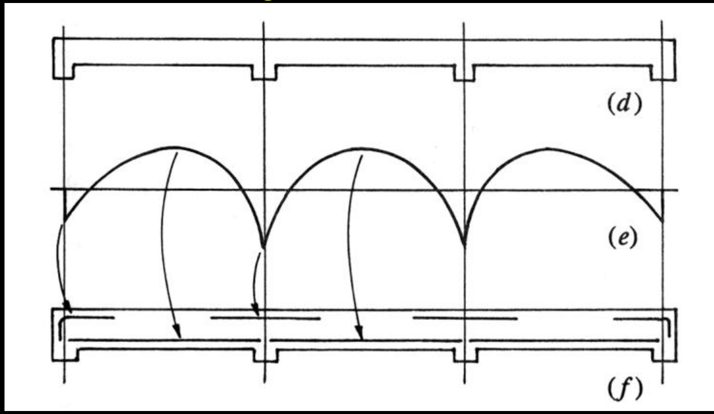


Fig. R11.6.3.6(b)—Definition of $\mathbf{A_{oh}}$

# Development Lengths

- required to allow steel to yield ($f_y$)
- standard hooks
  - moment at beam end



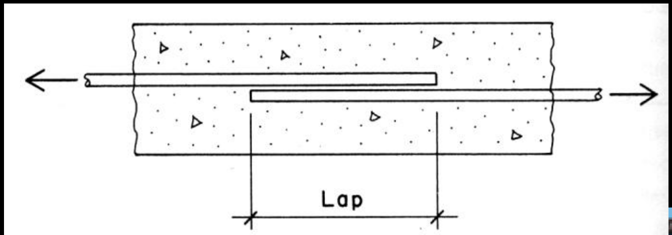
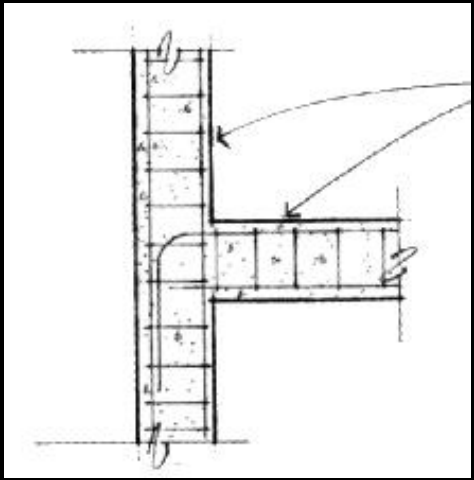



**Figure 13.24** The lapped splice for steel reinforcing bars.



- splices
  - lapped
  - mechanical connectors

# *Development Lengths*

- $l_d$ , *embedment required* ***both*** *sides*
- *proper cover, spacing:*
  - *No. 6 or smaller*

$$l_d = \frac{d_b F_y}{25\sqrt{f'_c}}$$

*or 12 in. minimum*

  - *No. 7 or larger*

$$l_d = \frac{d_b F_y}{20\sqrt{f'_c}}$$

*or 12 in. minimum*

# Development Lengths

- hooks
  - bend and extension

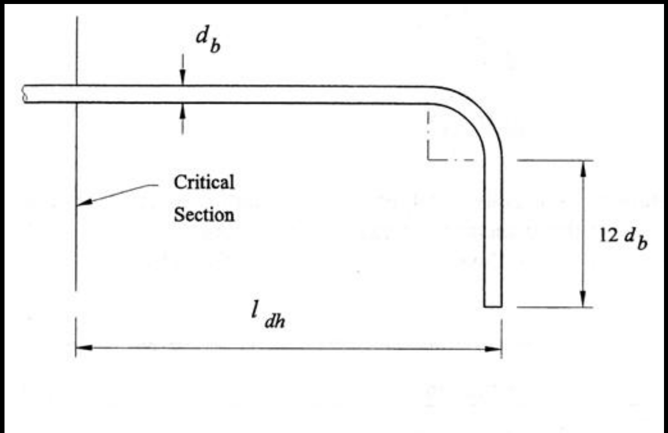



**Figure 9-17**: Minimum requirements for 90°bar hooks.

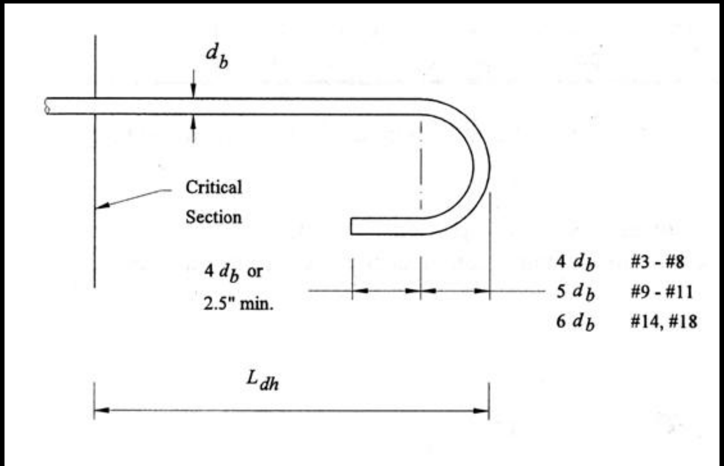



**Figure 9-18**: Minimum requirements for 180° bar hooks.

- minimum

$$l_{dh} = \frac{1200 d_b}{\sqrt{f'_c}}$$

# Development Lengths

- *bars in compression*

$$l_d = \frac{0.02 d_b F_y}{\sqrt{f'_c}} \le 0.0003 d_b F_y$$

- *splices*
  - *tension minimum is function of $l_d$ and splice classification*
  - *compression minimum*
  - *is function of $d_b$ and $F_y$*

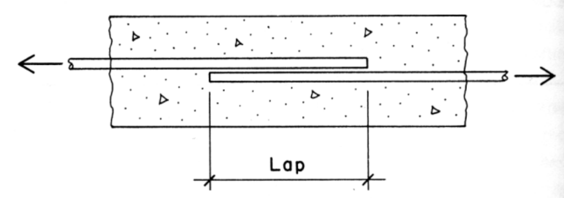



**Figure 13.24** The lapped splice for steel reinforcing bars.

# Concrete Deflections

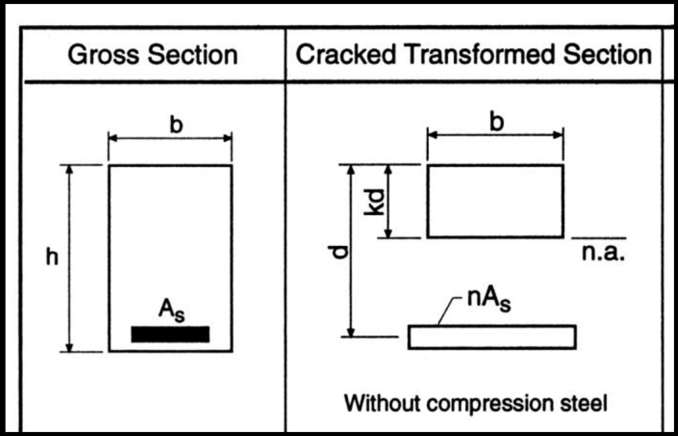
- elastic range
  - I transformed
  - $E_c$ (with $f'_c$ in psi)
    - normal weight concrete (~ 145 lb/ft³)

$$E_c = 57{,}000\sqrt{f'_c}$$

    - concrete between 90 and 160 lb/ft³

$$E_c = w_c^{1.5} 33\sqrt{f'_c}$$

- cracked
  - I cracked
  - E adjusted



# Deflection Limits

- *relate to whether or not beam supports or is attached to a damageable non-structural element*
- *need to check service live load and long term deflection against these*

| | |
|---|---|
| L/180 | roof systems (typical) – live |
| L/240 | floor systems (typical) – live + long term |
| L/360 | supporting plaster – live |
| L/480 | supporting masonry – live + long term |