

**ARCHITECTURAL STRUCTURES:  
FORM, BEHAVIOR, AND DESIGN**

**ARCH 331**

**DR. ANNE NICHOLS**

**SUMMER 2013**

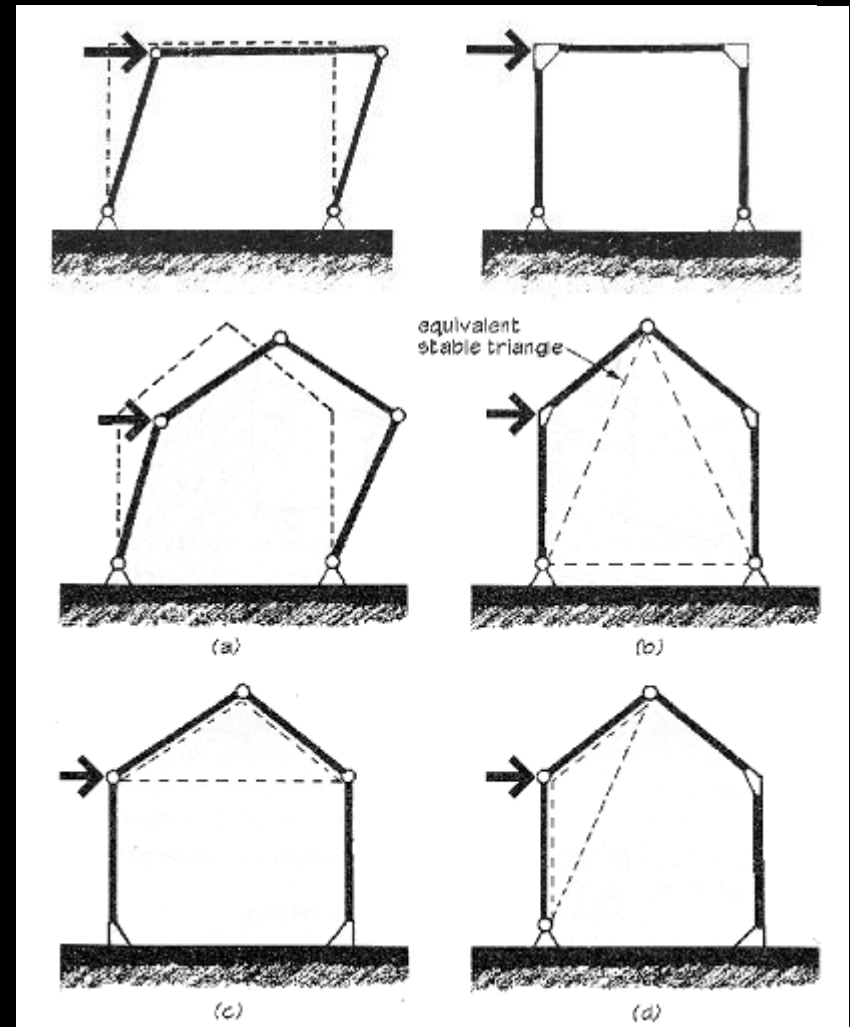
**lecture  
ten**

**rigid frames:  
compression & buckling**



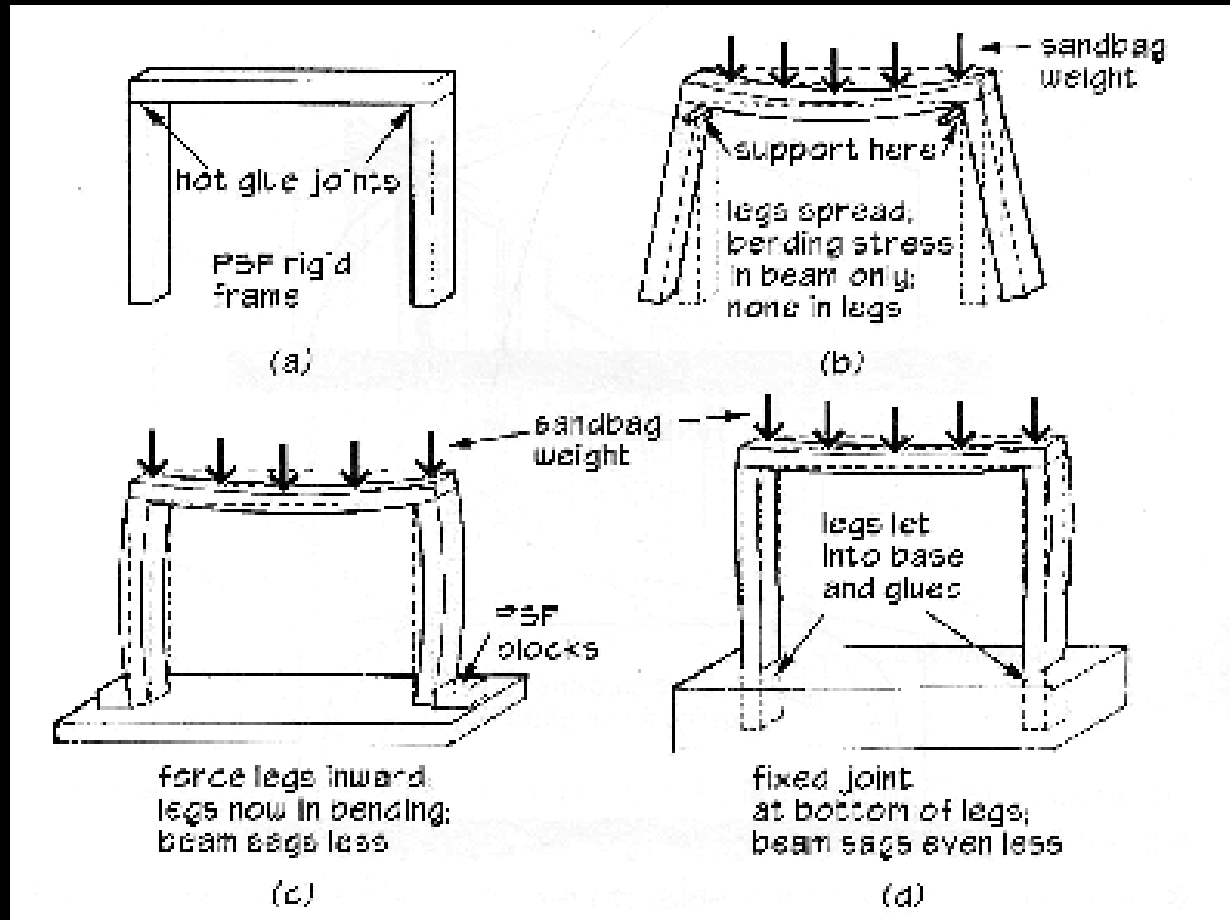
# Rigid Frames

- rigid frames have no pins
- frame is all one body
- joints transfer moments and shear
- typically statically indeterminate
- types
  - portal
  - gable



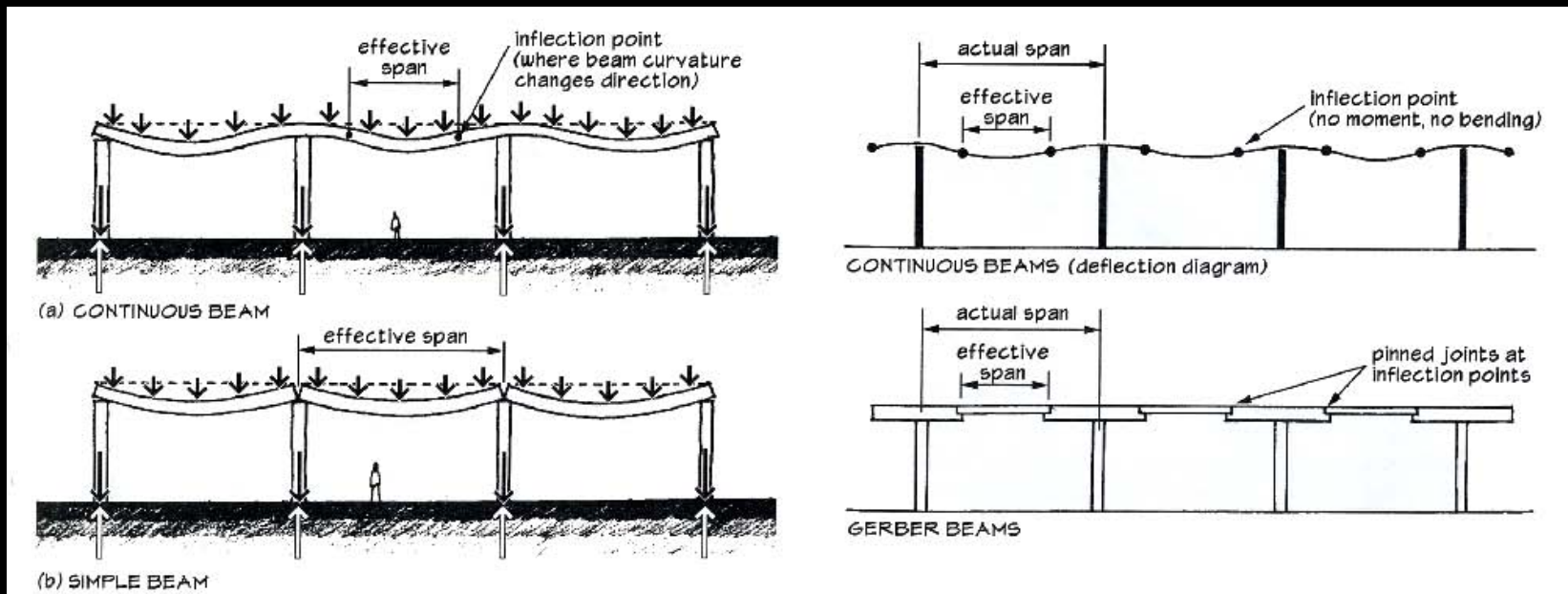
# Rigid Frames

- behavior



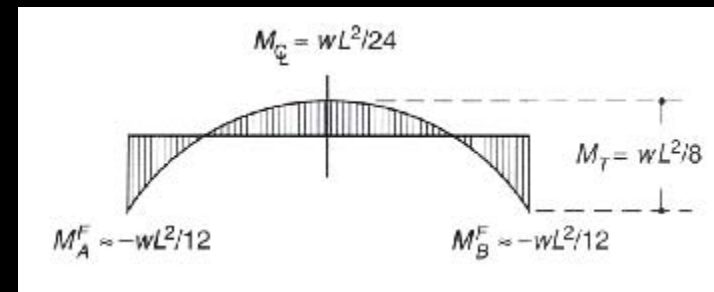
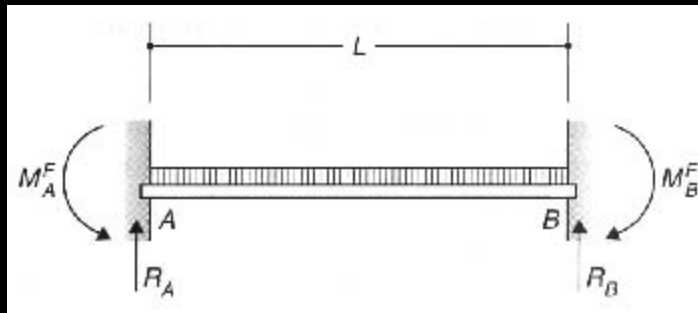
# Rigid Frames

- moments get redistributed
- deflections are smaller
- effective column lengths are shorter
- very sensitive to settling



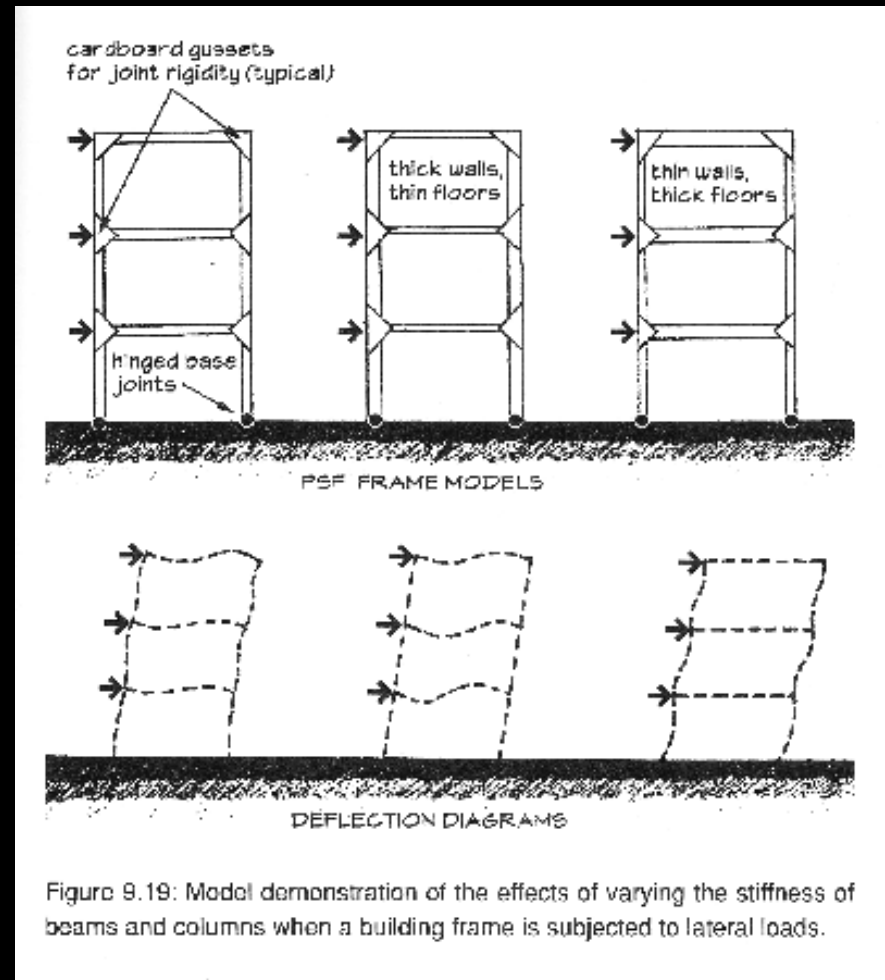
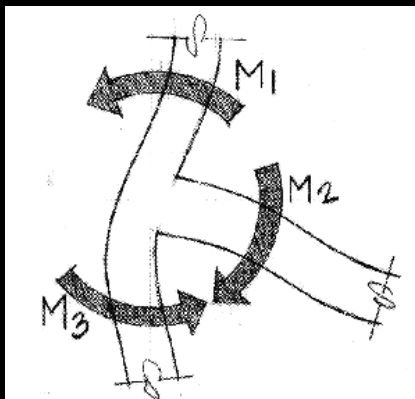
# Moment Redistribution

- continuous slabs & beams with uniform loading
  - joints similar to fixed ends, but can rotate
- change in moment to center =  $\frac{wL^2}{8}$ 
  - $M_{max}$  for simply supported beam



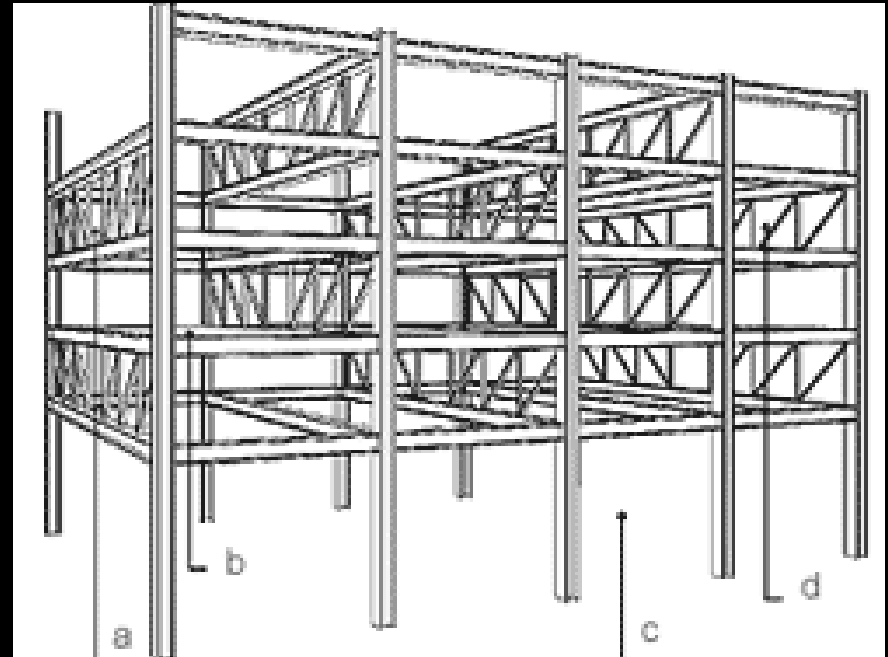
# Rigid Frames

- *resists lateral loadings*
- *shape depends on stiffness of beams and columns*
- *90° maintained*



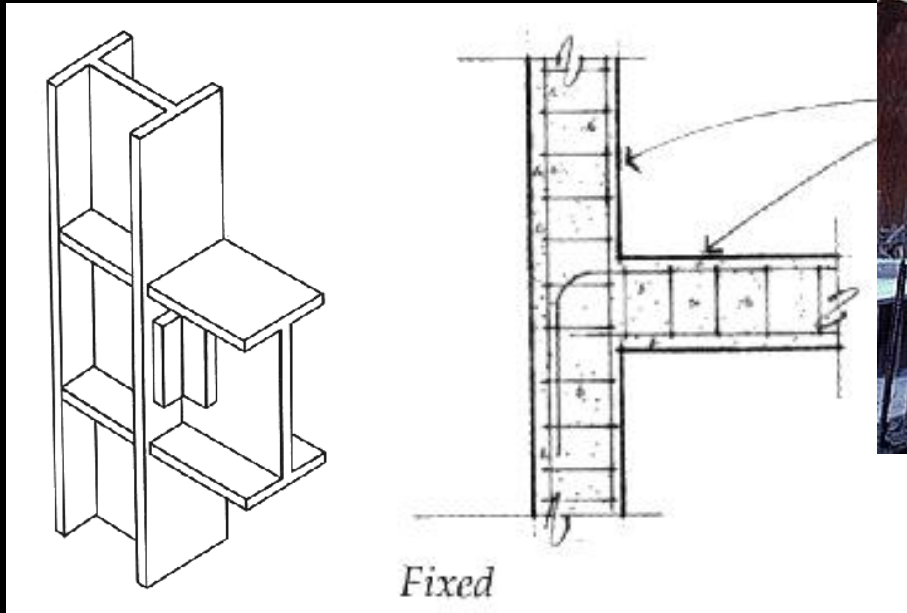
# Rigid Frames

- *staggered truss*
  - *rigidity*
  - *clear stories*



# Rigid Frames

- *connections*
  - *steel*
  - *concrete*



<http://nisee.berkeley.edu/godden>



# Braced Frames

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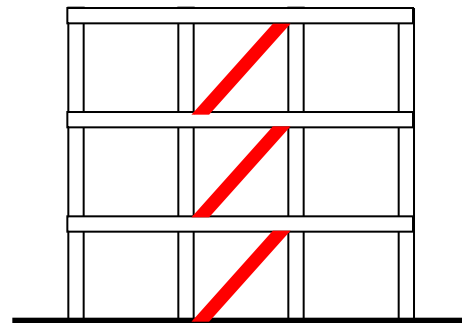
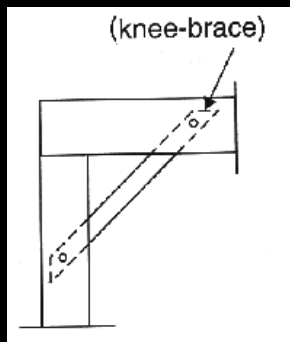
- *pin connections*
- *bracing to prevent lateral movements*



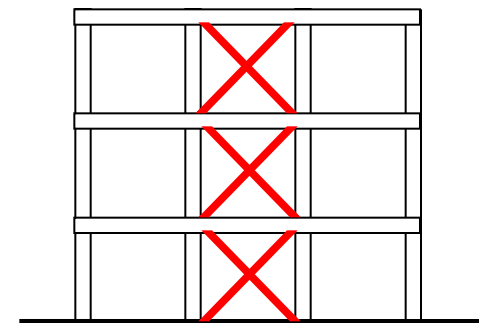
<http://nisee.berkeley.edu/godden>

# Braced Frames

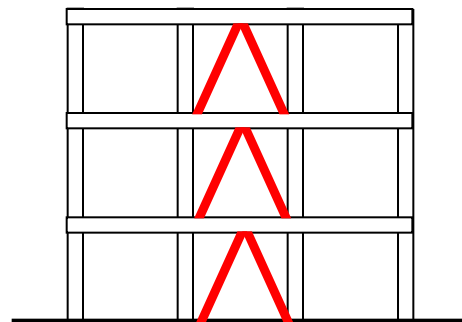
- *types of bracing*
  - *knee-bracing*
  - *diagonal*
  - *X*
  - *K or chevron*
  - *shear walls*



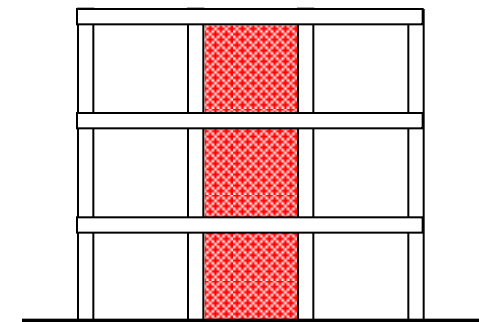
**diagonal**



**X**



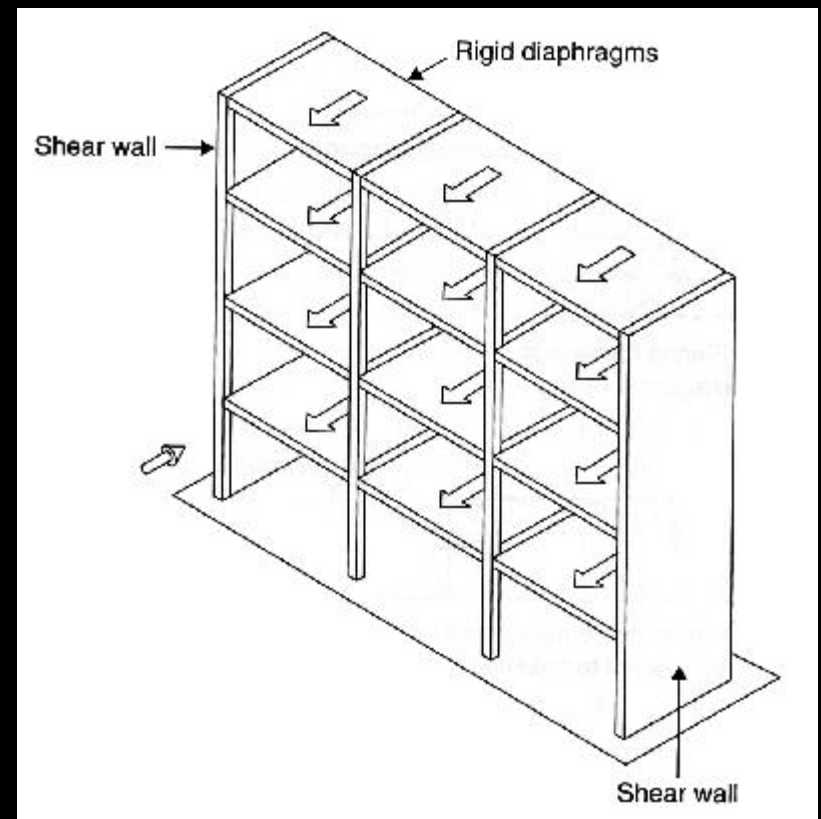
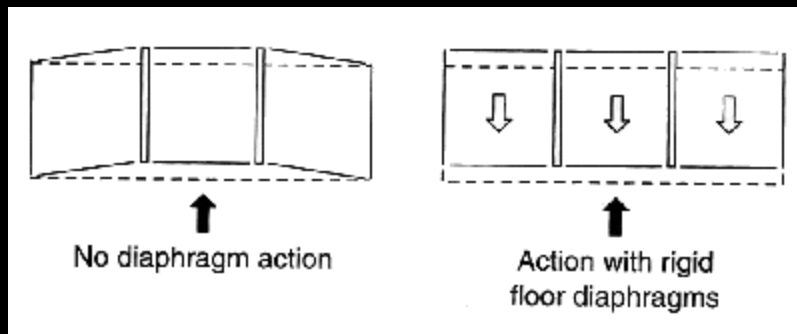
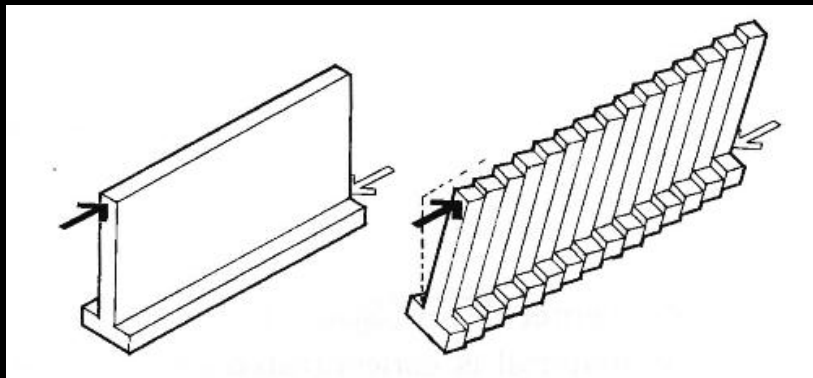
**K (chevron)**



**shear walls**

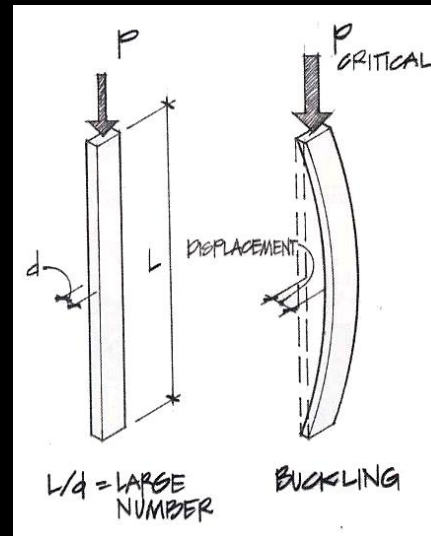
# Shear Walls

- *resist lateral load in plane with wall*



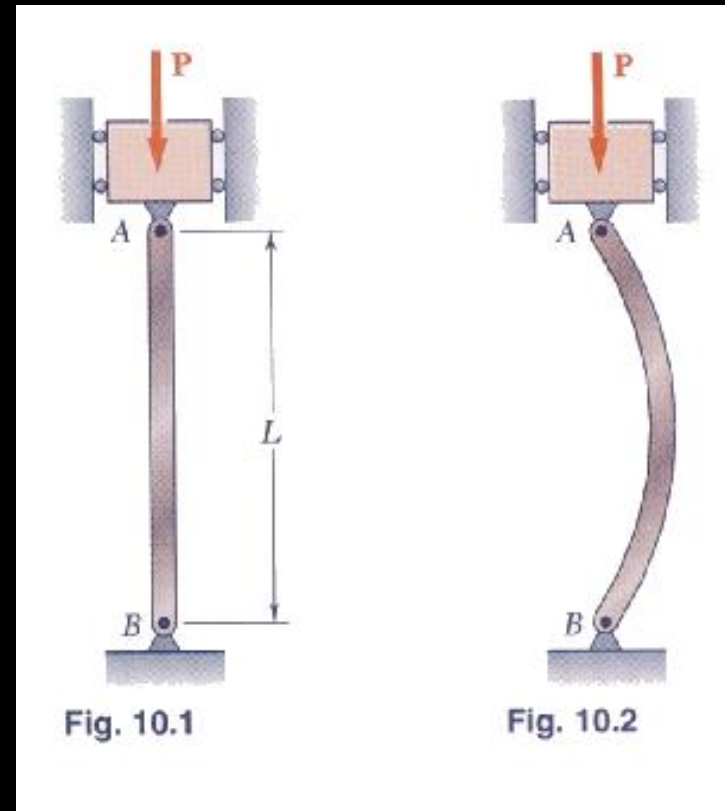
# Compression Members

- designed for strength & stresses
- designed for serviceability & deflection
- need to design for stability
  - ability to support a specified load without sudden or unacceptable deformations



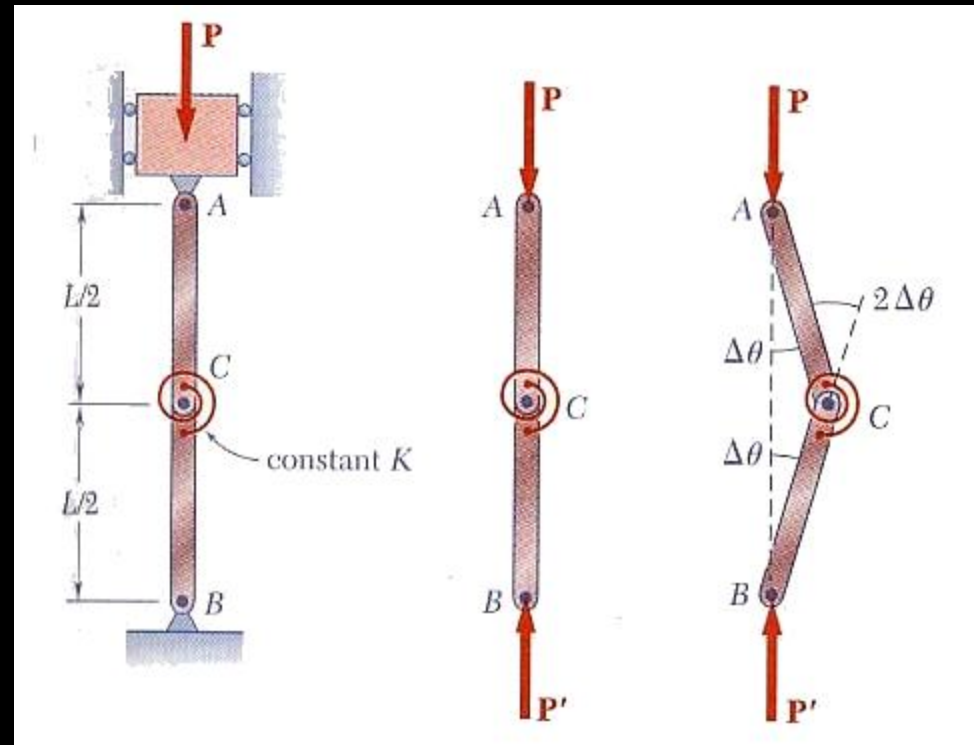
# Column Buckling

- axially loaded columns
- long & slender
  - unstable equilibrium = buckling
  - sudden and not good



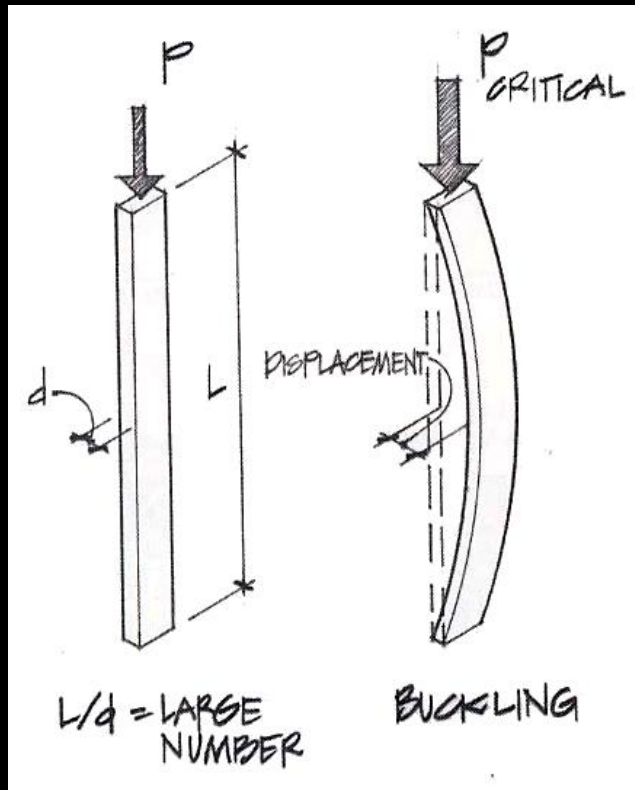
# Modeling

- can be modeled with a spring at mid-height
- when moment from deflection exceeds the spring capacity ... “boing”
- critical load  $P$

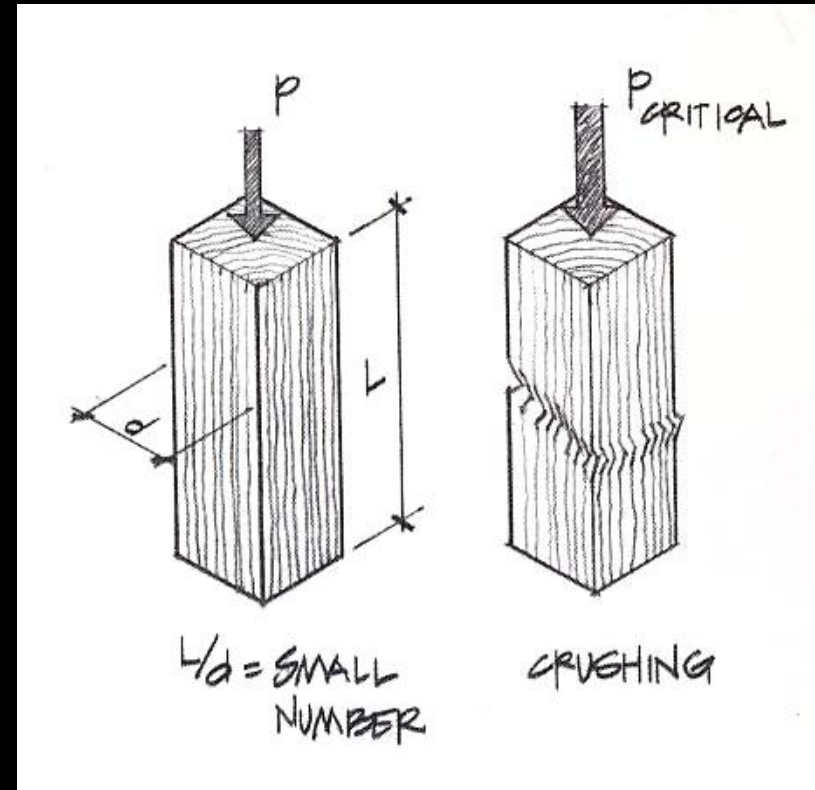


# Effect of Length

- long & slender



- short & stubby



# Buckling Load

- related to deflected shape ( $P\Delta$ )
- shape of sine wave
- Euler's Formula
- smallest  $I$  governs

$$P_{critical} = \frac{\pi^2 EI}{(L)^2}$$

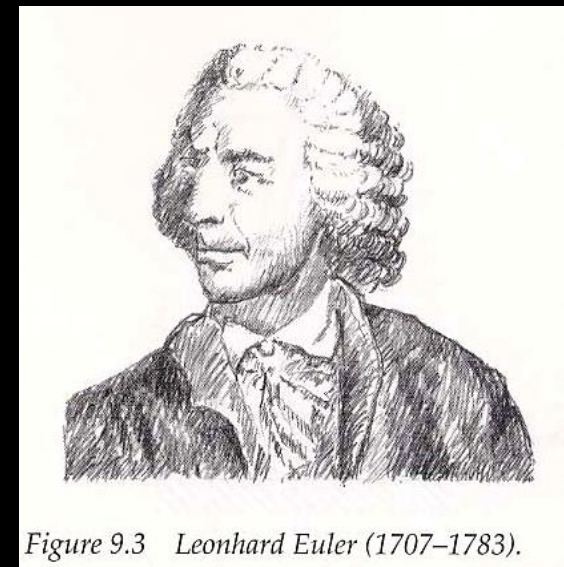
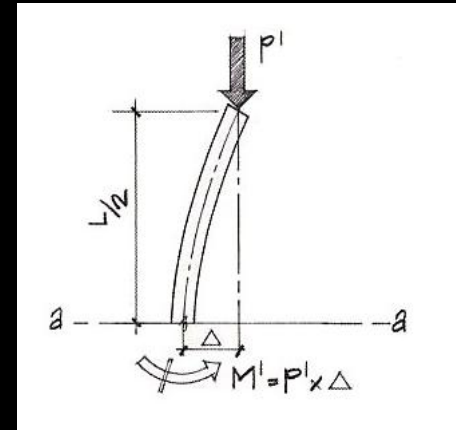


Figure 9.3 Leonhard Euler (1707–1783).



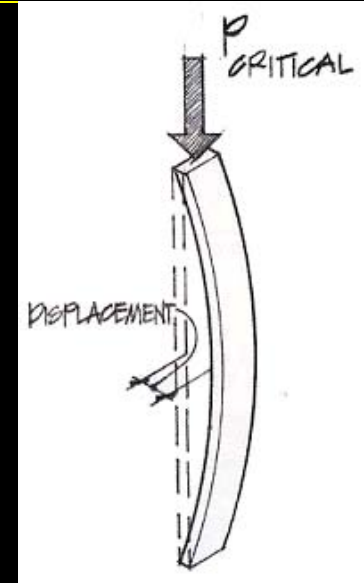
# Critical Stress

- short columns

$$f_{critical} = \frac{P_{actual}}{A} < F_a$$

- slenderness ratio =  $L_e/r$  ( $L/d$ )

- radius of gyration =  $r = \sqrt{\frac{I}{A}}$



weak axis

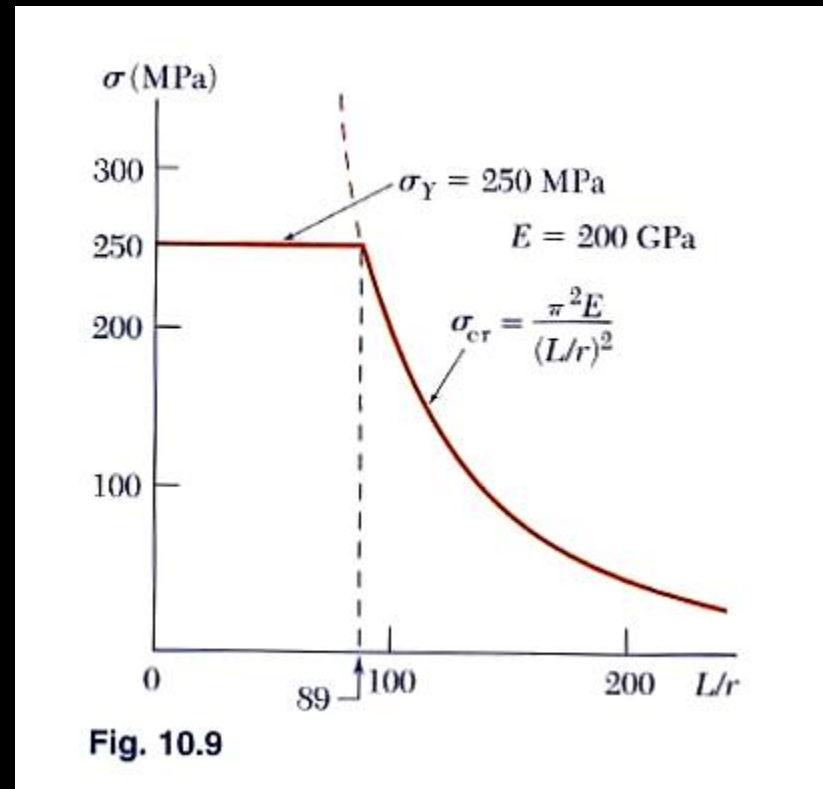
$$f_{critical} = \frac{P_{critical}}{A} = \frac{\pi^2 E A r^2}{A (L_e)^2} = \frac{\pi^2 E}{\left(\frac{L_e}{r}\right)^2}$$

$$P_{critical} = \frac{\pi^2 E A}{\left(\frac{L_e}{r}\right)^2}$$

# Critical Stresses

- when a column gets stubby,  $F_y$  will limit the load
- real world has loads with eccentricity
- $C_c$  for steel and allowable stress

$$\frac{L_e}{r} > C_c = \sqrt{\frac{2\pi^2 E}{F_y}}$$



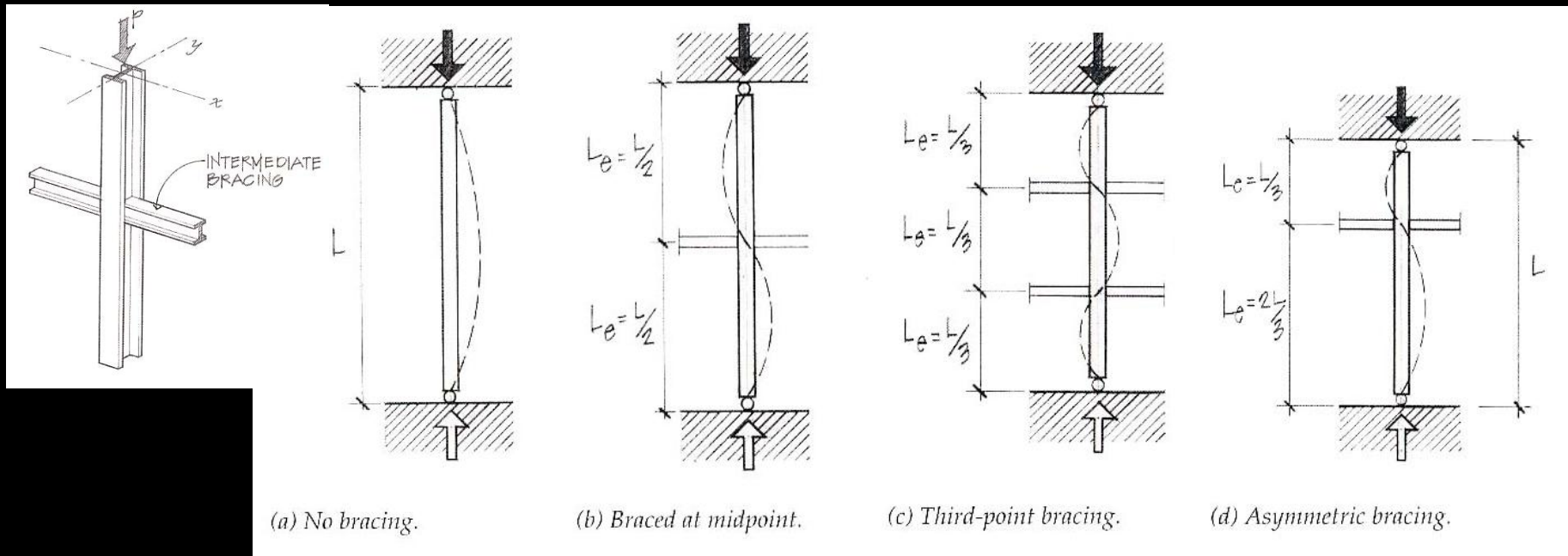
# Effective Length

- end conditions affect shape
- effective length factor,  $K$   $L_e = K \cdot L$

Buckled shape of column shown by dashed line	(a)	(b)	(c)	(d)	(e)	(f)
Theoretical $K$ value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design values when ideal conditions are approximated	0.65	0.80	1.0	1.2	2.10	2.0

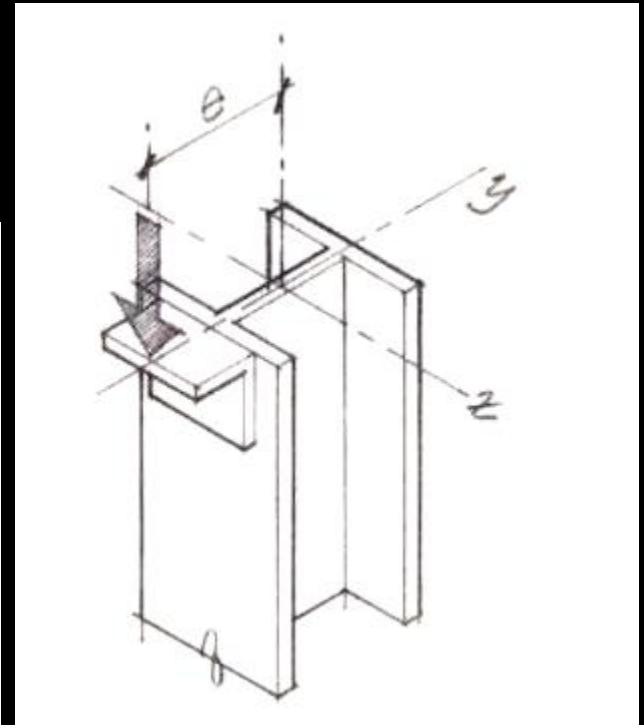
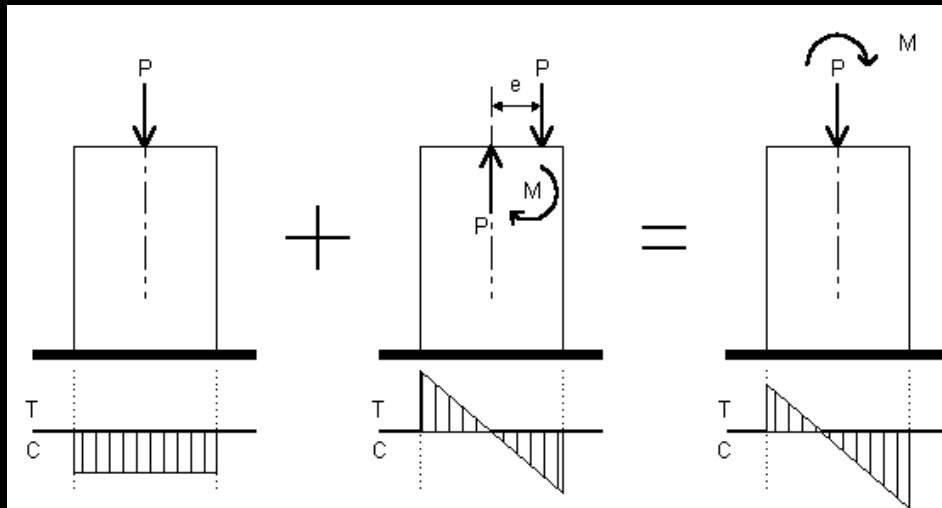
# Bracing

- *bracing affects shape of buckle in one direction*
- *both should be checked!*



# Centric & Eccentric Loading

- *centric*
  - allowable stress from strength or buckling
- *eccentric*
  - combined stresses



# Combined Stresses

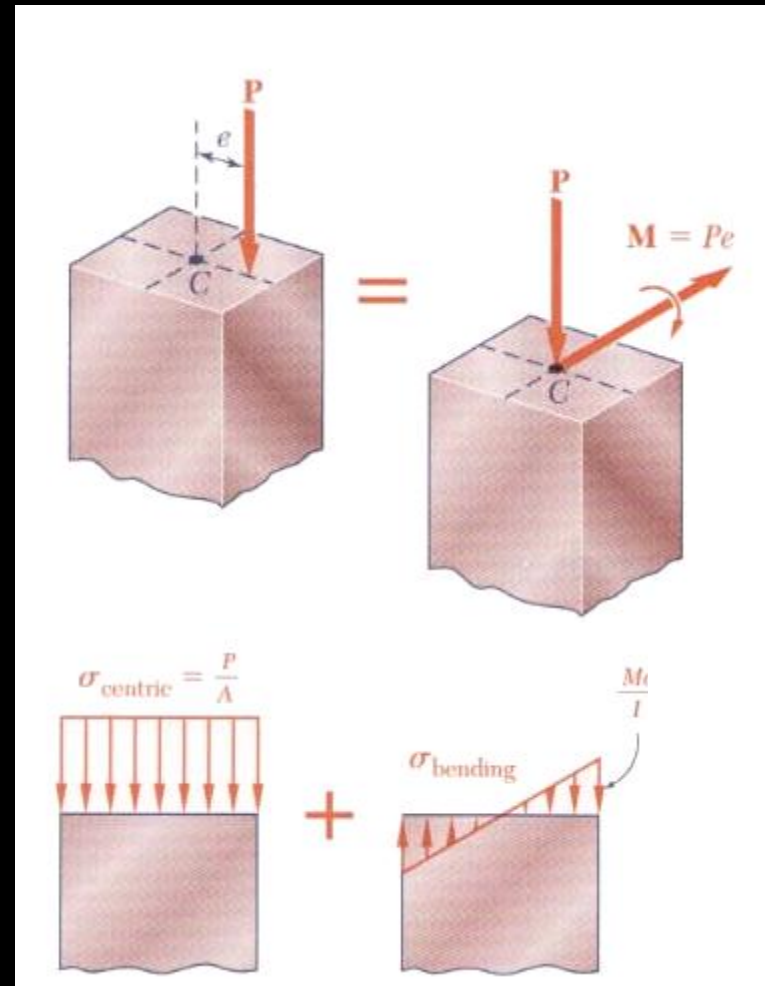
– axial + bending

$$f_{\max} = \frac{P}{A} + \frac{Mc}{I}$$

$$M = P \cdot e$$

– design

$$f_{\max} \leq F_{cr} = \frac{f_{cr}}{F.S.}$$



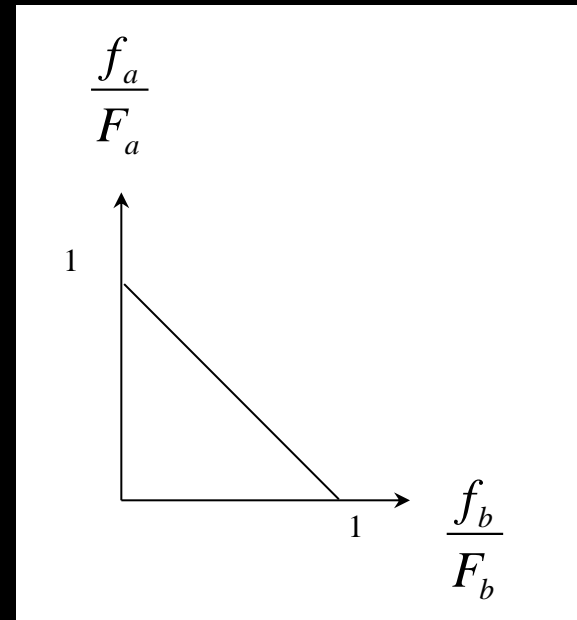
# Stress Limit Conditions

– ASD interaction formula

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1.0$$

– with biaxial bending

$$\frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0$$



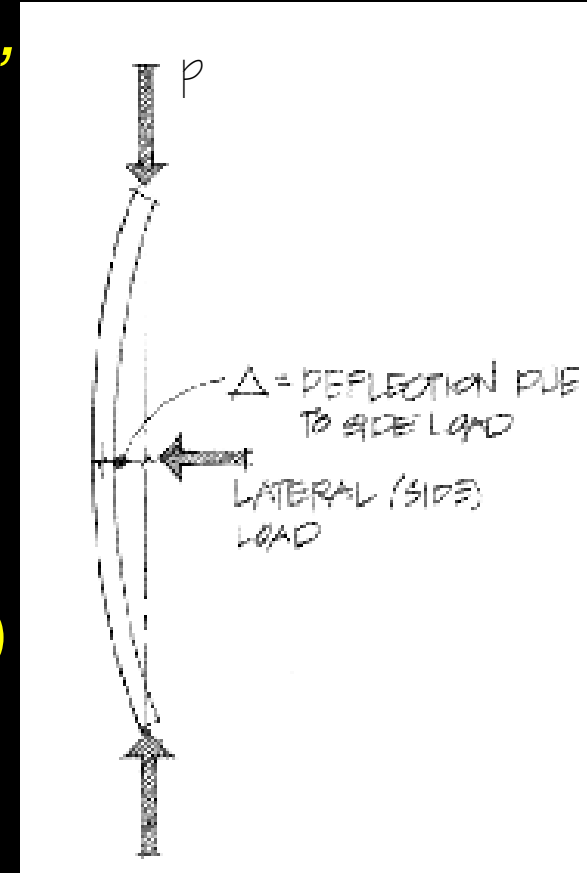
interaction diagram

# Stress Limit Conditions

– in reality, as the column flexes,  
the moment increases

– P-Δ effect

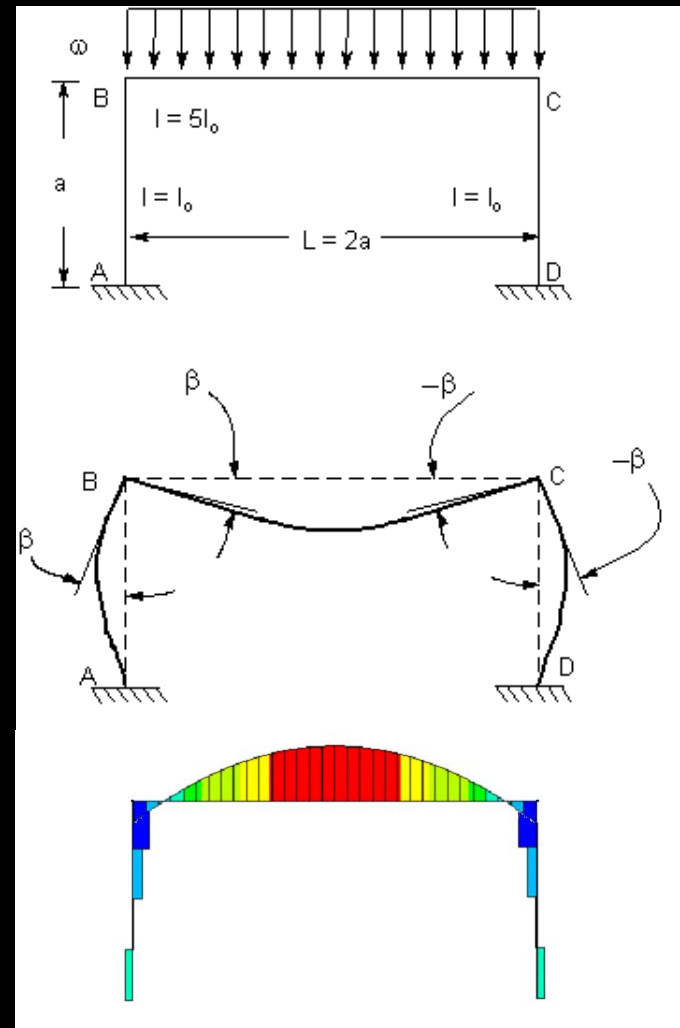
$$\frac{f_a}{F_a} + \frac{f_b \times (\text{Magnification factor})}{F_{bx}} \leq 1.0$$





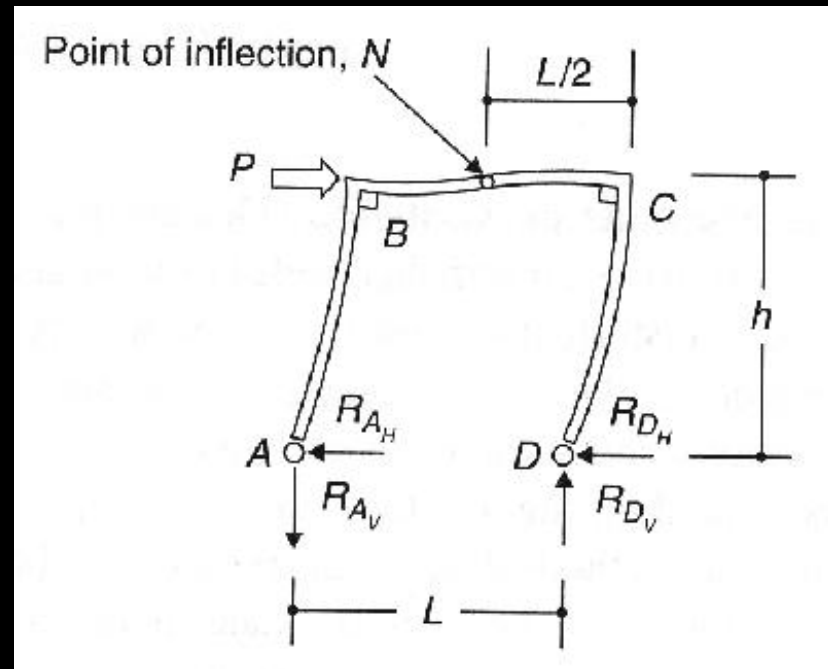
# Rigid Frame Analysis

- *members see*
  - *shear*
  - *axial force*
  - *bending*
- *V & M diagrams*
  - *plot on “outside”*



# Rigid Frame Analysis

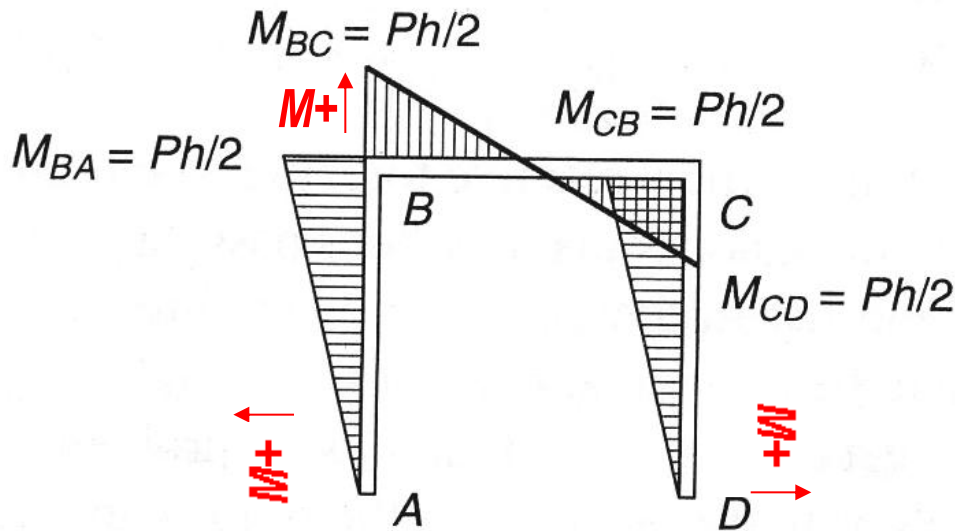
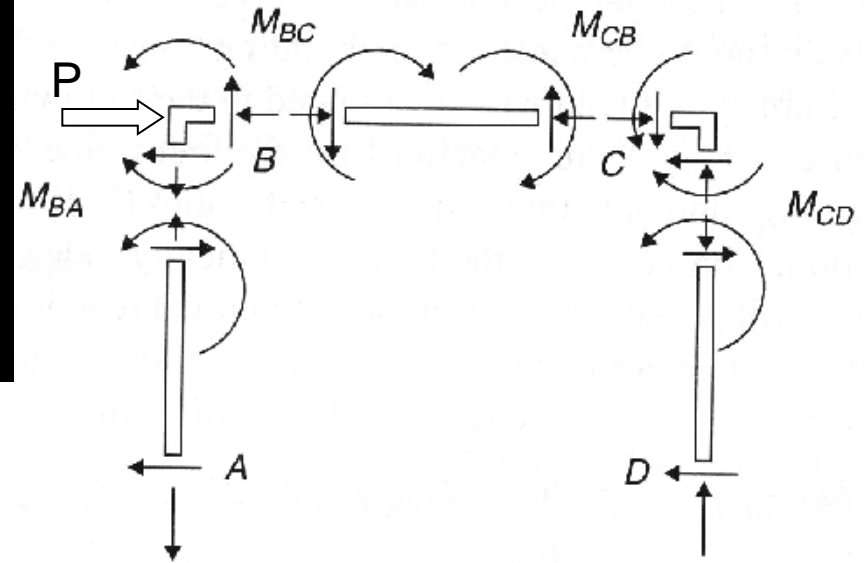
- need support reactions
- free body diagram each member
- end reactions are equal and opposite on next member
- “turn” member like beam
- draw  $V$  &  $M$



# Rigid Frame Analysis

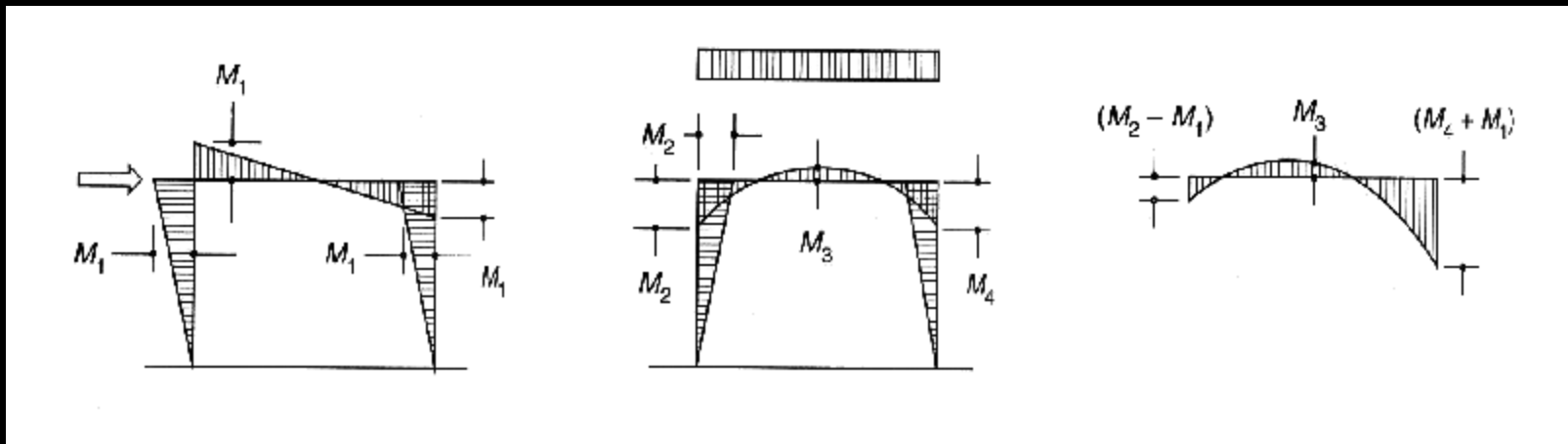
## – FBD & M

- opposite end reactions at joints



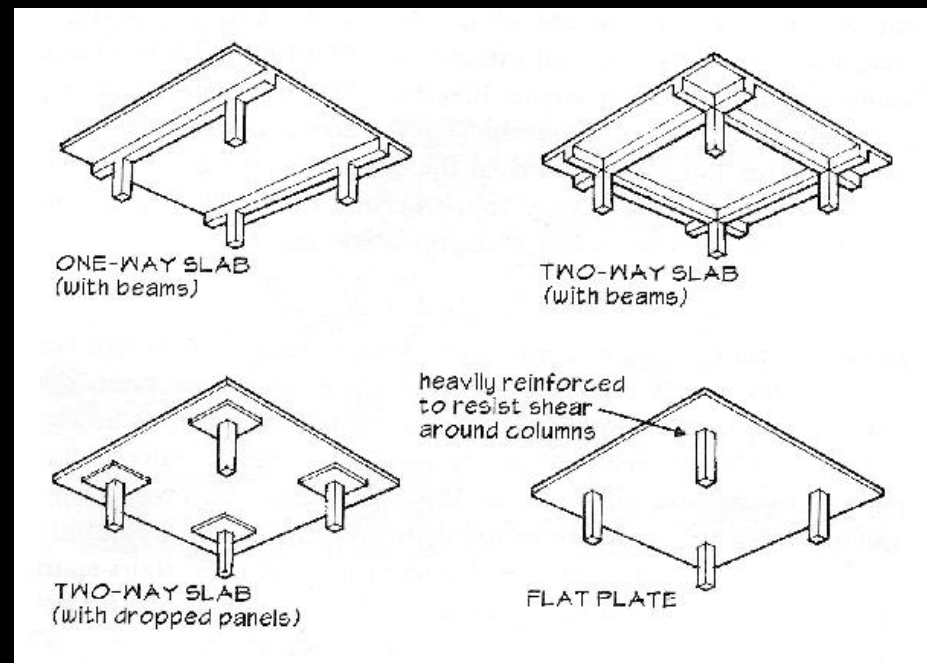
# Rigid Frame Design

- *loads and combinations*
  - *usually uniformly distributed gravity loads*
  - *worst case for largest moments...*
  - *wind direction can increase moments*



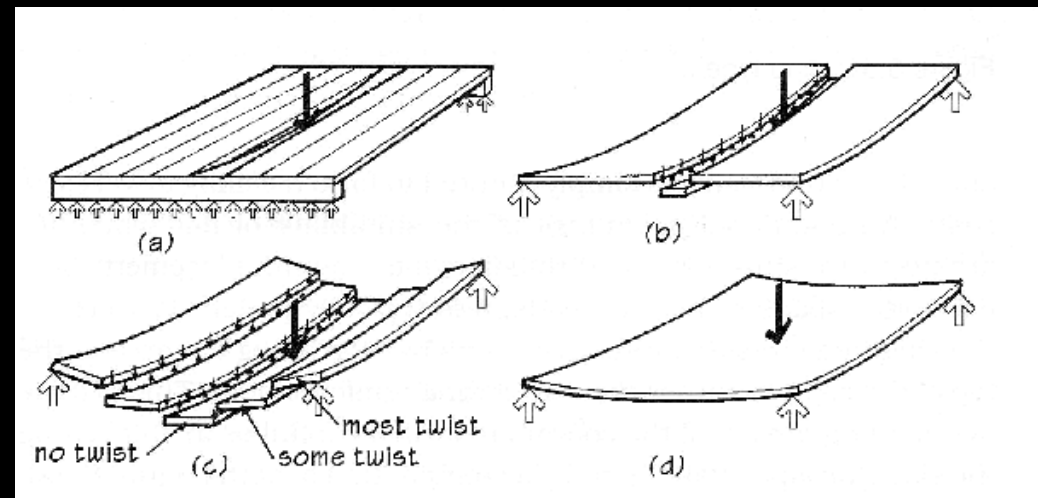
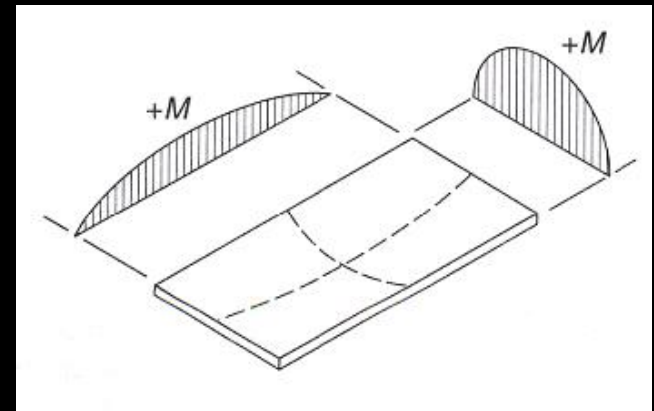
# Rigid Frame Design

- frames & floors
  - rigid frame can have slab floors or slab with connecting beams
- other
  - slabs or plates on columns



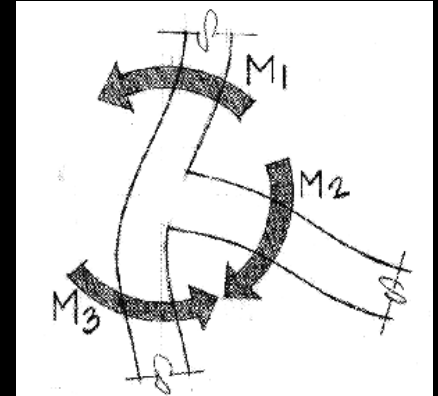
# Rigid Frame Design

- floors – plates & slabs
  - one-way behavior
    - side ratio  $> 1.5$
    - “strip” beam
  - two-way behavior
    - more complex



# Rigid Frame Design

- columns in frames
  - ends can be “flexible”
  - stiffness affected by beams and column =  $EI/L$



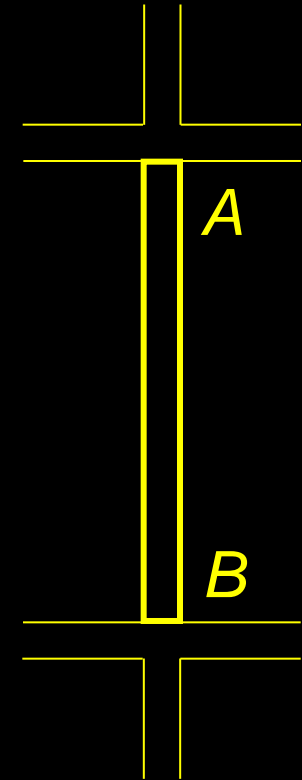
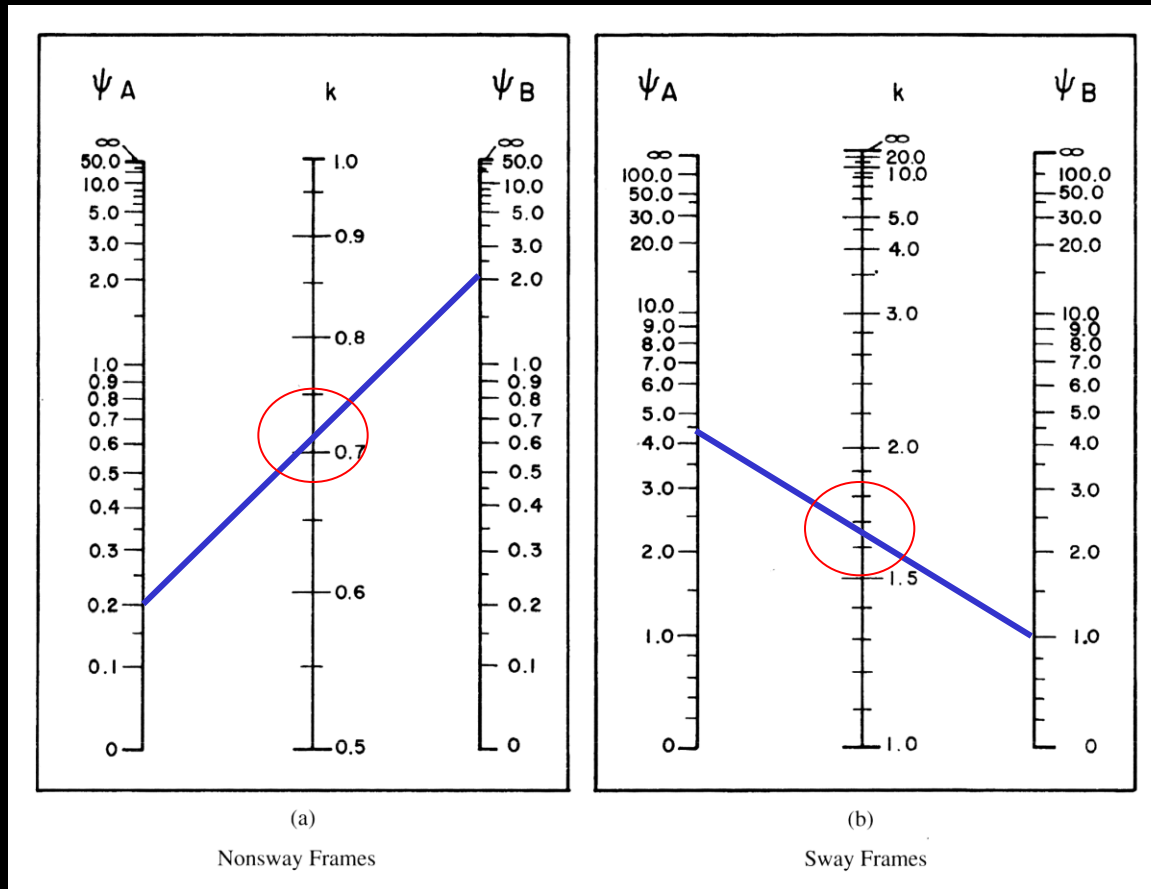
$$G = \Psi = \frac{\sum EI / l_c}{\sum EI / l_b}$$

- for the joint

- $l_c$  is the column length of each column
- $l_b$  is the beam length of each beam
- measured center to center

# Rigid Frame Design

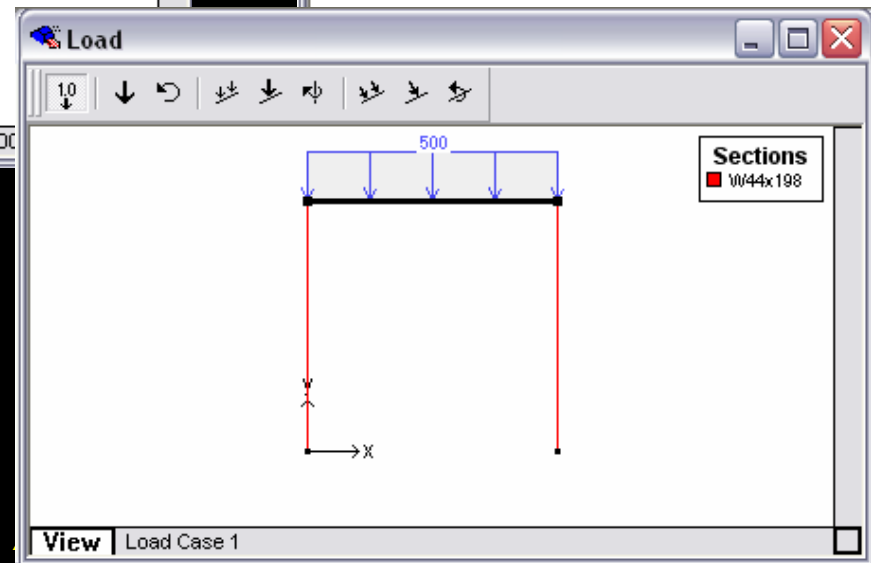
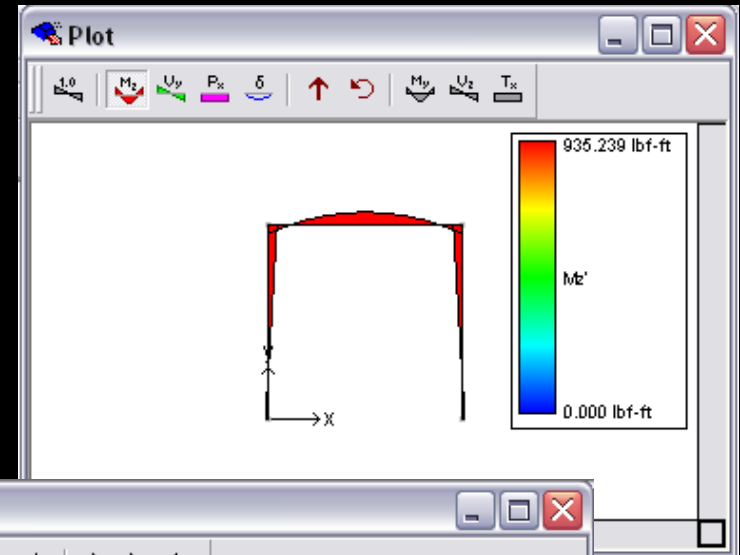
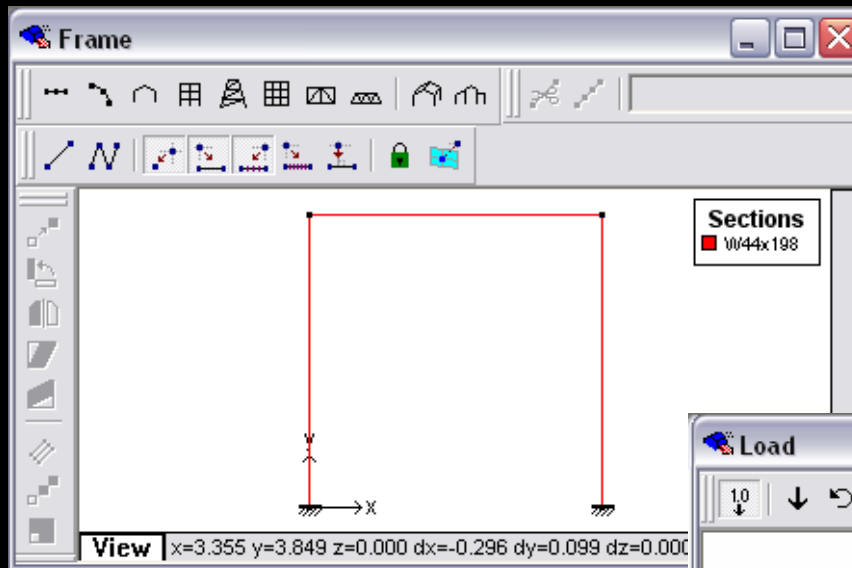
- column effective length,  $k$





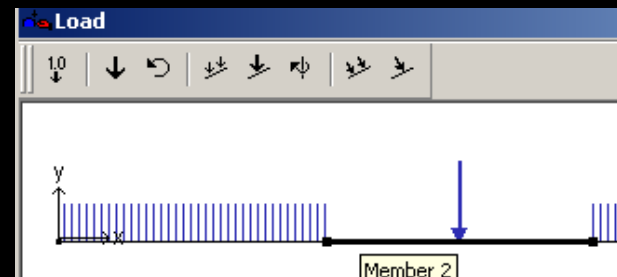
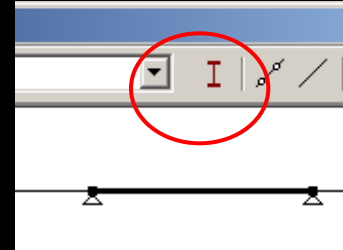
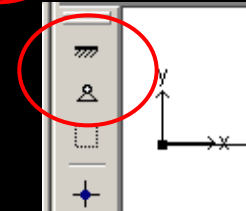
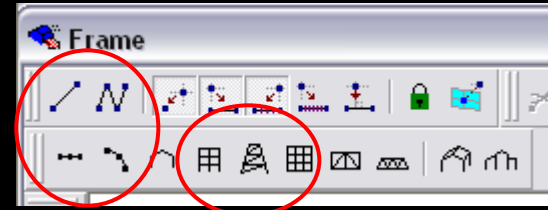
# Tools – Multiframe

- *in computer lab*



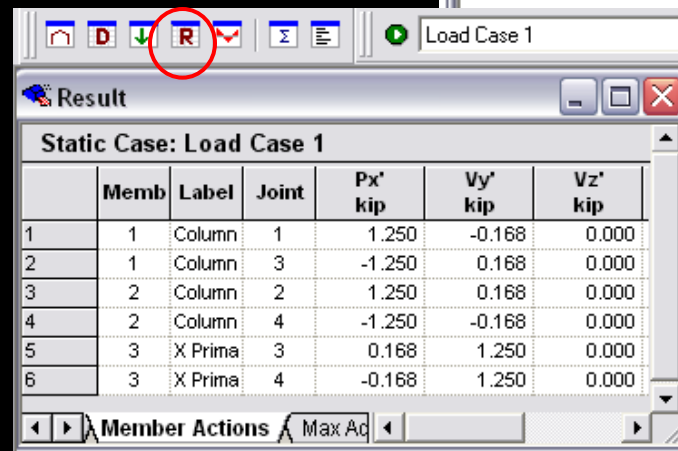
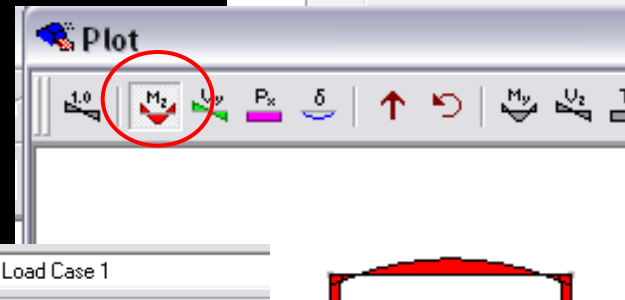
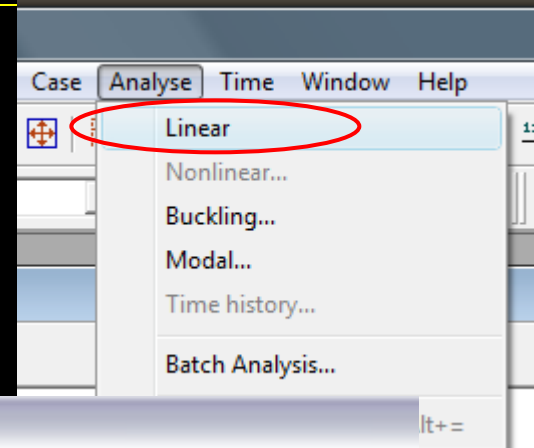
# Tools – Multiframe

- *frame window*
  - *define frame members*
    - *or pre-defined frame*
  - *select points, assign supports*
  - *select members, assign section*
  - *load window*
  - *select point or member, add point or distributed loads*



# Tools – Multiframe

- *to run analysis choose*
  - *Analyze menu*
    - *Linear*
- *plot*
  - *choose options*
- *results*
  - *choose options*



A screenshot of the 'Result' window in a software application. The window title is 'Load Case 1'. The toolbar has icons for 'D', 'R', and 'Σ'. The 'R' icon is circled in red. Below the toolbar is a table with the following data:

Static Case: Load Case 1						
	Memb	Label	Joint	Px' kip	Vy' kip	Vz' kip
1	1	Column	1	1.250	-0.168	0.000
2	1	Column	3	-1.250	0.168	0.000
3	2	Column	2	1.250	0.168	0.000
4	2	Column	4	-1.250	-0.168	0.000
5	3	X Prima	3	0.168	1.250	0.000
6	3	X Prima	4	-0.168	1.250	0.000