

Columns and Stability

Notation:

A = name for area $A36$ = designation of steel grade b = name for width C = symbol for compression C_c = column slenderness classification constant for steel column design d = name for dimension, or depth e = eccentric distance of application of a force (P) from the centroid of a cross section E = modulus of elasticity or Young's modulus f_a = axial stress f_b = bending stress $f_{critical}$ = critical buckling stress in column calculations from $P_{critical}$ f_x = total stress in the x axis direction F_a = allowable axial stress F_b = allowable bending stress F_y = yield stress I = moment of inertia I_{min} = moment of inertia that is critical to the calculation of slenderness ratio	K = effective length factor for columns L = name for length L_e = effective length that can buckle for column design, as is ℓ_e , $L_{effective}$ M = internal bending moment, as is M' $N.A.$ = shorthand for neutral axis P = name for axial force vector, as is P' P_{crit} = critical buckling load in column calculations, as is $P_{critical}$, P_{cr} r = radius of gyration T = symbol for compression W = designation for wide flange section y = vertical distance z = distance perpendicular to the x - y plane Δ = calculus symbol for small quantity = displacement due to bending θ = angle ϕ = diameter symbol π = pi (3.1415 radians or 180°) σ = engineering symbol for normal stress
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Design Criteria

Including strength (stresses) and servicability (including deflections), another requirement is that the structure or structural member be *stable*.

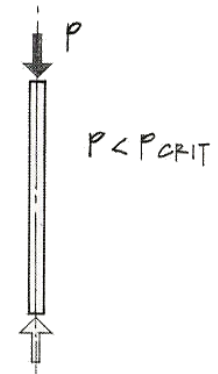
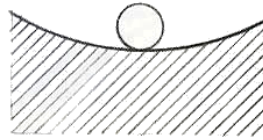
Stability is the ability of the structure to support a specified load without undergoing unacceptable (or sudden) deformations.

Physics

Recall that things like to be or *prefer* to be in their lowest energy state (potential energy). Examples include water in a water tank. The energy it took to put the water up there is stored until it is released and can flow due to gravity.

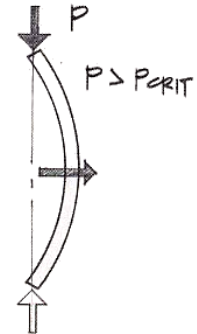
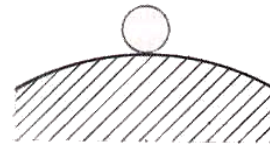
Stable Equilibrium

When energy is added to an object in the form of a push or disturbance, the object will return to it's original position. *Things don't change in the end.*



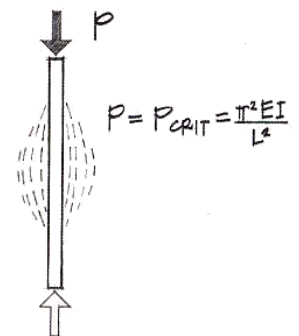
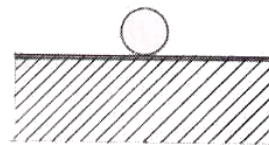
Unstable Equilibrium

When energy is added to an object, the object will move and get more "disturbed". *Things change rapidly.*



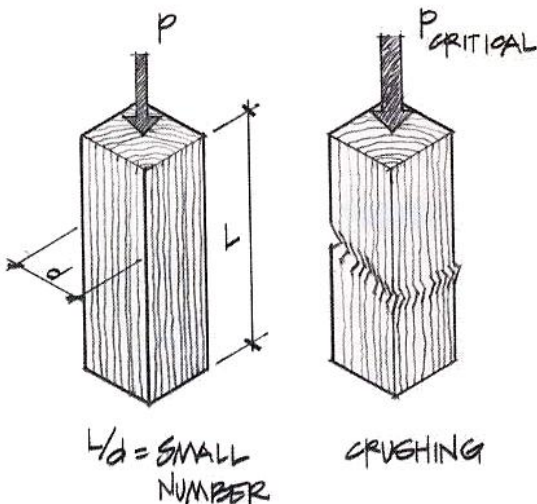
Neutral Equilibrium

When energy is added to an object, the object will move some then stop.. *Things change.*



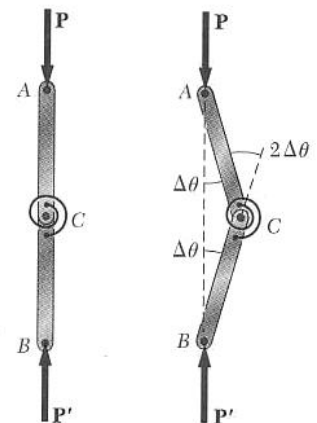
Column with Axial Loading

A column loaded centrally can experience unstable equilibrium, called *buckling*, because of how tall and slender they are. This instability is sudden and not good.



Buckling can occur in sheets (like my "memory metal" cookie sheet), pressure vessels or slender (narrow) beams not braced laterally.

Buckling can be thought of with the loads and motion of a column having a stiff spring at mid-height. There exists a load where the spring can't resist the moment in it any longer.

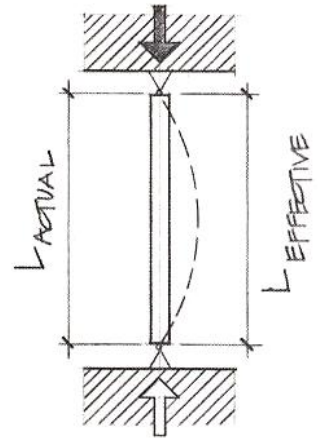


Short (stubby) columns will experience crushing before buckling.

Critical Buckling Load

The critical axial load to cause buckling is related to the deflected shape we could get (or determine from bending moment of $P \cdot \Delta$).

The buckled shape will be in the form of a *sine wave*.



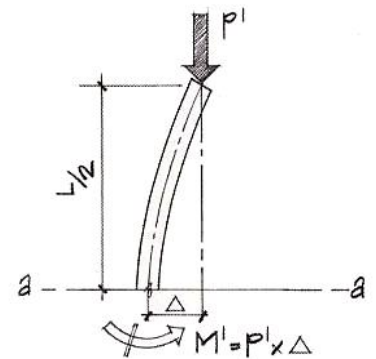
Euler Formula

Swiss mathematician Euler determined the relationship between the critical buckling load, the material, section and effective length (as long as the material stays in the elastic range):

$$P_{critical} = \frac{\pi^2 EI_{min}}{(L)^2} \quad \text{or} \quad P_{cr} = \frac{\pi^2 EI}{(L_e)^2} = \frac{\pi^2 EA}{\left(\frac{L_e}{r}\right)^2}$$

and the critical stress (if less than the normal stress) is:

$$f_{critical} = \frac{P_{critical}}{A} = \frac{\pi^2 EA r^2}{A(L_e)^2} = \frac{\pi^2 E}{\left(\frac{L_e}{r}\right)^2}$$



where $I=Ar^2$ and L_e/r is called the slenderness ratio. The smallest I of the section will govern.

Yield Stress and Buckling Stress

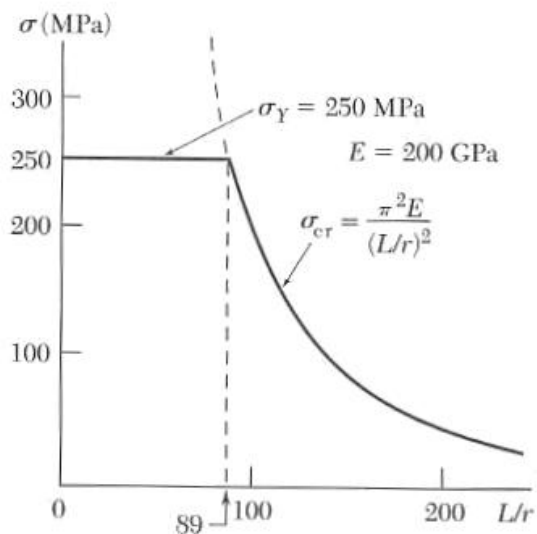
The two design criteria for columns are that they do not buckle and the strength is not exceeded. Depending on slenderness, one will control over the other.

But, because in the real world, things are rarely perfect – and columns will not actually be loaded concentrically, but will see eccentricity – Euler’s formula is used only if the critical stress is less than half of the yield point stress:

$$P_{critical} = \frac{\pi^2 EI_{min}}{(L)^2}; \quad f_{critical} = \frac{P_{critical}}{A} < \frac{F_y}{2}$$

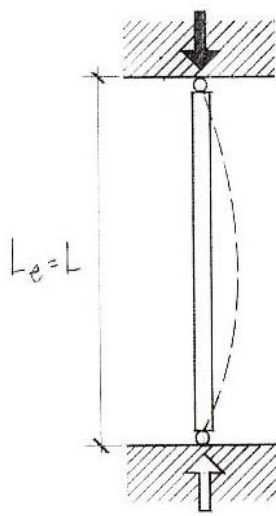
to be used for $\frac{L_e}{r} > C_c = \sqrt{\frac{2\pi^2 E}{F_y}}$

where C_c is the column slenderness classification constant and is the slenderness ratio of a column for which the critical stress is equal to half the yield point stress.

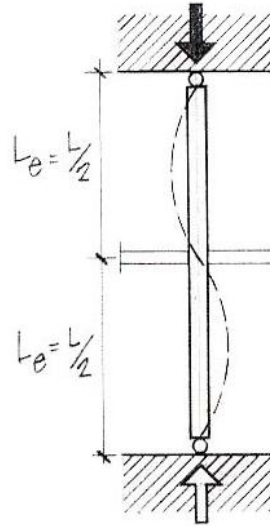


Effective Length and Bracing

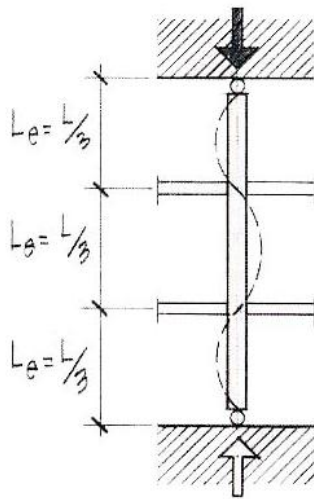
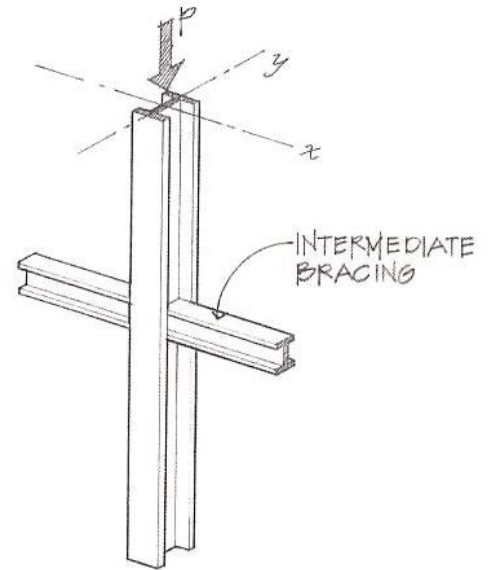
Depending on the end support conditions for a column, the effective length can be found from the deflected shape (elastic equations). If a very long column is braced intermittently along its length, the column length that will buckle can be determined. The effective length can be found by multiplying the column length by an effective length factor, K . $L_e = K \cdot L$



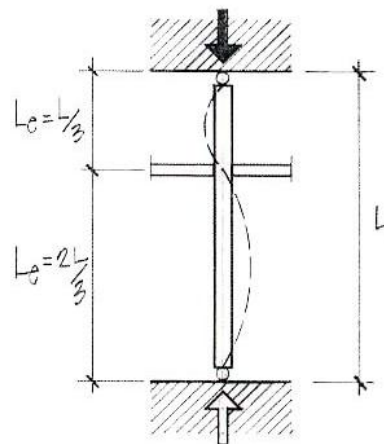
(a) No bracing.



(b) Braced at midpoint.



(c) Third-point bracing.



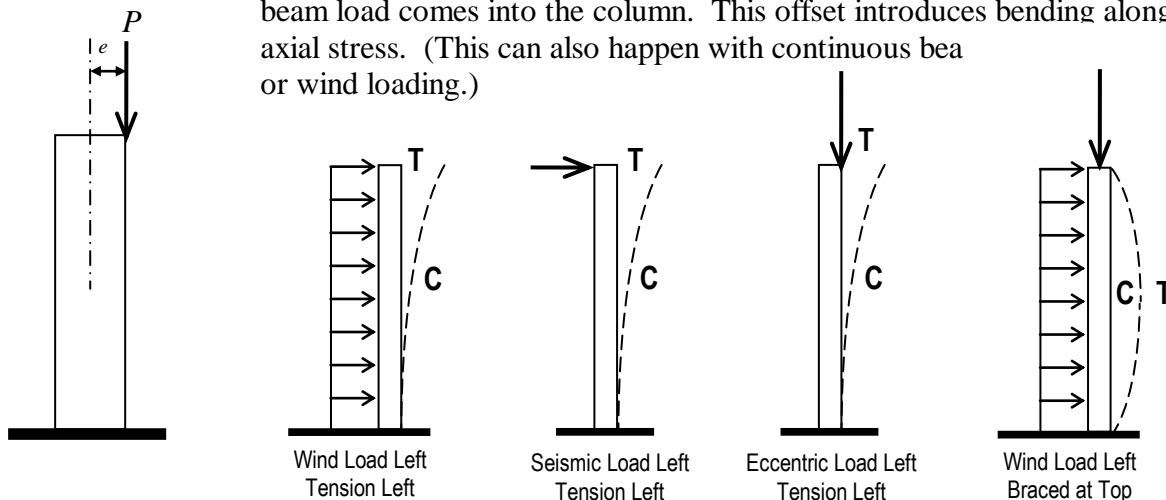
(d) Asymmetric bracing.

Buckled shape of column shown by dashed line	(a)	(b)	(c)	(d)	(e)	(f)
	0.5	0.7	1.0	1.0	2.0	2.0
	0.65	0.80	1.0	1.2	2.10	2.0
Theoretical <i>K</i> value						
Recommended design values when ideal conditions are approximated						
End conditions code	<ul style="list-style-type: none"> Rotation fixed, Translation fixed Rotation free, Translation fixed Rotation fixed, Translation free Rotation free, Translation free 					

Loading Location

Centric loading: The load is applied at the centroid of the cross section. The limiting allowable stress is determined from strength (P/A) or buckling.

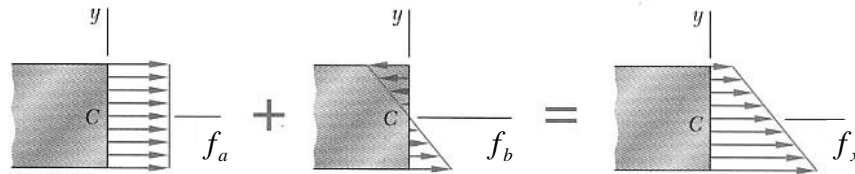
Eccentric loading: The load is offset from the centroid of the cross section because of how the beam load comes into the column. This offset introduces bending along with axial stress. (This can also happen with continuous beam or wind loading.)



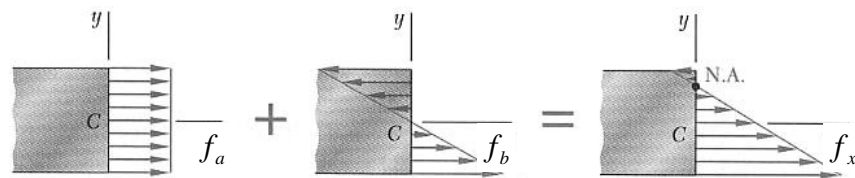
Eccentric Loading

The eccentricity causes bending stresses by a moment of value $P \times e$. Within the elastic range (linear stresses) we can *superposition* or add up the normal and bending stresses:

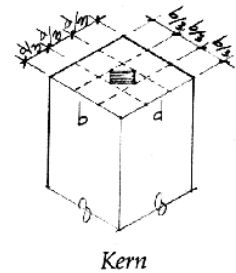
$$f_x = f_a + f_b = \frac{P}{A} + \frac{My}{I}$$



The resulting stress distribution is still *linear*. And the n.a. *moves* (if there is one).



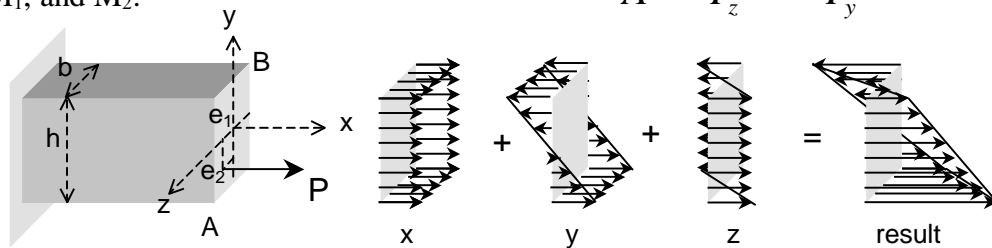
The value of e (or location of P) that causes the stress at an edge to become zero is at the edge of the **kern**. As long as P stays within the kern, there will *not* be any tension stress.



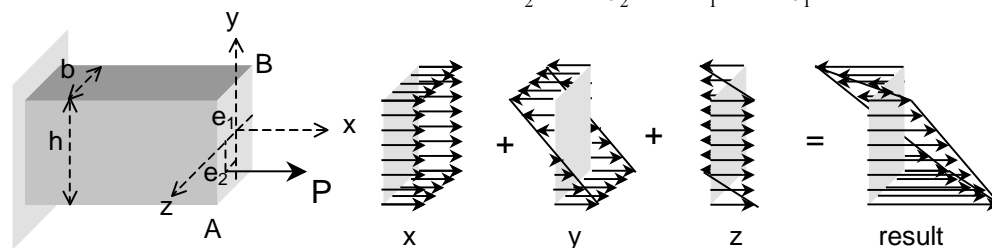
If there is bending in two directions (**bi-axial** bending), there will be one more bending stress added to the total:

$$f_x = f_a + f_{bx} + f_{by} = \frac{P}{A} + \frac{M_1 y}{I_z} + \frac{M_2 z}{I_y}$$

With P , M_1 , and M_2 :



$$M_2 = P \cdot e_2 \quad M_1 = P \cdot e_1$$

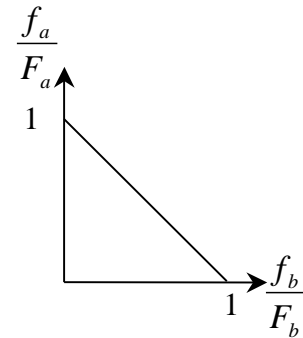


$$M_2 = P \cdot e_2 \quad M_1 = P \cdot e_1$$

Eccentric Loading Design

Because there are combined stresses, we can't just compare the axial stress to a limit axial stress or a bending stress to a limit bending stress. We use a limit called the **interaction diagram**. The diagram can be simplified as a straight line from the ratio of axial stress to allowable stress = 1 (no bending) to the ratio of bending stress to allowable stress = 1 (no axial load).

The interaction diagram can be more sophisticated (represented by a curve instead of a straight line). These type of diagrams take the effect of the bending moment increasing because the beam deflects. This is called the **P-Δ (P-delta)** effect.

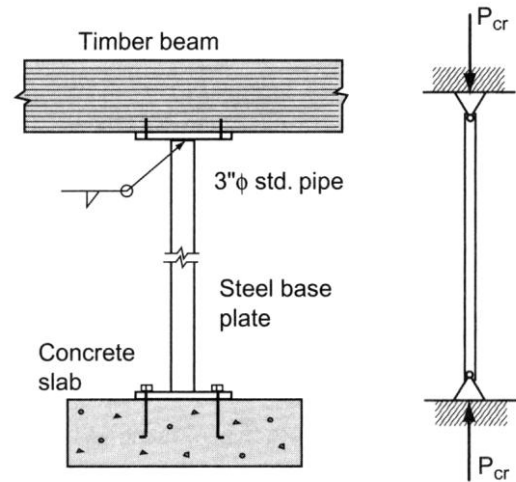


Limit Criteria Methods

- 1) $\frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1.0$ interaction formula (bending in one direction)
- 2) $\frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0$ interaction formula (biaxial bending)
- 3) $\frac{f_a}{F_a} + \frac{f_b \times (\text{Magnification factor})}{F_b} \leq 1.0$ interaction formula (P-Δ effect)

Example 1 (pg 346)**Example Problem 10.1: Short and Long Columns—
Modes of Failure (Figures 10.11 and 10.12)**

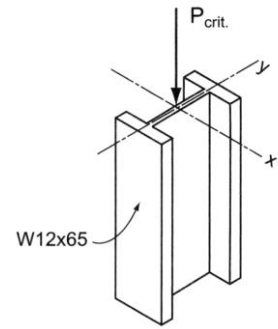
Determine the critical buckling load for a 3" ϕ standard weight steel pipe column that is 16 ft. tall and pin connected. Assume that $E = 29 \times 10^6$ psi



Example 2 (pg 346)**Example Problem 10.2 (Figure 10.13)**

Determine the critical buckling stress for a 30-foot-long, W12x65 steel column. Assume simple pin connections at the top and bottom.

$$F_y = 36 \text{ ksi (A36 steel)}; \quad E = 29 \times 10^3 \text{ ksi}$$



Example 3 (pg357)

Example Problem 10.8 (Figures 10.33 and 10.34a, b)

Determine the buckling load capacity of a 2×4 stud 12 feet high if blocking is provided at midheight. Assume $E = 1.2 \times 10^6$ psi.

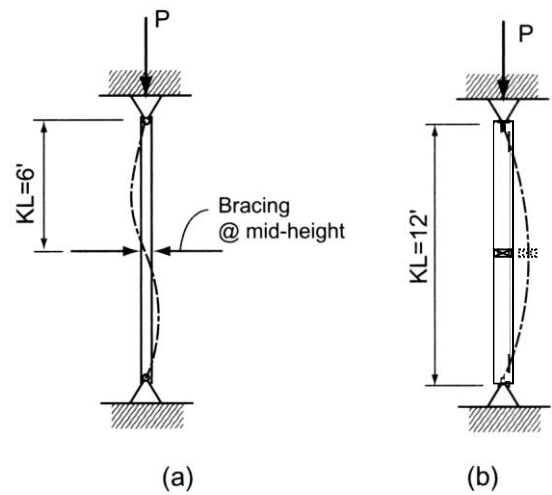
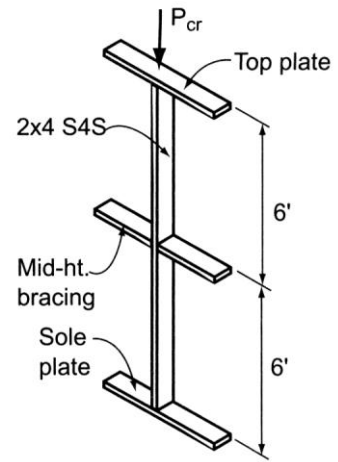


Figure 10.34 (a) Weak axis. (b) Strong axis.