



beam sections geometric properties

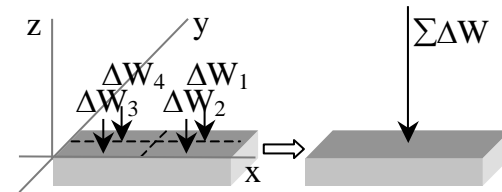
Sections 1
Lecture 9

Architectural Structures
ARCH 331

F2009abn

Center of Gravity

- location of equivalent weight
- determined with calculus



- sum element weights $W = \int dW$

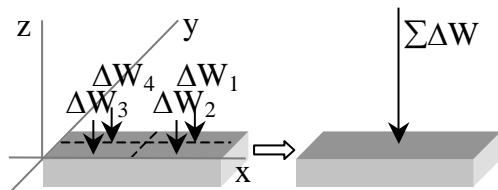
Sections 2
Lecture 9

Foundations Structures
ARCH 331

F2008abn

Center of Gravity

- “average” x & y from moment



$$\sum M_y = \sum_{i=1}^n x_i \Delta W_i = \bar{x} W \Rightarrow \bar{x} = \frac{\sum(x \Delta W)}{W}$$

$$\sum M_x = \sum_{i=1}^n y_i \Delta W_i = \bar{y} W \Rightarrow \bar{y} = \frac{\sum(y \Delta W)}{W}$$

“bar” means average

Centroid

- “average” x & y of an area
- for a volume of constant thickness
 - $\Delta W = \gamma \Delta A$ where γ is weight/volume
 - center of gravity = centroid of area

$$\bar{x} = \frac{\sum(x \Delta A)}{A}$$

$$\bar{y} = \frac{\sum(y \Delta A)}{A}$$



Sections 3
Lecture 9

Foundations Structures
ARCH 331

F2008abn

Sections 4
Lecture 9

Foundations Structures
ARCH 331

F2008abn

Centroid

- for a line, sum up length

$$\bar{x} = \frac{\sum(x\Delta L)}{L}$$

$$\bar{y} = \frac{\sum(y\Delta L)}{L}$$



Sections 5
Lecture 9

Foundations Structures
ARCH 331

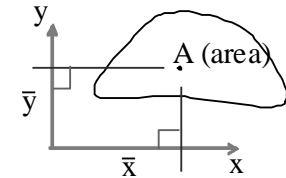
F2008abn

1st Moment Area

- math concept
- the moment of an area about an axis

$$Q_x = \bar{y}A$$

$$Q_y = \bar{x}A$$



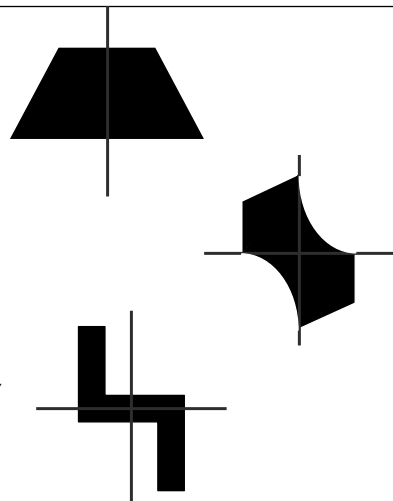
Sections 6
Lecture 9

Foundations Structures
ARCH 331

F2008abn

Symmetric Areas

- symmetric about an axis
- symmetric about a center point
- mirrored symmetry



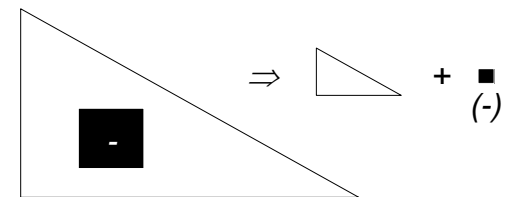
Sections 7
Lecture 9

Foundations Structures
ARCH 331

F2008abn

Composite Areas

- made up of basic shapes
- areas can be negative
- (centroids can be negative for any area)



Sections 8
Lecture 9

Foundations Structures
ARCH 331

F2008abn

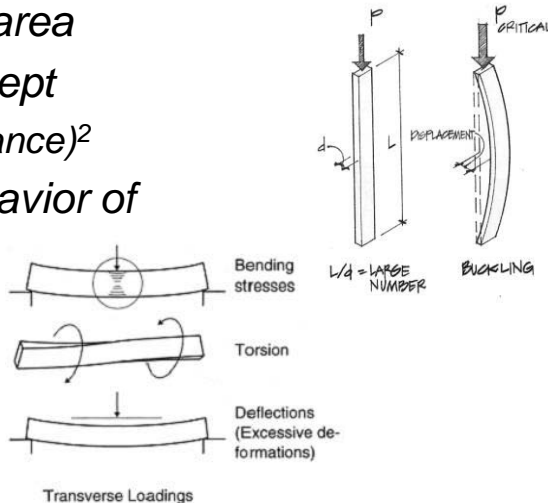
Basic Procedure

1. Draw reference origin (if not given)
2. Divide into basic shapes (+/-)
3. Label shapes
4. Draw table
5. Fill in table
6. Sum necessary columns
7. Calculate \hat{x} and \hat{y}

Component	Area	\bar{x}	$\bar{x}A$	\bar{y}	$\bar{y}A$
Σ					

Moments of Inertia

- 2nd moment area
 - math concept
 - area x (distance)²
- need for behavior of
 - beams
 - columns



Area Centroids

- Table 7.1 – pg. 242

Centroids of Common Shapes of Areas and Lines

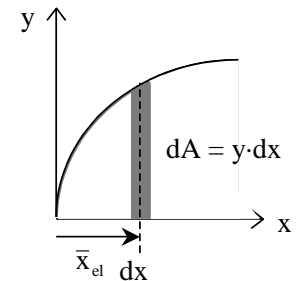
Shape		\bar{x}	\bar{y}
Triangular area		$\frac{b}{3}$	$\frac{h}{3}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$
Semicircular area		0	$\frac{4r}{3\pi}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$
Parabolic area		0	$\frac{3h}{5}$

Moment of Inertia

- about any reference axis
- can be negative

$$I_y = \int x^2 dA$$

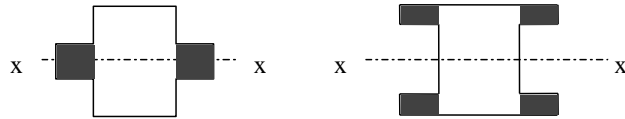
$$I_x = \int y^2 dA$$



- resistance to bending and buckling

Moment of Inertia

- same area moved away a distance
– larger I



Sections 13
Lecture 9

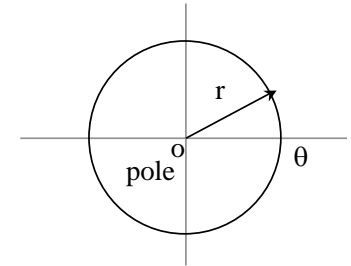
Foundations Structures
ARCH 331

F2008abn

Polar Moment of Inertia

- for roundish shapes
- uses polar coordinates (r and θ)
- resistance to twisting

$$J_o = \int r^2 dA$$



Sections 14
Lecture 9

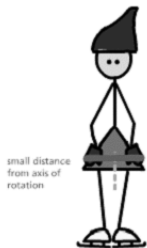
Foundations Structures
ARCH 331

F2008abn

Radius of Gyration

- measure of inertia with respect to area

$$r_x = \sqrt{\frac{I_x}{A}}$$



small distance
from axis of
rotation

When a figure skater changes position, he or she is redistributing his or her mass. Thus, every position has its own unique rotational inertia.



arms & hands
further
from axis
of rotation

The rotational inertia of the figure skater increases when her arms are raised because more of her mass is redistributed further from her axis of rotation.

Sections 15
Lecture 9

Foundations Structures
ARCH 331

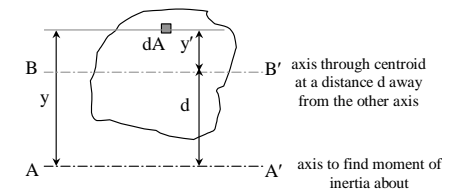
F2008abn

Parallel Axis Theorem

- can find composite I once composite centroid is known (basic shapes)

$$I_x = I_{cx} + Ad_y^2$$

$$= \bar{I}_x + Ad_y^2$$



$$I = \sum \bar{I} + \sum Ad^2$$

$$\bar{I} = I - Ad^2$$

Sections 16
Lecture 9

Foundations Structures
ARCH 331

F2008abn

Basic Procedure

1. Draw reference origin (if not given)
2. Divide into basic shapes (+/-)
3. Label shapes
4. Draw table with $A, \bar{x}, \bar{x}A, \bar{y}, \bar{y}A, \bar{I}'s, d's,$
and $Ad^2's$
5. Fill in table and get \hat{x} and \hat{y} for composite
6. Sum necessary columns
7. Sum $\bar{I}'s$ and $Ad^2's$

$$\begin{pmatrix} d_x = \hat{x} - \bar{x} \\ d_y = \hat{y} - \bar{y} \end{pmatrix}$$

Sections 17
Lecture 9

Foundations Structures
ARCH 331

F2008abn

Area Moments of Inertia

- Table 7.2 – pg. 252: (bars refer to centroid)

- x, y
- x', y'
- C

Rectangle		$\bar{I}_x = \frac{1}{12}bh^3$ $\bar{I}_y = \frac{1}{12}b^3h$ $I_x = \frac{1}{3}bh^3$ $I_y = \frac{1}{3}b^3h$ $J_C = \frac{1}{12}bh(b^2 + h^2)$
Triangle		$\bar{I}_x = \frac{1}{36}bh^3$ $I_x = \frac{1}{12}bh^3$
Circle		$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$ $J_O = \frac{1}{2}\pi r^4$
Semicircle		$I_x = I_y = \frac{1}{8}\pi r^4$ $J_O = \frac{1}{4}\pi r^4$
Quarter circle		$I_x = I_y = \frac{1}{16}\pi r^4$ $J_O = \frac{1}{4}\pi r^4$
Ellipse		$\bar{I}_x = \frac{1}{4}\pi ab^3$ $\bar{I}_y = \frac{1}{4}\pi a^3b$ $J_O = \frac{1}{4}\pi ab(a^2 + b^2)$

Sections 18
Lecture 9

F2008abn