ARCHITECTURAL STRUCTURES:
FORM, BEHAVIOR, AND DESIGN
ARCH 331
DR. ANNE NICHOLS
FALL 2013

lecture
SIX

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mechanics of materials

Mechanics of Materials

- MECHANICS

- MATERIALS

Mechanics of Materials

- *external loads and their effect on deformable bodies*
- *use it to answer question if structure meets requirements of*
  - *stability and equilibrium*
  - *strength and stiffness*
- *other principle building requirements*
  - *economy, functionality and aesthetics*

Knowledge Required

- *material properties*
- *member cross sections*
- *ability of a material to resist breaking*
- *structural elements that resist excessive*
  - *deflection*
  - *deformation*

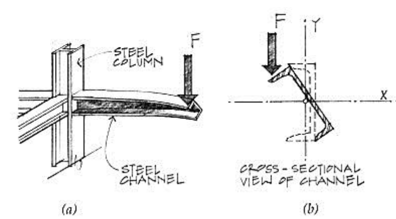
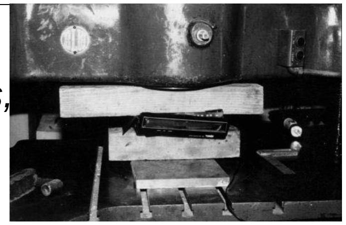


Figure 2.34 An example of torsion on a cantilever beam.

Problem Solving



1. STATICS:

   equilibrium of external forces, internal forces, stresses

2. GEOMETRY:

   cross section properties, deformations and conditions of geometric fit, strains

3. MATERIAL PROPERTIES:

   stress-strain relationship for each material obtained from testing

Stress

- stress is a term for the intensity of a force, like a pressure
- internal or applied
- force per unit area

$$stress = f = \frac{P}{A}$$

Design

- materials have a critical stress value where they could break or yield
  - ultimate stress
  - yield stress
  - compressive stress
  - fatigue strength
  - (creep & temperature)

acceptance vs. failure

Design (cont)

- we'd like

$$f_{actual} << F_{allowable}$$

- stress distribution may vary: average
- uniform distribution exists IF the member is loaded axially (concentric)

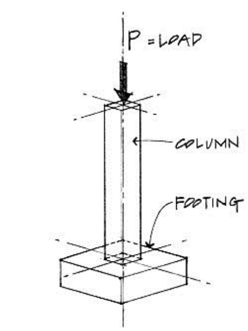
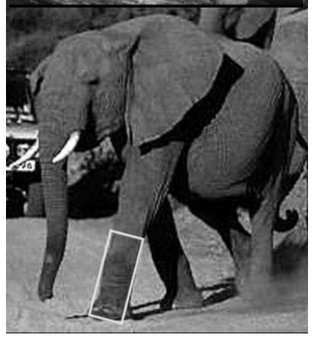


Figure 5.3 Centric loads.

Scale Effect

- *model scale*
  - *material weights by volume, small section areas*
- *structural scale*
  - *much more material weight, bigger section areas*
- *scale for strength is not proportional:*

$$\frac{\gamma L^3}{L^2} = \gamma L$$



Normal Stress (direct)

- *<u>normal</u> stress is normal to the cross section*
  - *stressed area is perpendicular to the load*

$$f_{t\ or\ c} = \frac{P}{A}$$
$$(\sigma)$$

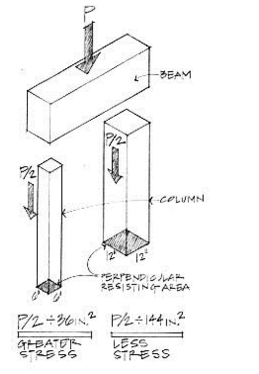


Figure 5.7 Two columns with the same load, different stress.

Shear Stress

- *stress parallel to a surface*

$$f_v = \frac{P}{A} = \frac{P}{td}$$
$$(\tau_{ave})$$

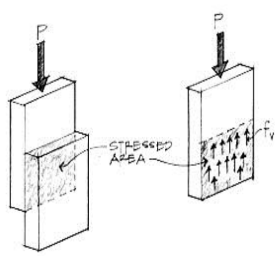


Figure 5.10 Shear stress between two glued blocks.

Bearing Stress

- *stress on a surface by <u>contact</u> in compression*

$$f_p = \frac{P}{A} = \frac{P}{td}$$
$$(\sigma)$$

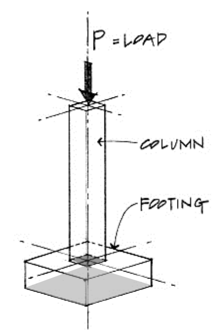


Figure 5.3 Centric loads.

Bending Stress

- *normal stress caused by bending*

$$f_b = \frac{Mc}{I} = \frac{M}{S}$$

$$(\sigma)$$

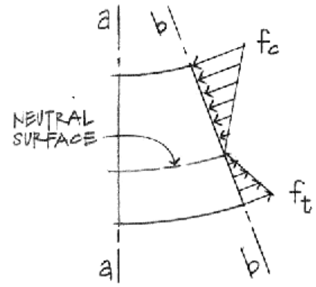


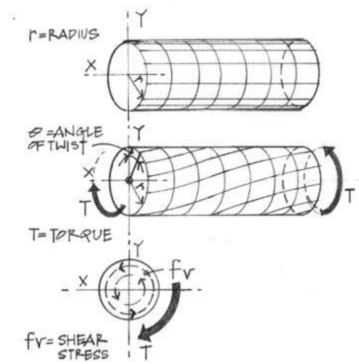
Figure 8.8 Bending stresses on section b-b.

Torsional Stress

- *shear stress caused by twisting*

$$f_v = \frac{T\rho}{J}$$

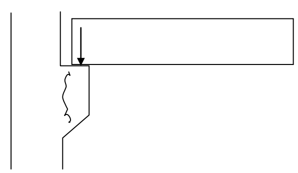
$$(\tau)$$

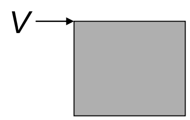


Structures and Shear

- *what structural elements see shear?*
  - *beams*
  - *bolts*
  - *splices*
  - *slabs*
  - *footings*
  - *walls*
    - *wind*
    - *seismic loads*

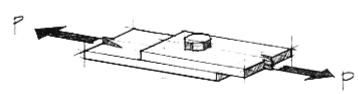
beams, bolts, splices: connections



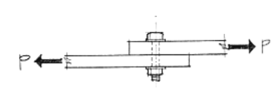


Bolts

- *connected members in tension cause shear stress*

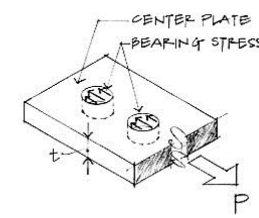


(a) Two steel plates bolted using one bolt.



(b) Elevation showing the bolt in

- *connected members in compression cause bearing stress*



Bearing stress on plate.

Single Shear

- seen when 2 members are connected

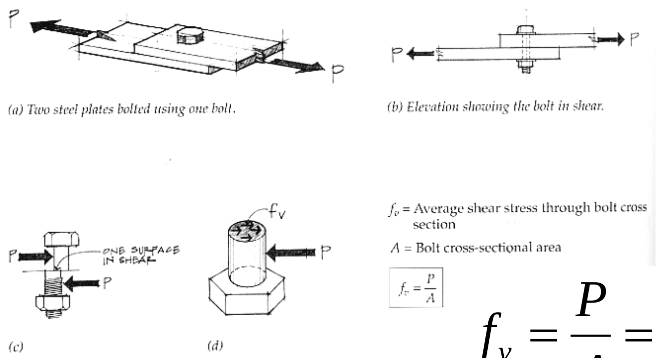


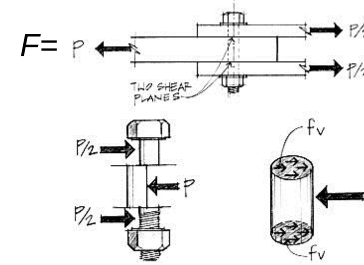
Figure 5.11 A bolted connection—single shear.

$$f_v = \frac{P}{A} = \frac{P}{\pi \, d^2/4}$$

Double Shear

- seen when 3 members are connected
- <u>two</u> areas

F=



Free-body diagram of middle section of the bolt in shear.

Figure 5.12 A bolted connection in double shear.

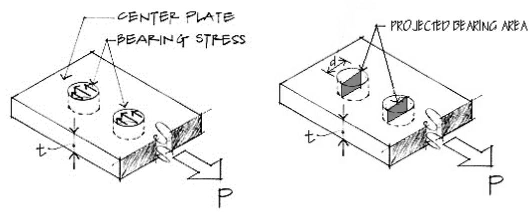
$f_v = \frac{P}{2A}$

(two shear planes)

$$f_v = \frac{P}{2A} = \frac{P/2}{A} = \frac{P/2}{\pi \, d^2/4}$$

Bolt Bearing Stress

- compression & contact
- projected area

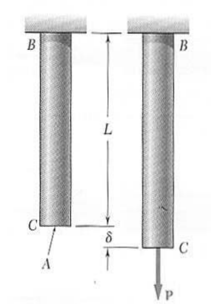


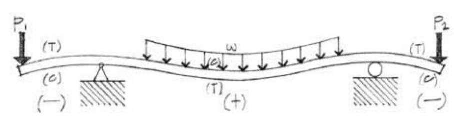
Bearing stress on plate.

$$f_p = \frac{P}{A_{projected}} = \frac{P}{td}$$

Strain

- materials deform
- axially loaded materials change length
- bending materials deflect



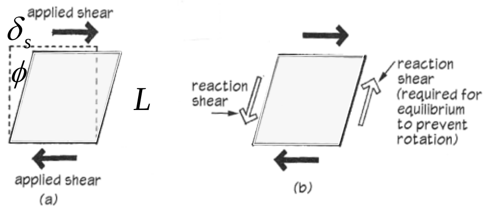


- STRAIN:
  - change in length over length + UNITLESS

$$strain = \varepsilon = \frac{\Delta L}{L}$$

Shearing Strain

- *deformations with shear*
- *parallelogram*
- *change in angles*
- *stress:* τ
- *strain:* γ
  - *unitless (radians)*

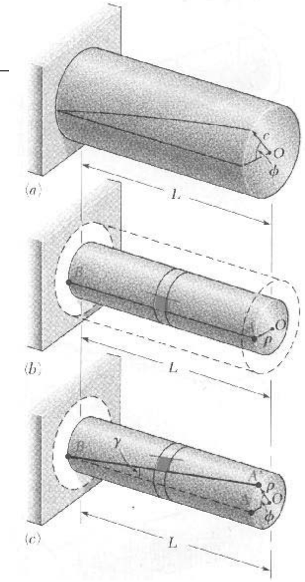


$$\gamma = \frac{\delta_s}{L} = \tan\phi \cong \phi$$

Shearing Strain

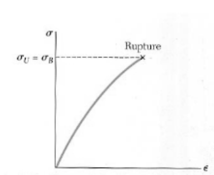
- *deformations with torsion*
- *twist*
- *change in angle of line*
- *stress:* τ
- *strain:* γ
  - *unitless (radians)*

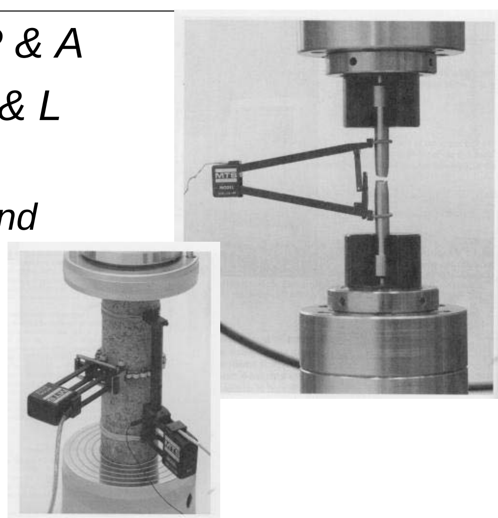
$$\gamma = \frac{\rho\phi}{L}$$



Load and Deformation

- *for stress, need P & A*
- *for strain, need* δ *& L*
  - *how?*
  - *TEST with load and measure*
  - *plot P/A vs.* ε





Material Behavior

- *every material has its own response*
  - *10,000 psi*
  - *L = 10 in*
  - *Douglas Fir vs. steel?*

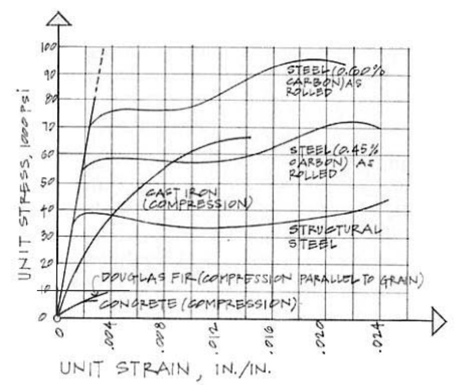


Figure 5.20 Stress-strain diagram for various materials.

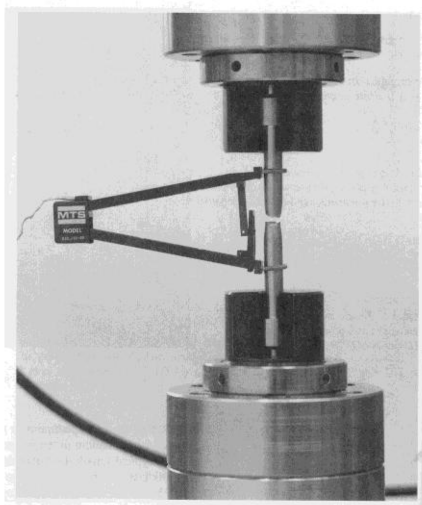
Behavior Types

- ductile - "necking"
- true stress

$$f = \frac{P}{A}$$

- engineering stress
  - (simplified)

$$f = \frac{P}{A_o}$$



Behavior Types

- brittle

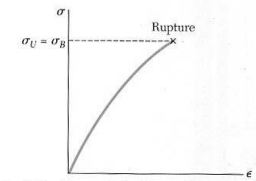


Fig. 2.11 Stress-strain diagram for a typical brittle material.

- semi-brittle

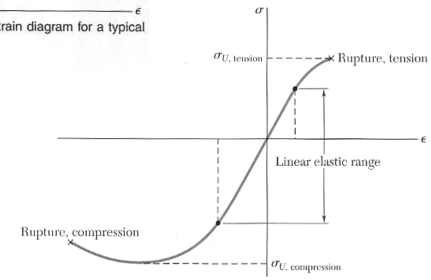


Fig. 2.14 Stress-strain diagram for concrete.

Stress to Strain

- important to us in f-ε diagrams:
  - straight section
  - LINEAR-ELASTIC
  - recovers shape (no permanent deformation)

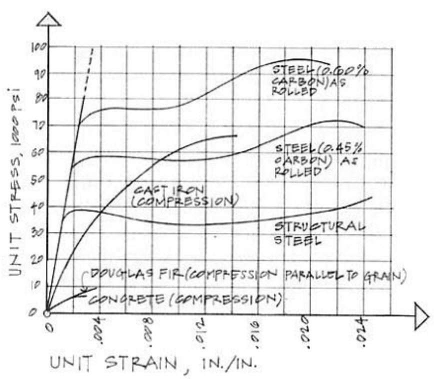
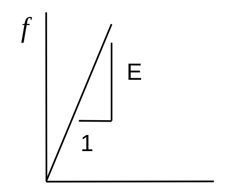


Figure 5.20 Stress-strain diagram for various materials.

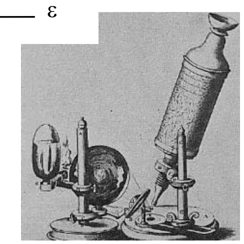
Hooke's Law

- straight line has constant slope
- Hooke's Law

$$f = E \cdot \varepsilon$$



- E
  - Modulus of elasticity
  - Young's modulus
  - units just like stress



Stiffness

- ability to resist strain
- steels
  - same E
  - different <u>yield points</u>
  - different <u>ultimate strength</u>

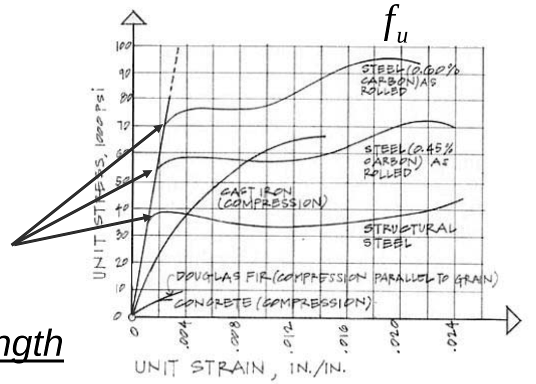
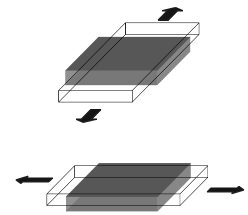


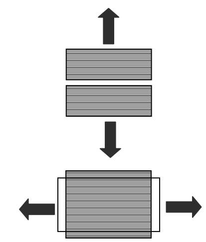
Figure 5.20 Stress-strain diagram for various materials.

Isotropy & Anisotropy

- ISOTROPIC
  - materials with E same at any direction of loading
  - ex. steel



- ANISOTROPIC
  - materials with different E at any direction of loading
  - ex. wood is <u>orthotropic</u>



Elastic, Plastic, Fatigue

- elastic springs back
- plastic has permanent deformation
- fatigue caused by reversed loading cycles

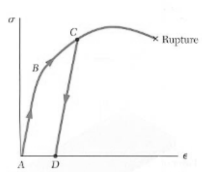


Fig. 2.18

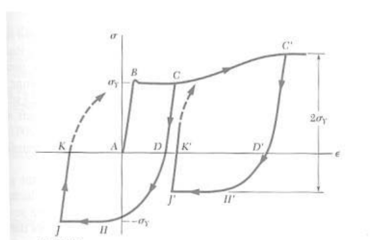


Fig. 2.20

Plastic Behavior

- ductile

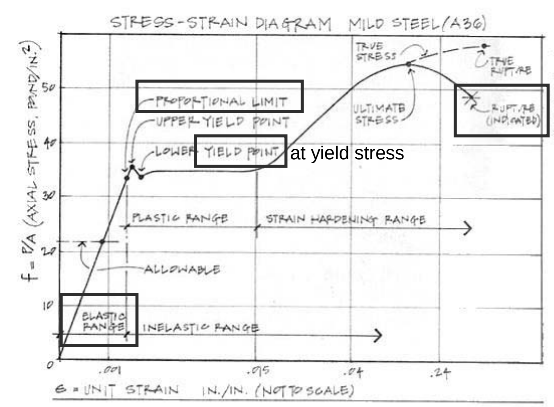


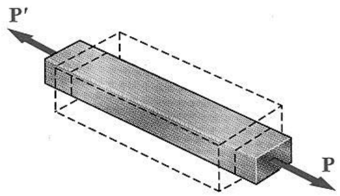
Figure 5.22 Stress-strain diagram for mild steel (A36) with key points highlighted.

Lateral Strain

- or "what happens to the cross section with axial stress"

$$\varepsilon_x = \frac{f_x}{E}$$

$$f_y = f_z = 0$$



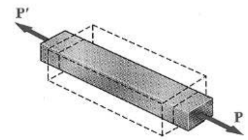
- strain in lateral direction
  – negative
  – equal for isometric materials $\quad \varepsilon_y = \varepsilon_z$

Poisson's Ratio

- constant relationship between longitudinal strain and lateral strain

$$\mu = -\frac{\text{lateral strain}}{\text{axial strain}} = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}$$

$$\varepsilon_y = \varepsilon_z = -\frac{\mu f_x}{E}$$



- sign! $\quad 0 < \mu < 0.5$

Calculating Strain

- from Hooke's law

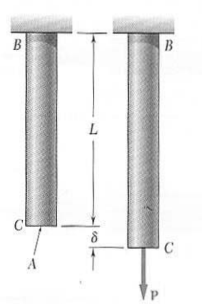
$$f = E \cdot \varepsilon$$

- substitute

$$\frac{P}{A} = E \cdot \frac{\delta}{L}$$

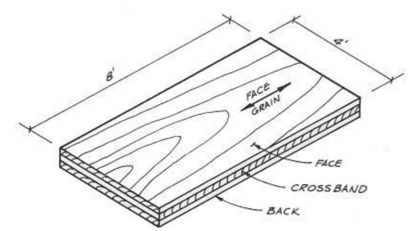
- get $\Rightarrow$

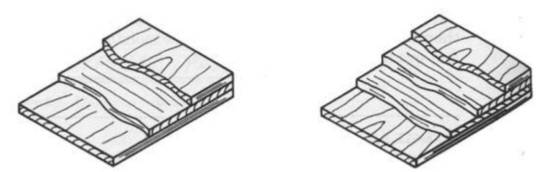
$$\delta = \frac{PL}{AE}$$



Orthotropic Materials

- non-isometric
- directional values of E and μ
- ex:
  – plywood
  – laminates
  – polymer composites



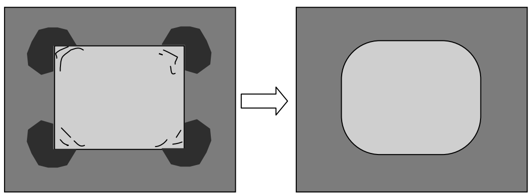


3 LAYER 3 PLY CONSTRUCTION

3 LAYER 4 PLY CONSTRUCTION

Stress Concentrations

- why we use f_{ave}
- increase in stress at changes in geometry
  - sharp notches
  - holes
  - corners



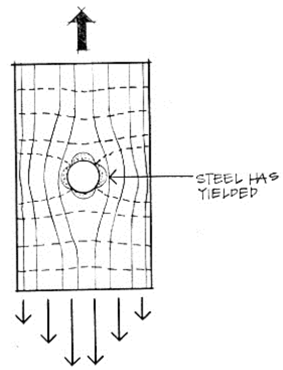
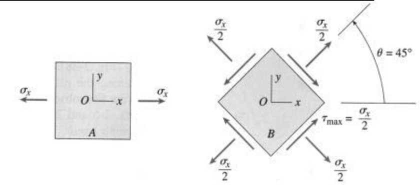


Figure 5.35 Stress trajectories around a hole.

Maximum Stresses

- if we need to know where max f and f_v happen:



$$\theta = 0^\circ \rightarrow \cos\theta = 1 \qquad f_{max} = \frac{P}{A_o}$$

$$\theta = 45^\circ \rightarrow \cos\theta = \sin\theta = \sqrt{0.5}$$

$$f_{v-max} = \frac{P}{2A_o} = \frac{f_{max}}{2}$$

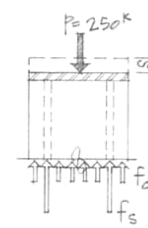
Maximum Stresses

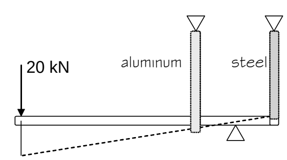
FIG. 2-37 Shear failure along a 45° plane of a wood block loaded in compression

FIG. 2-38 Slip bands (or Lüders' bands) in a polished steel specimen loaded in tension

Deformation Relationships

- physical movement
  - axially (same or zero)
  - rotations from axial changes

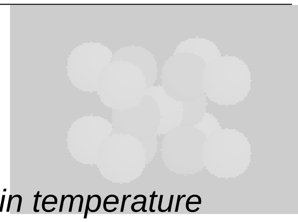


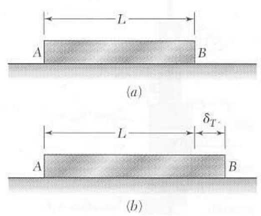


- $\delta = \dfrac{PL}{AE}$ relates δ to P

Deformations from Temperature

- atomic chemistry reacts to changes in energy
- solid materials
  - can contract with decrease in temperature
  - can expand with increase in temperature
- linear change can be measured per degree





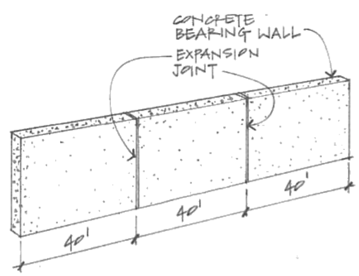
Thermal Deformation

- α - the rate of strain per degree
- UNITS : $\dfrac{}{°F}$, $\dfrac{}{°C}$
- length change: $\delta_T = \alpha(\Delta T)L$
- thermal strain: $\varepsilon_T = \alpha(\Delta T)$

  – <u>no stress</u> when movement allowed

Coefficients of Thermal Expansion

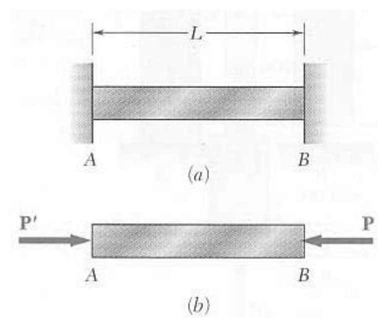
Material	Coefficients (α) [in./in./°F]
Wood	3.0×10^{-6}
Glass	4.4×10^{-6}
Concrete	5.5×10^{-6}
Cast Iron	5.9×10^{-6}
Steel	6.5×10^{-6}
Wrought Iron	6.7×10^{-6}
Copper	9.3×10^{-6}
Bronze	10.1×10^{-6}
Brass	10.4×10^{-6}
Aluminum	12.8×10^{-6}



Stresses and Thermal Strains

- if thermal movement is restrained <u>stresses</u> are induced

1. bar pushes on supports
2. support pushes back
3. reaction causes internal stress $f = \dfrac{P}{A} = \dfrac{\delta}{L}E$

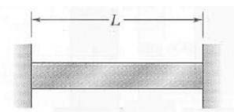


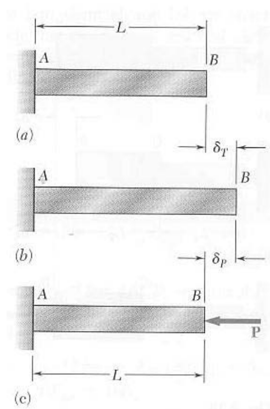
Superposition Method

– can remove a support to make it look determinant

– replace the support with a reaction

– enforce the geometry constraint





Superposition Method

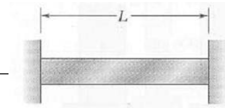
– total length change restrained to <u>zero</u>

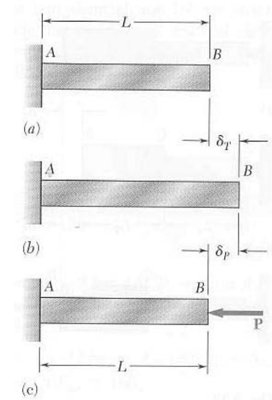
constraint: $\delta_P + \delta_T = 0$

$$\delta_p = -\frac{PL}{AE} \qquad \delta_T = \alpha(\Delta T)L$$

sub: $$-\frac{PL}{AE} + \alpha(\Delta T)L = 0$$

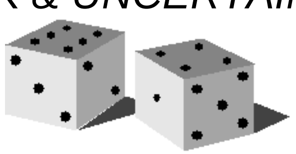
$$f = -\frac{P}{A} = -\alpha(\Delta T)E$$





Design of Members

- beyond allowable stress...
- materials aren't uniform 100% of the time
  - – ultimate strength or capacity to failure may be different and some strengths hard to test for
- RISK & UNCERTAINTY



$$f_u = \frac{P_u}{A}$$

Factor of Safety

- accommodate uncertainty with a safety factor:

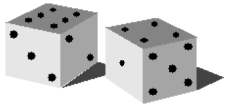
$$\text{allowable load} = \frac{\text{ultimate load}}{F.S}$$

- with linear relation between load and stress:

$$F.S = \frac{\text{ultimate load}}{\text{allowable load}} = \frac{\text{ultimate stress}}{\text{allowable stress}}$$

Load and Resistance Factor Design

- *loads on structures are*
  - *not constant*
  - *can be more influential on failure*
  - *happen more or less often*
  - *UNCERTAINTY*



$$R_u = \gamma_D R_D + \gamma_L R_L \leq \phi R_n$$

ϕ *- resistance factor*

γ *- load factor for (D)ead & (L)ive load*