ARCHITECTURAL STRUCTURES:
FORM, BEHAVIOR, AND DESIGN
ARCH 331
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FALL 2013

lecture
three

forces and moments

Structural Math

- *quantify environmental loads*
  - *how big is it?*
- *evaluate geometry and angles*
  - *where is it?*
  - *what is the scale?*
  - *what is the size in a particular direction?*
- *quantify what happens in the structure*
  - *how big are the internal forces?*
  - *how big should the beam be?*

Structural Math

- *physics takes observable phenomena and relates the measurement with rules: mathematical relationships*
- *need*
  - *reference frame*
  - *measure of length, mass, time, direction, velocity, acceleration, work, heat, electricity, light*
  - *calculations & geometry*

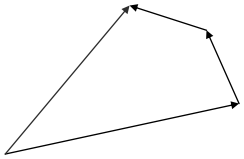
Physics for Structures

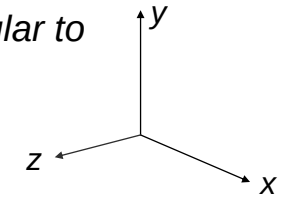
- *measures*
  - *US customary & SI*

Units	*US*	*SI*
Length	*in, ft, mi*	*mm, cm, m*
Volume	*gallon*	*liter*
Mass	*lb mass*	*g, kg*
Force	*lb force*	*N, kN*
Temperature	*F*	*C*

Physics for Structures

- *scalars – any quantity*
- *vectors - quantities with direction*
  - *like displacements*
  - *summation results in the "straight line path" from start to end*
  - *<u>normal</u> vector is perpendicular to something*





Language

- *symbols for operations: +,-, /, x*
- *symbols for relationships: (), =, <, >*
- *algorithms*
  - *cancellation*
  - *factors*
  - *signs*
  - *ratios and proportions*
  - *power of a number*
  - *conversions, ex. 1X = 10 Y*
  - *operations on <u>both sides</u> of equality*

$$\frac{2}{5}\times\frac{5}{6}=\frac{2}{6}=\frac{2}{2\times 3}=\frac{1}{3}$$

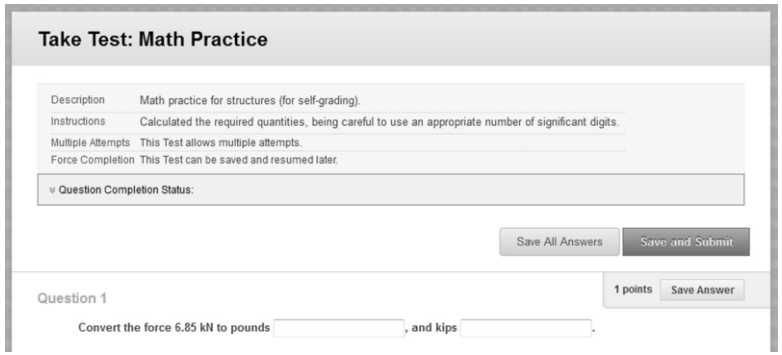
$$\frac{x}{6}=\frac{1}{3}$$

$$10^3=1000$$

$$\frac{10Y}{1X}\ or\ \frac{1X}{10Y}=1$$

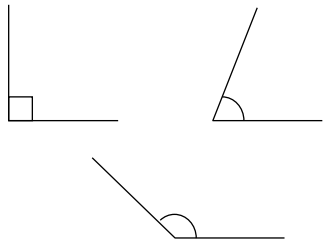
On-line Practice

- *eCampus / Study Aids*



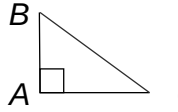
Geometry

- *angles*
  - *right = 90º*
  - *acute < 90º*
  - *obtuse > 90º*
  - *π = 180º*



- *triangles*
  - *area*
  - *hypotenuse*
  - *total of angles = 180º*

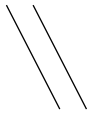
$$=\frac{b\times h}{2}$$



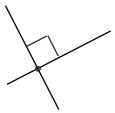
$$AB^2+AC^2=BC^2$$

Geometry

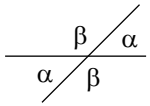
- *lines and relation to angles*
  - *parallel lines can't intersect*



  - *perpendicular lines cross at 90º*
  - *intersection of two lines is a point*

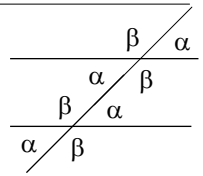


  - *opposite angles are equal when two lines cross*

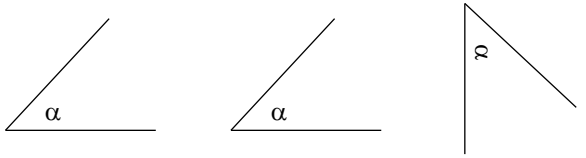


Geometry

  - *intersection of a line with parallel lines results in identical angles*



  - *two lines intersect in the same way, the angles are identical*

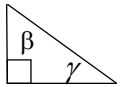


Geometry

  - *sides of two angles are parallel and intersect opposite way, the angles are <u>supplementary</u> - the sum is 180°*

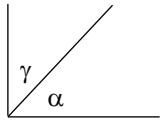
  - *two angles that sum to 90° are said to be <u>complimentary</u>*

$$\beta + \gamma = 90°$$



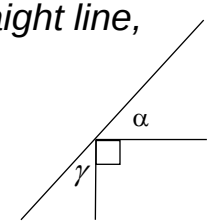
Geometry

  - *sides of two angles bisect a right angle (90°), the angles are <u>complimentary</u>*



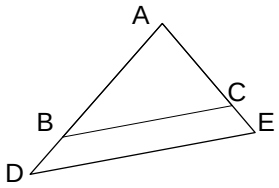
$$\alpha + \gamma = 90°$$

  - *right angle bisects a straight line, remaining angles are <u>complimentary</u>*

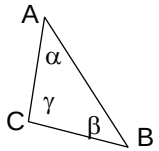


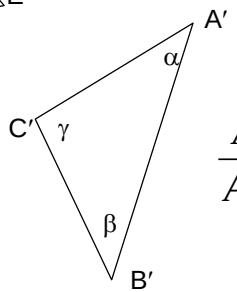
Geometry

– *similar triangles have proportional sides*



$$\frac{AB}{AD} = \frac{AC}{AE} = \frac{BC}{DE}$$

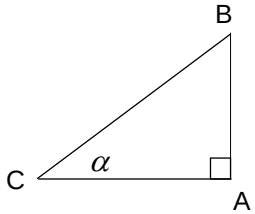




$$\frac{AB}{A'B'} = \frac{AC}{A'C'} = \frac{BC}{B'C'}$$

Trigonometry

- *for right triangles*



$$\sin = \frac{opposite\ side}{hypotenuse} = \sin\alpha = \frac{AB}{CB}$$

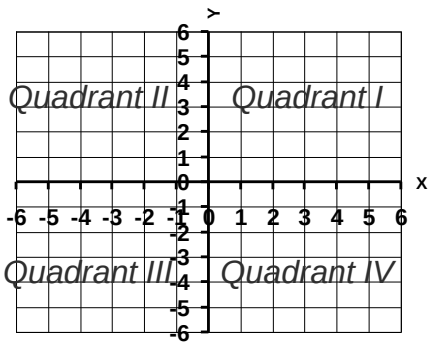
$$\cos = \frac{adjacent\ side}{hypotenuse} = \cos\alpha = \frac{AC}{CB}$$

$$\tan = \frac{opposite\ side}{adjacent\ side} = \tan\alpha = \frac{AB}{AC}$$

SOHCAHTOA

Trigonometry

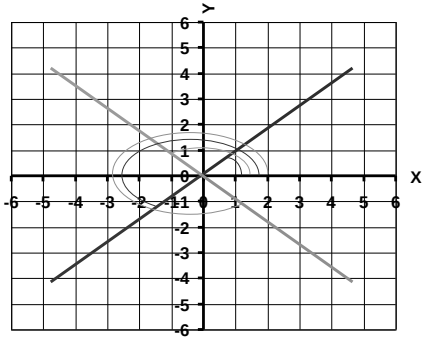
- *cartesian coordinate system*
  - *origin at 0,0*
  - *coordinates in (x,y) pairs*
  - *x & y have signs*



Trigonometry

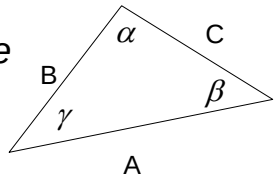
- *for angles starting at <u>positive x</u>*
  - sin *is y side*
  - cos *is x side*

sin<0 for 180-360°
cos<0 for 90-270°
tan<0 for 90-180°
tan<0 for 270-360°



Trigonometry

- *for all triangles*
  - *sides A, B & C are opposite angles α, β & γ*



  - *LAW of SINES*

$$\frac{\sin\alpha}{A} = \frac{\sin\beta}{B} = \frac{\sin\gamma}{C}$$

  - *LAW of COSINES*

$$A^2 = B^2 + C^2 - 2BC\cos\alpha$$

Algebra

- *equations (something = something)*
- *constants*
  - *real numbers or shown with a, b, c...*
- *unknown terms, variables*
  - *names like R, F, x, y*
- *linear equations*
  - *unknown terms have no exponents*
- *simultaneous equations*
  - *variable set satisfies <u>all equations</u>*

Algebra

- *solving one equation*
  - *only works with one variable*
  - *ex:* $2x-1=0$
    - *add to both sides* $2x-1+1=0+1$

$$2x=1$$

    - *divide both sides* $\frac{2x}{2}=\frac{1}{2}$
    - *get x by itself on a side* $x=\frac{1}{2}$

Algebra

- *solving one equations*
  - *only works with one variable*
  - *ex:* $2x-1=4x+5$
    - *subtract from both sides*

$$2x-1-2x=4x+5-2x$$

    - *subtract from both sides*

$$-1-5=2x+5-5$$

    - *divide both sides* $\frac{-6}{2}=\frac{-3\cdot 2}{2}=\frac{2x}{2}$
    - *get x by itself on a side*

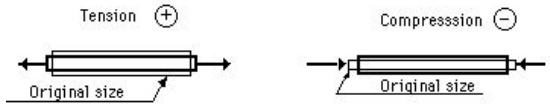
$$x=-3$$

Algebra

- *solving two equation*
  - *only works with two variables*
  - *ex:* $2x+3y=8$ / $12x-3y=6$
    - *look for term similarity*
    - *can we add or subtract to eliminate one term?*
    - *add* $2x+3y+12x-3y=8+6$
    - $14x=14$
    - *get x by itself on a side* $\frac{14x}{14}=\frac{14}{14}=x=1$

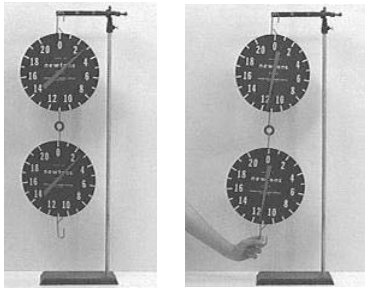
Forces

- *statics*
  - *physics of forces and reactions on bodies and systems*
  - *equilibrium (bodies at rest)*
- *forces*
  - *something that exerts on an object:*
    - *motion*
    - *tension*
    - *compression*



Force

- *"action of one body on another that affects the state of motion or rest of the body"*
- *Newton's 3rd law:*
  - *for every force of action there is an equal and opposite reaction along the same line*



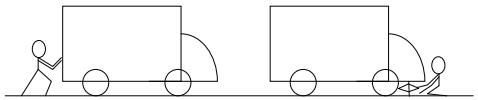
http://www.physics.umd.edu

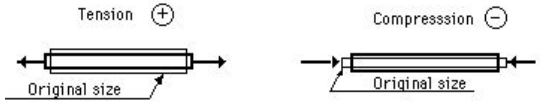
Force Characteristics

- *applied at a point*
- *magnitude*
  - *Imperial units: lb, k (kips)*
  - *SI units: N (newtons), kN*
- *direction*

Forces on Rigid Bodies

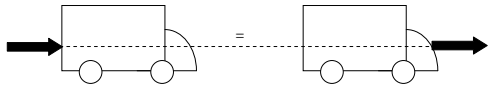
- *for statics, the bodies are ideally rigid*
- *can translate and rotate*
- *internal forces are*
  - *in bodies*
  - *between bodies (connections)*
- *external forces act on bodies*





Transmissibility

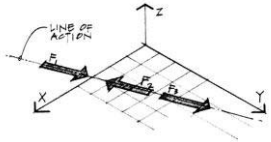
- *the force stays on the same line of action*
- *truck can't tell the difference*



- *only valid for EXTERNAL forces*

Force System Types

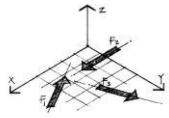
- *collinear*



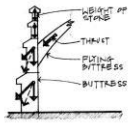
Collinear—*All forces acting along the same straight line.*
Figure 2.17(a) Particle or rigid body.

Force System Types

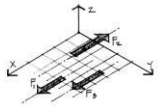
- *coplanar*



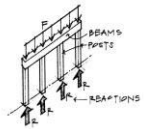
Coplanar—*All forces acting in the same plane.*
Figure 2.17(b) Rigid bodies.



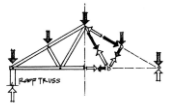
Forces in a buttress system.



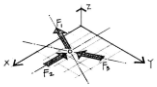
Coplanar, parallel—*All forces are parallel and act in the same plane.*
Figure 2.17(c) Rigid bodies.



A beam supported by a series of columns.



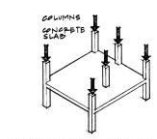
Loads applied to a roof truss.



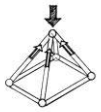
Coplanar, concurrent—*All forces intersect at a common point and lie in the same plane.*
Figure 2.17(d) Particle or rigid body.

Force System Types

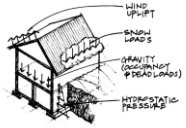
- *space*



Column loads in a concrete building.



One component of a three-dimensional space frame.



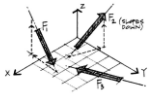
Array of forces acting simultaneously on a house.

Noncoplanar, parallel—All forces are parallel to each other, but not all lie in the same plane.

Figure 2.17(e) Rigid bodies.

Noncoplanar, concurrent—All forces intersect at a common point but do not all lie in the same plane.

Figure 2.17(f) Particle or rigid bodies.

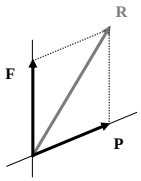


Noncoplanar, nonconcurrent—All forces are skewed.

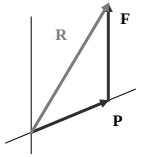
Figure 2.17(g) Rigid bodies.

Adding Vectors

- *graphically*
  - *parallelogram law*
    - *diagonal*
    - *long for 3 or more vectors*



  - *tip-to-tail*
    - *more convenient with lots of vectors*



Force Components

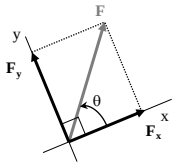
- *convenient to resolve into 2 vectors*
- *at right angles*
- *in a "nice" coordinate system*
- *θ is between F_x and F <u>from F_x</u>*

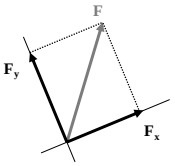
$$F_x = F\cos\theta$$

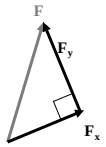
$$F_y = F\sin\theta$$

$$F = \sqrt{F_x^2 + F_y^2}$$

$$\tan\theta = \frac{F_y}{F_x}$$

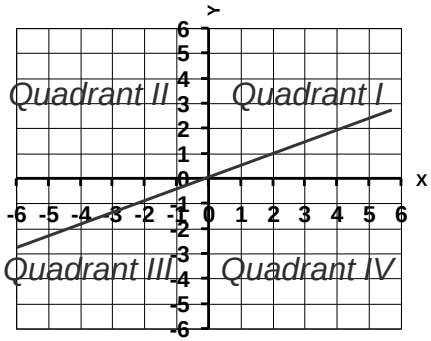






Trigonometry

- *F_x is negative*
  - *90° to 270°*
- *F_y is negative*
  - *180° to 360°*
- *tan is positive*
  - *quads I & III*
- *tan is negative*
  - *quads II & IV*

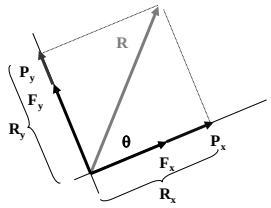


Component Addition

- *find all x components*
- *find all y components*
- *find sum of x components, R_x (resultant)*
- *find sum of y components, R_y*

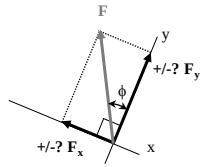
$$R = \sqrt{R_x^2 + R_y^2}$$

$$\tan\theta = \frac{R_y}{R_x}$$



Alternative Trig for Components

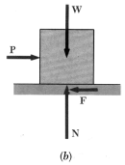
- *doesn't relate angle to axis direction*
- *ϕ is "small" angle between F and <u>EITHER F_x or F_y</u>*
- *no sign out of calculator!*
- *have to choose RIGHT trig function, resulting direction (sign) and component axis*

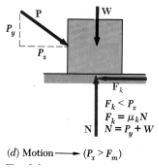


Friction

- *resistance to movement*
- *contact surfaces determine μ*
- *proportion of normal force ($\perp$)*
  - *opposite to slide direction*
  - *static > kinetic*

$$F = \mu N$$





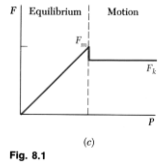
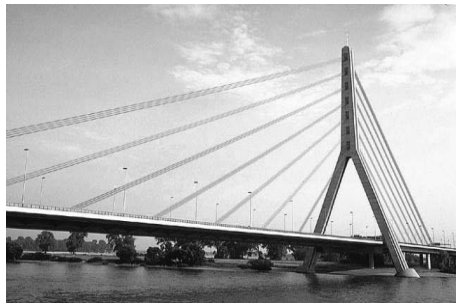


Fig. 8.1

Cables

- *simple*
- *uses*
  - *suspension bridges*
  - *roof structures*
  - *transmission lines*
  - *guy wires, etc.*



http:// nisee.berkeley.edu/godden

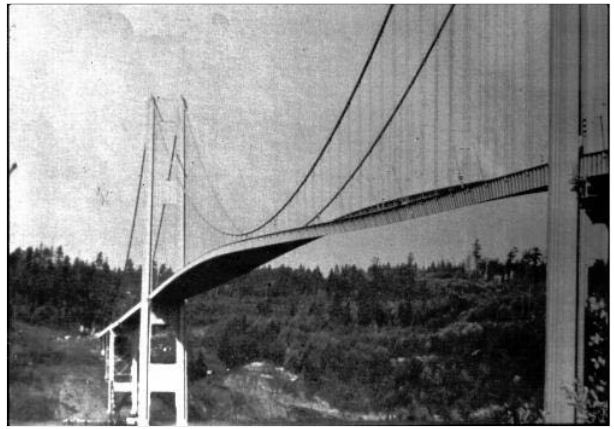
- *have same tension all along*
- *can't stand compression*

Cables Structures

- *use high-strength steel*
- *need*
  - *towers*
  - *anchors*
- *don't want movement*

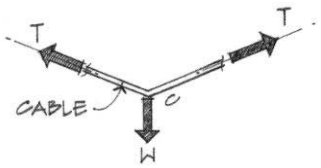
http:// nisee.berkeley.edu/godden

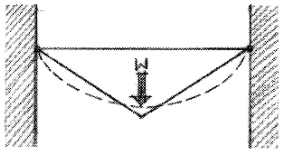
Cable Structures



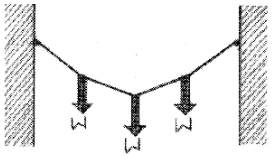
Cable Loads

- *straight line between forces*
- *with one force*
  - *concurrent*
  - *symmetric*





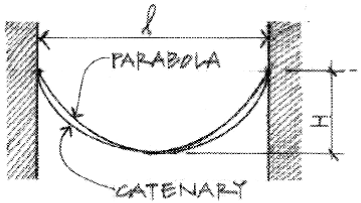
(a) Simple concentrated load—triangle.



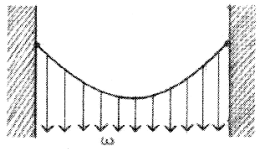
(b) Several concentrated loads—polygon.

Cable Loads

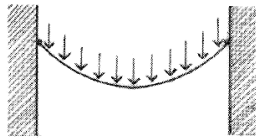
- *shape directly related to the distributed load*



(e) Comparison of a parabolic and a catenary curve.



(c) Uniform loads (horizontally)—parabola.



(d) Uniform loads (along the cable length)—catenary.

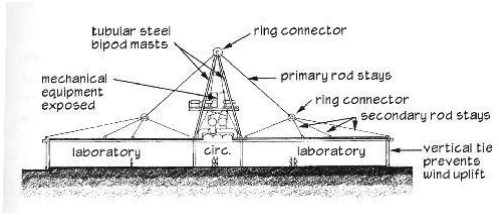
Cable-Stayed Structures

- *diagonal cables support horizontal spans*
- *typically symmetrical*
- *Patcenter, Rogers 1986*

www.columbia.edu

Patcenter, Rogers 1986

- *column free space*
- *roof suspended*
- *solid steel ties*
- *steel frame supports masts*



Patcenter, Rogers 1986

- *dashes – cables pulling*

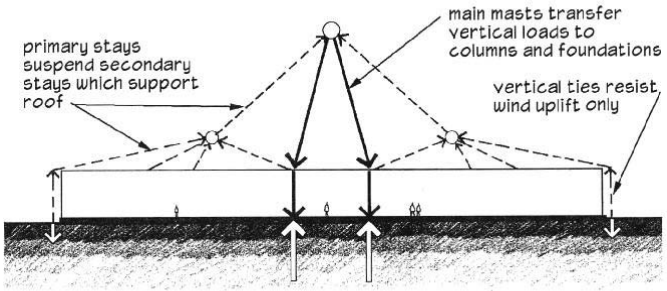
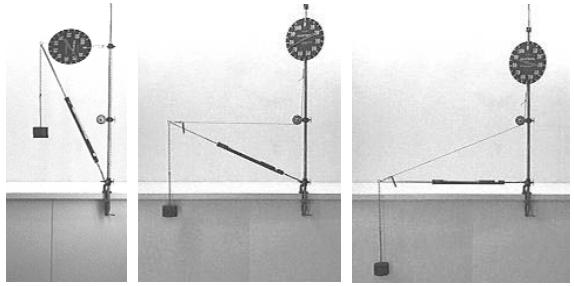


Figure 3.5: Patcenter, load path diagram.

Moments

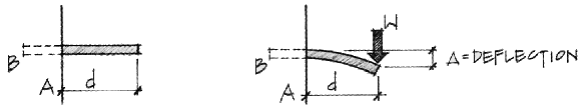
- *forces have the tendency to make a body rotate about an axis*



http://www.physics.umd.edu

– same translation but different rotation

Moments



(a) Unloaded. (b) Loaded.

Figure 2.33 Moment on a cantilever beam.

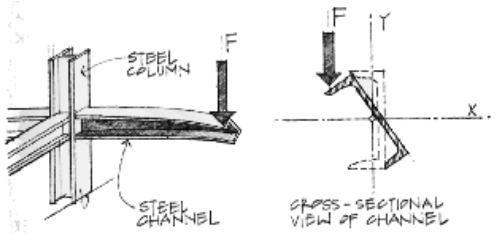
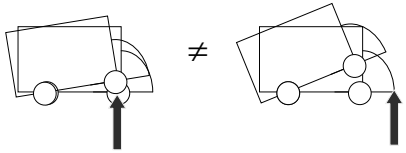


Figure 2.34 An example of torsion on a cantilever beam.

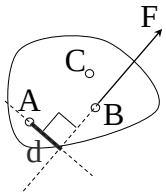
Moments

- *a force acting at a different point causes a different moment:*



Moments

- *defined by magnitude and direction*
- *units: N·m, k·ft*
- *direction:*
  - *+ ccw (right hand rule)*
  - *- cw*
- *value found from F and ⊥ distance*



$$M = F \cdot d$$

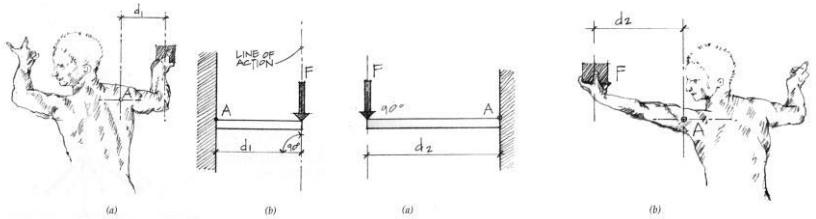
- *d also called "lever" or "moment" arm*

Moments

- *with same F:*

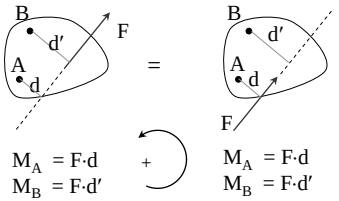
$$M_A = F \cdot d_1 < M_A = F \cdot d_2$$

(bigger)



Moments

- *additive with sign convention*
- *can still move the force*
  <u>along the line of action</u>

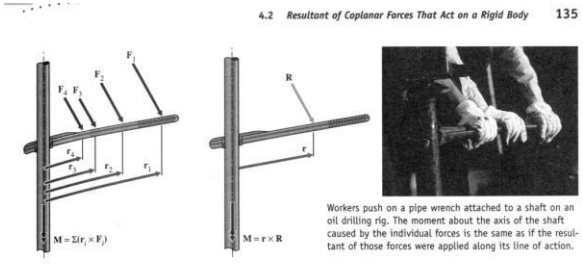


Moments

- *Varignon's Theorem*
  - *resolve a force into components at a point and finding perpendicular distances*
  - *calculate sum of moments*
  - *equivalent to original moment*
- *makes life easier!*
  - *geometry*
  - *when component runs through point, d=0*

Moments of a Force

- *moments of a force*
  - *introduced in Physics as "Torque Acting on a Particle"*
  - *and used to satisfy rotational equilibrium*



Physics and Moments of a Force

- *my Physics book:*

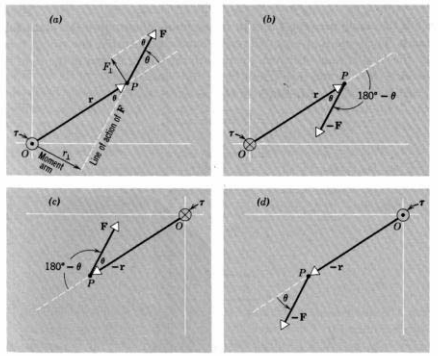
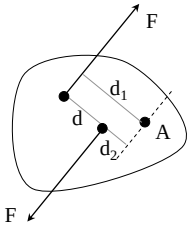


FIGURE 11–2 The plane shown is that defined by **r** and **F** in Fig. 11–1. (*a*) The magnitude of **τ** is given by $rF_\perp$ (Eq. 11–2*b*) or by $rF_\perp$ (Eq. 11–2*c*). (*b*) Reversing **F** reverses the direction of **τ**. (*c*) Reversing **r** reverses the direction of **τ**. (*d*) Reversing **F** and **r** leaves the direction of **τ** unchanged. The directions of **τ** are represented by ⊙ (perpendicularly out of the figure, the symbol representing the tip of an arrow) and by ⊗ (perpendicularly into the figure, the symbol representing the tail of an arrow).

Moment Couples

- *2 forces*
  - *same size*
  - *opposite direction*
  - *distance d apart*
  - *cw or ccw*



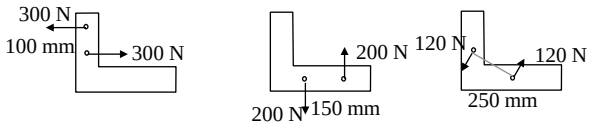
$$M = F \cdot d$$

  - *not dependant on point of application*

$$M = F \cdot d_1 - F \cdot d_2$$

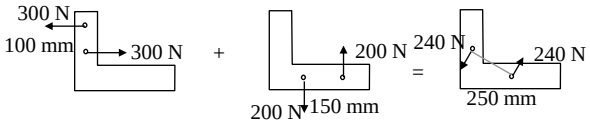
Moment Couples

- *equivalent couples*
  - *same magnitude and direction*
  - *F & d may be different*



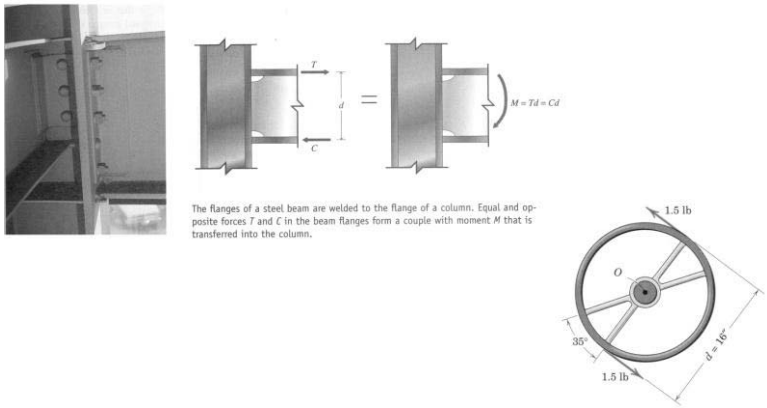
Moment Couples

- *added just like moments caused by one force*
- *can <u>replace</u> two couples with a single couple*



Moment Couples

- *moment couples in structures*

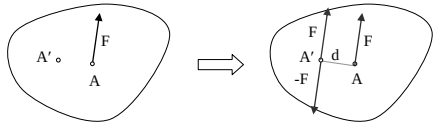


Equivalent Force Systems

- *two forces at a point is equivalent to the resultant at a point*
- *resultant is equivalent to two components at a point*
- *resultant of equal & opposite forces at a point is zero*
- *put equal & opposite forces at a point (sum to 0)*
- *transmission of a force along action line*

Force-Moment Systems

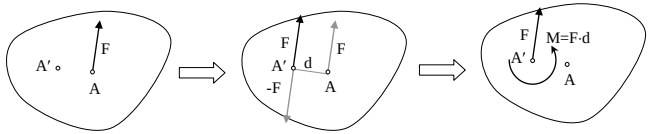
- *single force causing a moment can be replaced by the same force at a different point by providing the moment that force caused*



- *moments are shown as arched arrows*

Force-Moment Systems

- *a force-moment pair can be replaced by a force at another point causing the original moment*



Parallel Force Systems

- *forces are in the same direction*
- *can find resultant force*
- *need to find <u>location</u> for equivalent moments*