ARCHITECTURAL STRUCTURES:
FORM, BEHAVIOR, AND DESIGN
ARCH 331
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FALL 2013

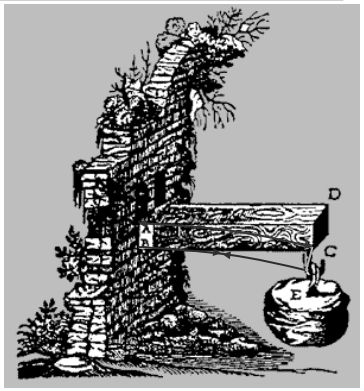
lecture
ten


# beams:
# bending and shear stress



## Beam Bending

- Galileo
  - relationship between stress and depth$^2$
- can see
  - top squishing
  - bottom stretching

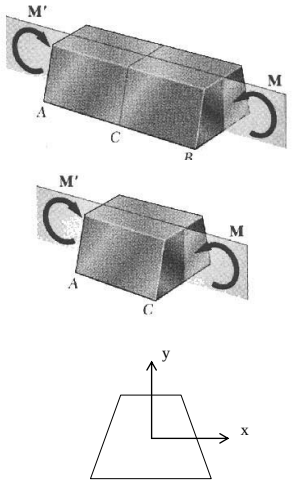



- what are the stress across the section?



## Pure Bending

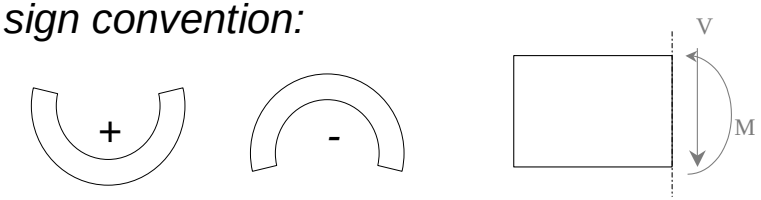
- bending only
- no shear
- axial normal stresses from bending can be found in
  - homogeneous materials
  - plane of symmetry
  - follow Hooke's law






## Bending Moments

- sign convention:




- size of maximum internal moment will govern our design of the section

## Normal Stresses

- *geometric fit*
  - *plane sections remain plane*
  - *stress varies linearly*

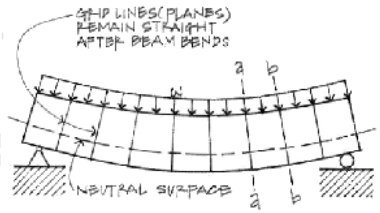



Figure 8.5(b) Beam bending under load.

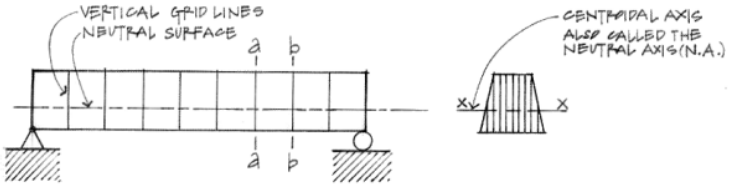



Figure 8.5(a) Beam elevation before loading. Beam cross section.



## Neutral Axis

- *stresses vary linearly*
- *zero stress occurs at the centroid*
- *<u>neutral axis</u> is line of centroids (n.a.)*

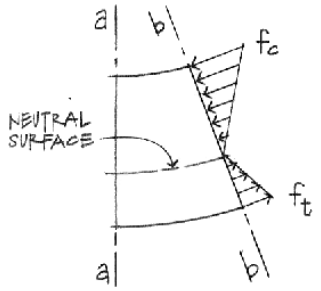



Figure 8.8 Bending stresses on section b-b.

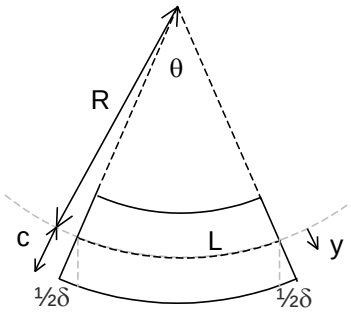


## Derivation of Stress from Strain

- *pure bending = arc shape*

$$L = R\theta$$

$$L_{outside} = (R + y)\theta$$




$$\varepsilon = \frac{\delta}{L} = \frac{L_{outside} - L}{L} = \frac{(R+y)\theta - R\theta}{R\theta} = \frac{y}{R}$$



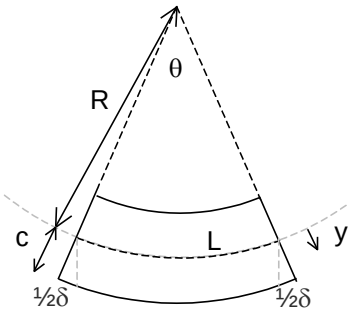
## Derivation of Stress

- *zero stress at n.a.*

$$f = E\varepsilon = \frac{Ey}{R}$$

$$f_{\max} = \frac{Ec}{R}$$

$$f = \frac{y}{c} f_{\max}$$



## Bending Moment

- *resultant moment from stresses = bending moment!*

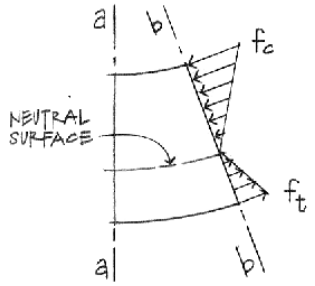



Figure 8.8 Bending stresses on section b-b.

$$M = \Sigma f y \Delta A$$

$$= \Sigma \frac{y f_{max}}{c} y \Delta A = \frac{f_{max}}{c} \underline{\Sigma y^2 \Delta A} = \frac{f_{max}}{c} I = f_{max} S$$



## Bending Stress Relations

$$\frac{1}{R} = \frac{M}{EI}$$

*curvature*

$$f_b = \frac{My}{I}$$

*general bending stress*

$$S = \frac{I}{c}$$

*section modulus*

$$f_b = \frac{M}{S}$$

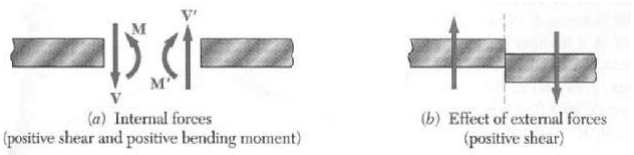
*maximum bending stress*

$$S_{required} \geq \frac{M}{F_b}$$

*required section modulus for design*



## Transverse Loading and Shear




- *perpendicular loading*
- *internal shear*
- *along with bending moment*



## Bending vs. Shear in Design

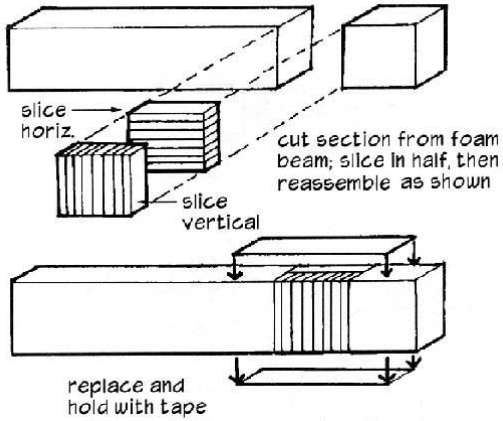
- *bending stresses dominate*


- *shear stresses exist horizontally with shear*


- *<u>no shear stresses</u> with pure bending*

## Shear Stresses

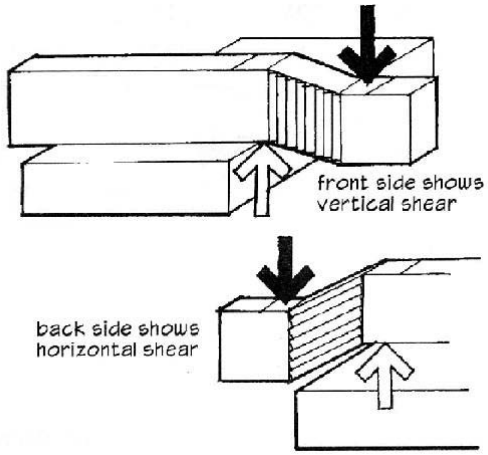
- *horizontal & vertical*






## Shear Stresses

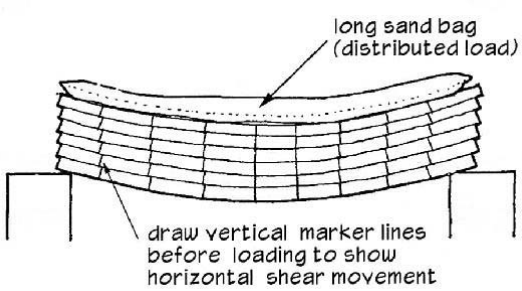
- *horizontal & vertical*






## Beam Stresses

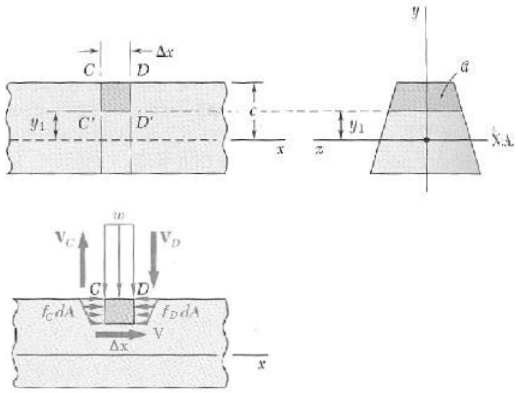
- *horizontal with bending*






## Equilibrium

- *horizontal force V needed*

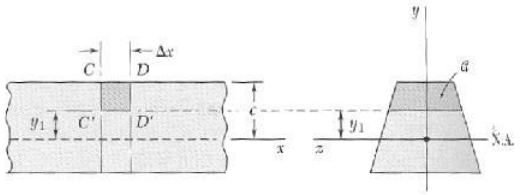


$$V_{longitudinal} = \frac{V_T Q}{I} \Delta x$$

- *Q is a moment area*

## Moment of Area

- *Q is a moment area with respect to the n.a. of area <u>above or below</u> the horizontal*
- $Q_{max}$ *at y=0 (neutral axis)*




- *q is shear flow:*

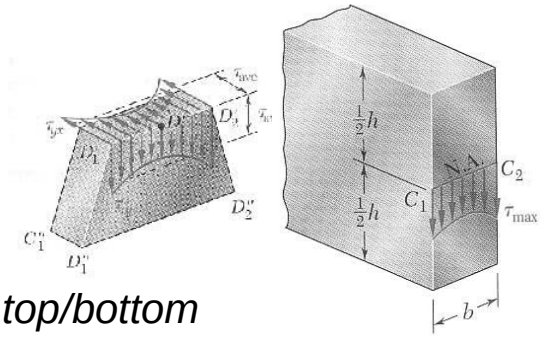
$$q = \frac{V_{longitudinal}}{\Delta x} = \frac{V_T Q}{I}$$



## Shearing Stresses

$$f_v = \frac{V}{\Delta A} = \frac{V}{b \cdot \Delta x}$$

$$f_{v-ave} = \frac{VQ}{Ib}$$




- $f_{v-ave} = 0$ *on the top/bottom*
- *b min may not be with Q max*
- *with h/4* $\geq$*b,* $f_{v\text{-}max} \leq 1.008\, f_{v\text{-}ave}$

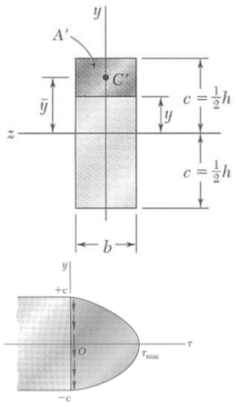


## Rectangular Sections

$$I = \frac{bh^3}{12} \qquad Q = A\bar{y} = bh^2/8$$

$$f_v = \frac{VQ}{Ib} = \frac{3V}{2A}$$

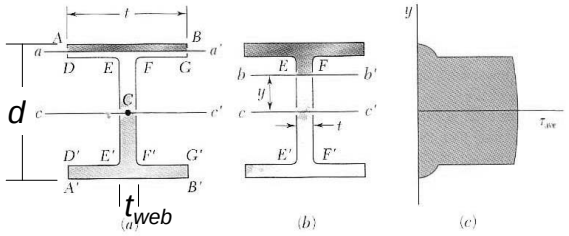
- $f_{v\text{-}max}$ *occurs at <u>n.a.</u>*






## Steel Beam Webs

- *W and S sections*
  - *b varies*



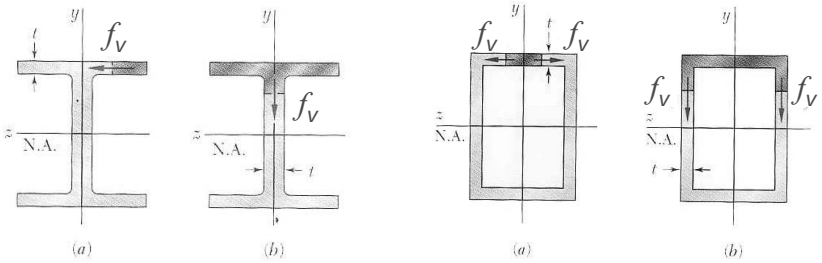

  - *stress in flange negligible*
  - *presume constant stress in web*

$$f_{v-\max} = \frac{3V}{2A} \approx \frac{V}{A_{web}}$$

## Shear Flow

- *loads applied in plane of symmetry*
- *cut made perpendicular*

$$q = \frac{VQ}{I}$$

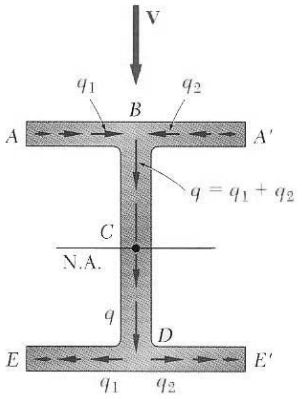





## Shear Flow Quantity

- *sketch from Q*

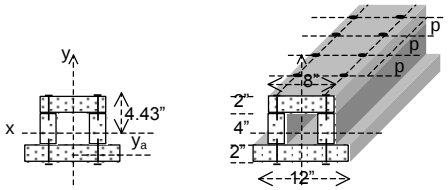
$$q = \frac{VQ}{I}$$






## Connectors Resisting Shear

- *plates with*
  - *nails*
  - *rivets*
  - *bolts*
- *splices*




$$\frac{V_{longitudinal}}{p} = \frac{VQ}{I}$$

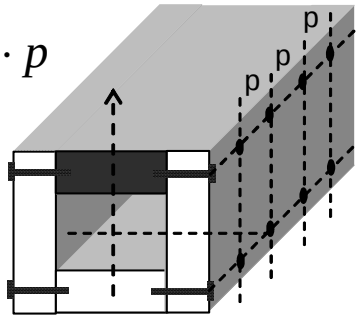
$$nF_{connector} \geq \frac{VQ_{connected\ area}}{I} \cdot p$$



## Vertical Connectors

- *isolate an area with vertical interfaces*

$$nF_{connector} \geq \frac{VQ_{connected\ area}}{I} \cdot p$$



# *Unsymmetrical Shear or Section*

- *member can bend and twist*
  - *not symmetric*
  - *shear not in that plane*
- *shear center*
  - *moments balance*