ARCHITECTURAL STRUCTURES:
FORM, BEHAVIOR, AND DESIGN

ARCH 331

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FALL 2013
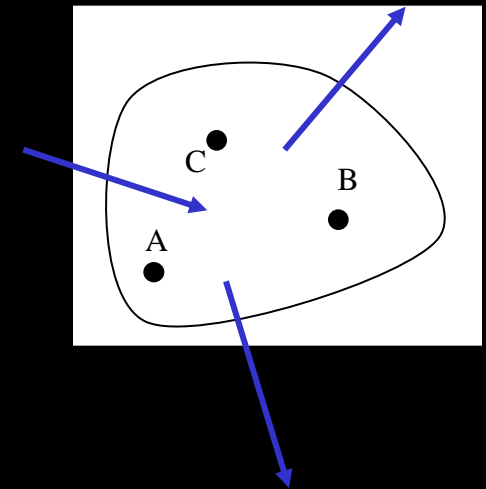
lecture
five

rigid body equilibrium

Equilibrium

- *rigid body*
  - *doesn't deform*
  - *coplanar force systems*
- *static:*

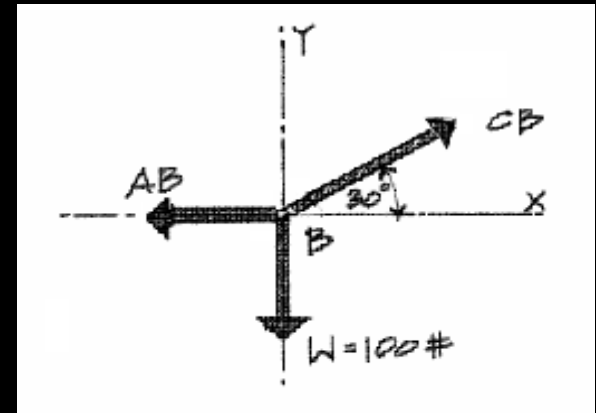$$R_x = \sum F_x = 0$$

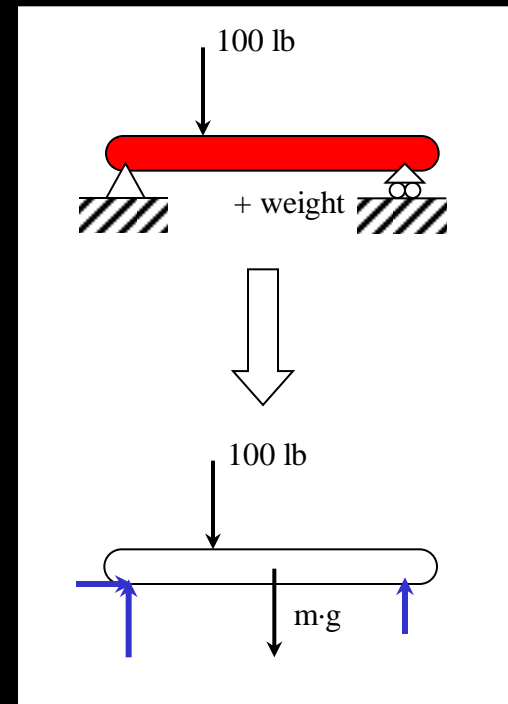$$R_y = \sum F_y = 0$$

$$\boxed{M = \sum M = 0}$$

Free Body Diagram

- *FBD (sketch)*

- *tool to see all forces on a body or a point including*
  - *external forces*
  - *weights*
  - *force reactions*
  - *external moments*
  - *moment reactions*
  - *internal forces*

Free Body Diagram

- *determine body*
- *FREE it from:*
  - *ground*
  - *supports & connections*
- *draw all external forces acting ON the body*
  - *reactions*
  - *applied forces*
  - *gravity*

Free Body Diagram

- *sketch FBD with relevant geometry*
- *resolve each force into components*
  - *known & unknown <u>angles</u> – name them*
  - *known & unknown <u>forces</u> – name them*
  - *known & unknown <u>moments</u> – name them*
- *are any forces related to other forces?*
- *for the unknowns*
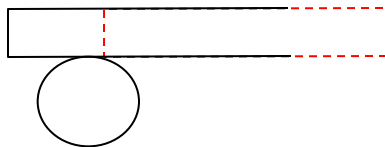- *write only as many equilibrium equations as needed*
- *solve up to 3 equations*

Free Body Diagram

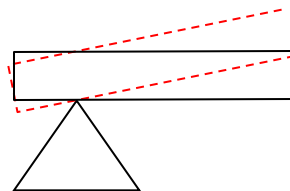- *solve equations*
  - *most times 1 unknown easily solved*
  - *plug into other equation(s)*

- *common to have unknowns of*
  - *force magnitudes*
  - *force angles*
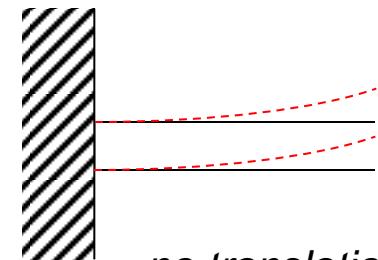 - *moment magnitudes*

Reactions on Rigid Bodies

- *result of applying force*

- *unknown size*

- *connection or support type*
  - *known direction*
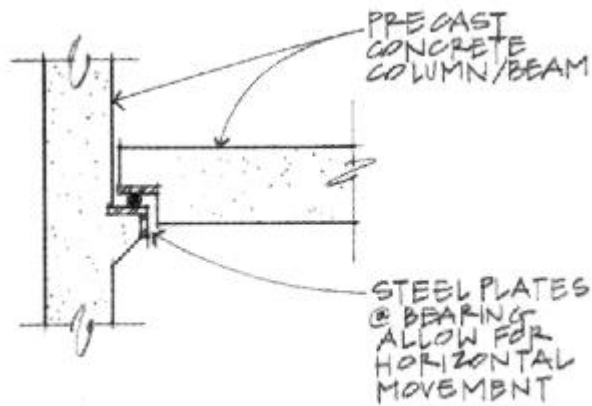 - *related to motion prevented*
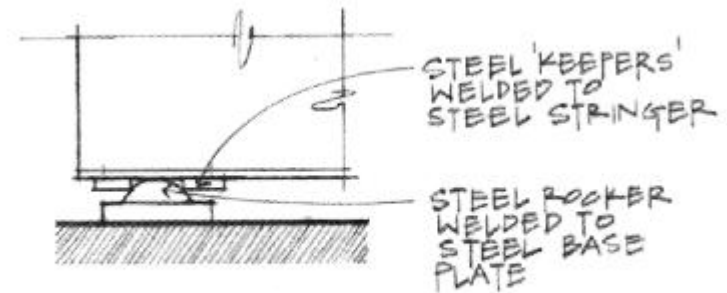
no vertical motion

no translation

no translation
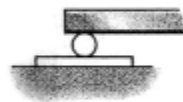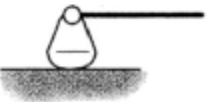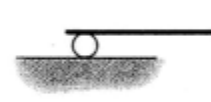*no rotation*

Supports and Connections

PRECAST CONCRETE COLUMN/BEAM

STEEL PLATES @ BEARING ALLOW FOR HORIZONTAL MOVEMENT

Roller

STEEL 'KEEPERS' WELDED TO STEEL STRINGER

STEEL ROCKER WELDED TO STEEL BASE PLATE

Rocker

rollers

rocker

F

Supports and Connections

STEEL COLUMN/
BEAM
CONNECTION,
BOLTED.

Pin

smooth pin or hinge

CAST-IN-PLACE
CONCRETE
COLUMN/BEAM
CONNECTION.

NOTE:
OVERLAPPING
STEEL
REINFORCEMENT
MAKES ASSEMBLY
MONOLITHIC.

fixed support

Fixed

F_y

M

F_x
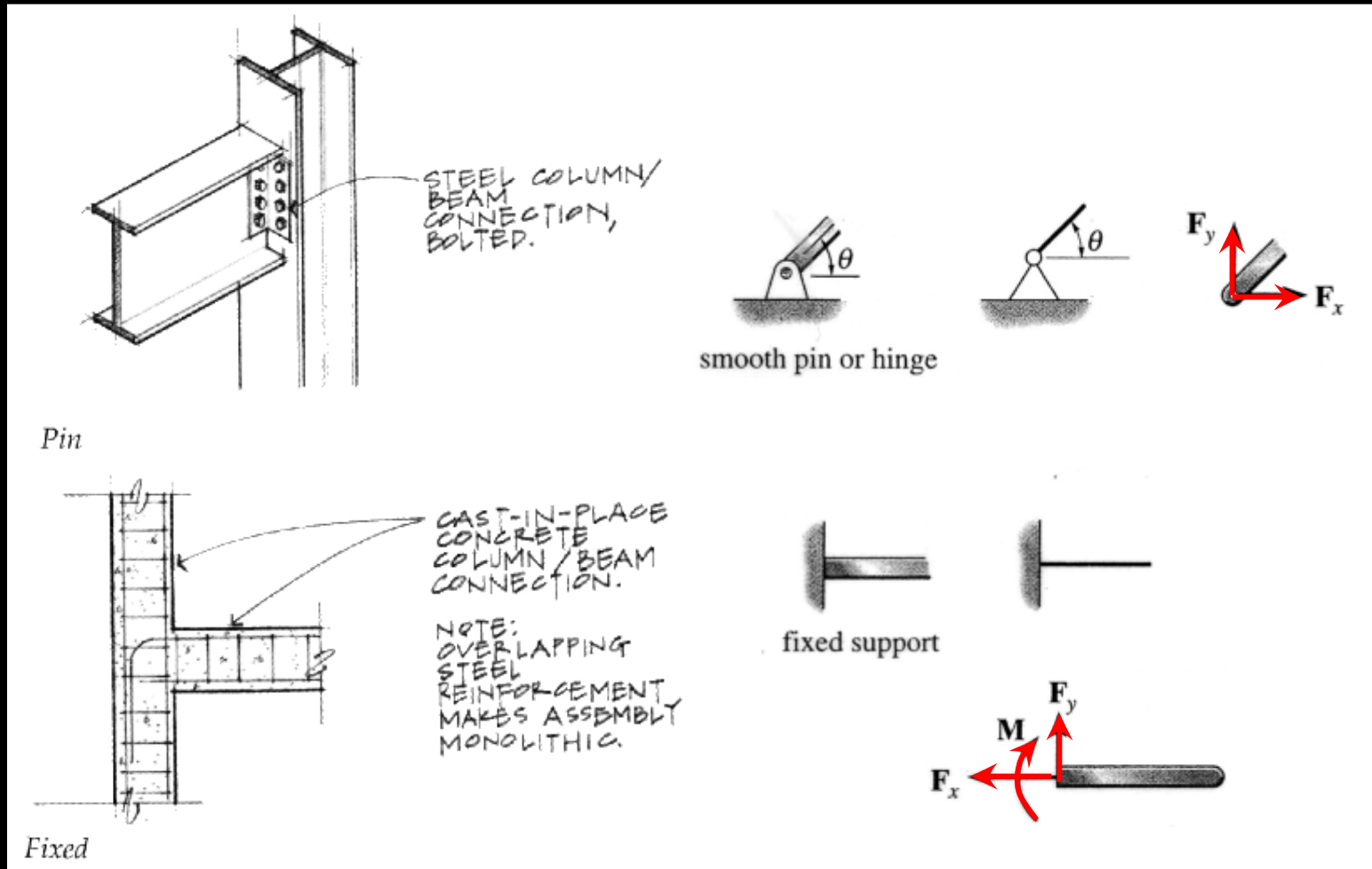
FBD Example

- *500 lb known*

- *pin – A_x, A_y*

- *smooth surface – B at 4:3*

- *3 equations*

- *sum moments at*
  - *A?*
  - *B?   (B_x)*
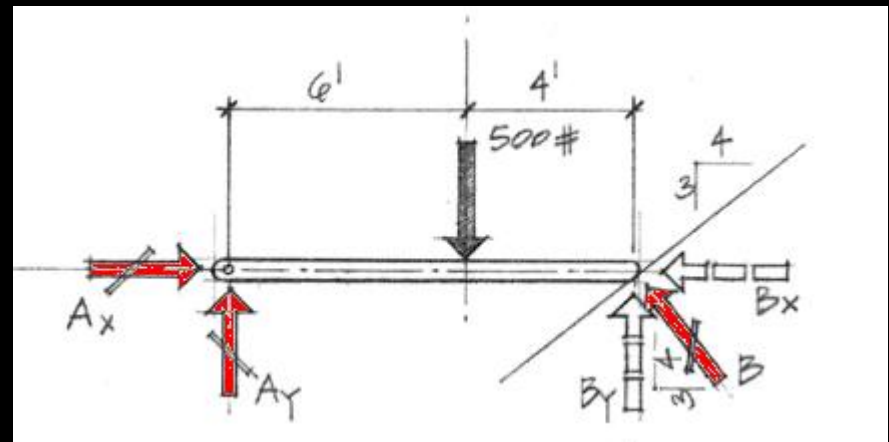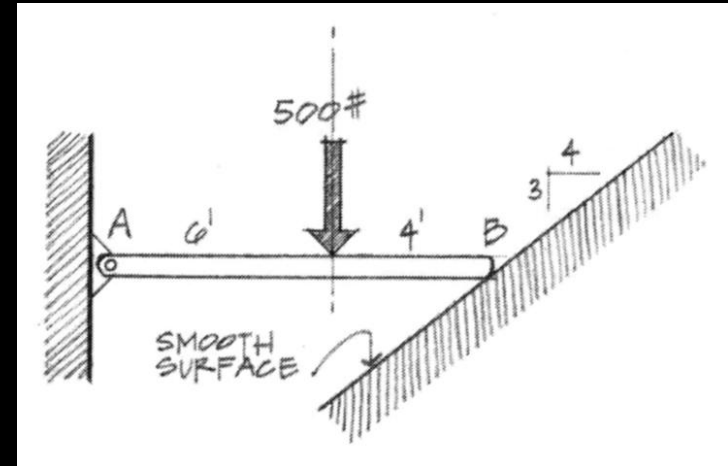
Moment Equations

- *sum moments at intersection where the most forces intersect*

- *multiple moment equations may not be useful*

- *combos:*

$$\sum F_x = 0 \qquad \sum F = 0 \qquad \sum M_1 = 0$$

$$\sum F_y = 0 \qquad \sum M_1 = 0 \qquad \sum M_2 = 0$$

$$\sum M_1 = 0 \qquad \sum M_2 = 0 \qquad \sum M_3 = 0$$

Recognizing Reactions

3
unknowns

3
unknowns

F

F

weight

m·g

Recognizing Reactions

$$\underline{3}$$
unknowns

weight

F_1 F_2

F_1 F_2
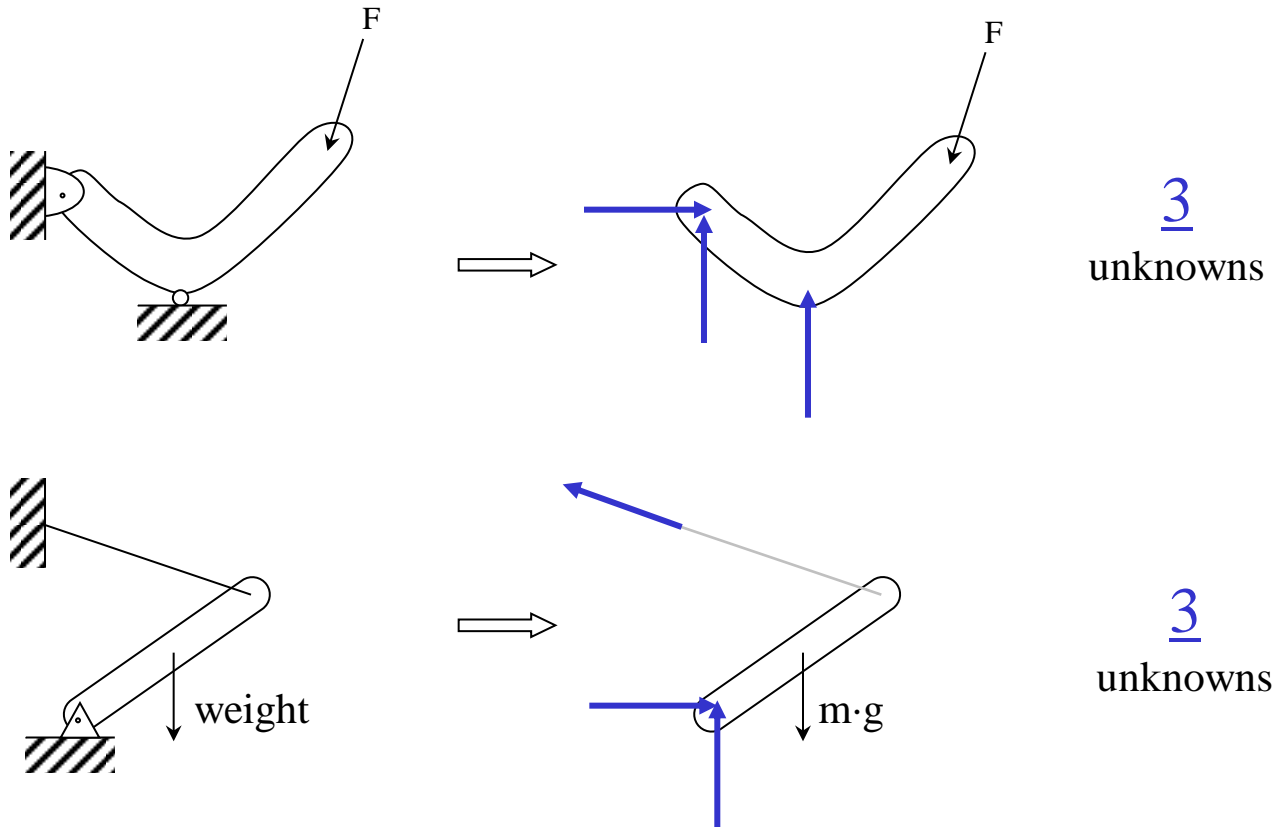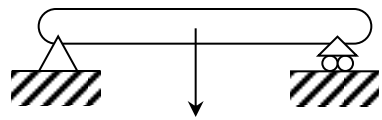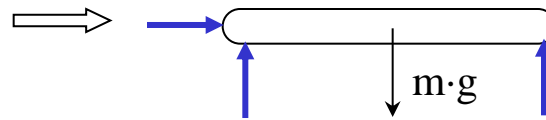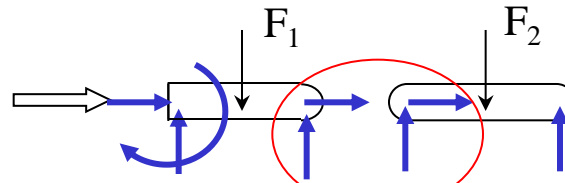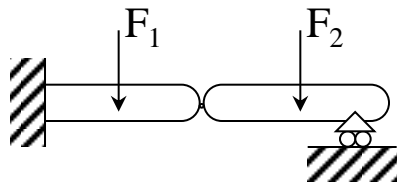
$$\underline{6}$$
unknowns for
2 bodies

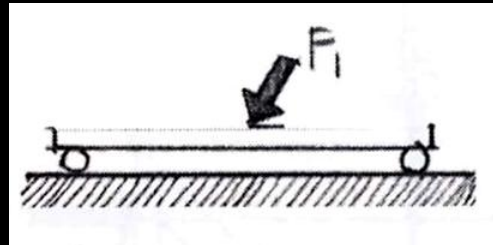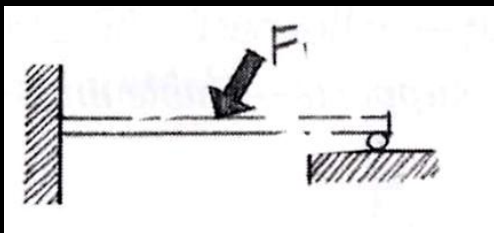*not independent*
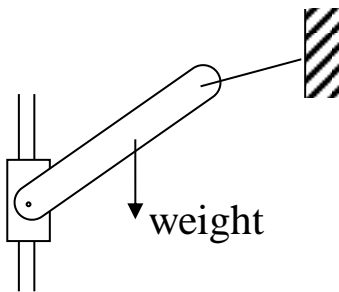
weight   m·g
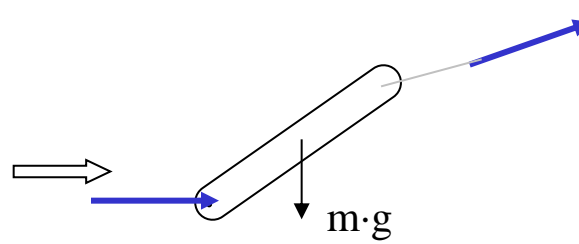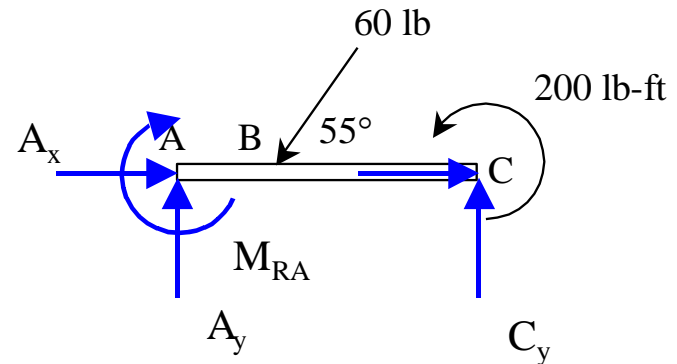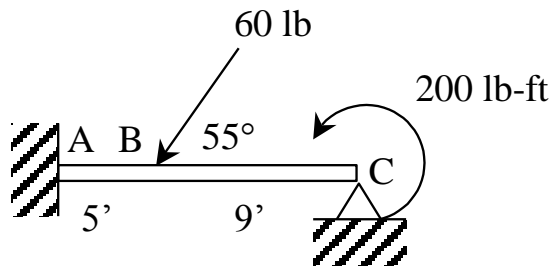
$$\underline{2}$$
unknowns

Constraints

- *completely constrained*
    - *doesn't move*
    - *may not be statically determinate*
- *improperly or partially constrained*
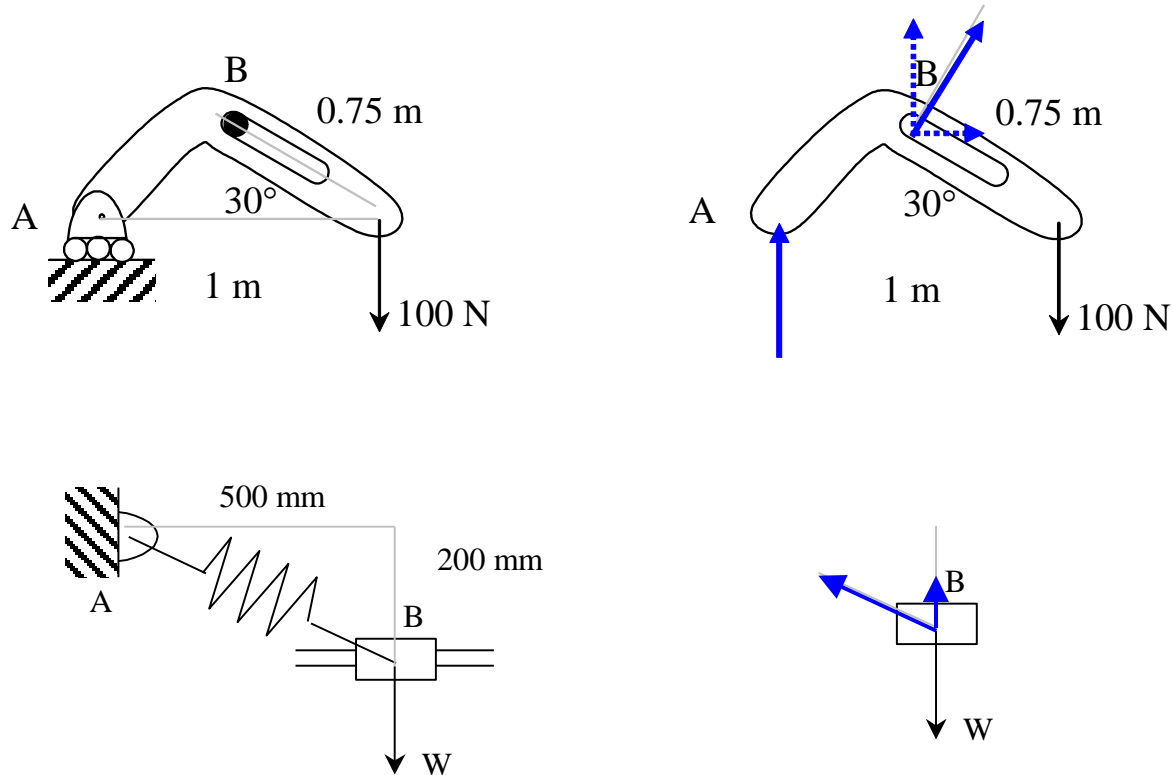   - *has $\leq$ unknowns*
    - *can't solve*
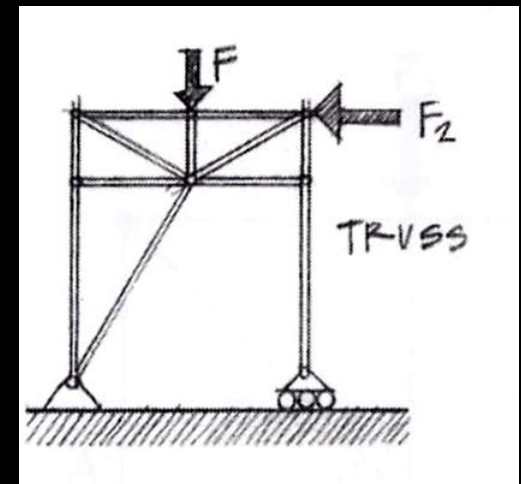
Constraints

- *overconstrained*
  - *won't move*
  - *can't be solved with statics*
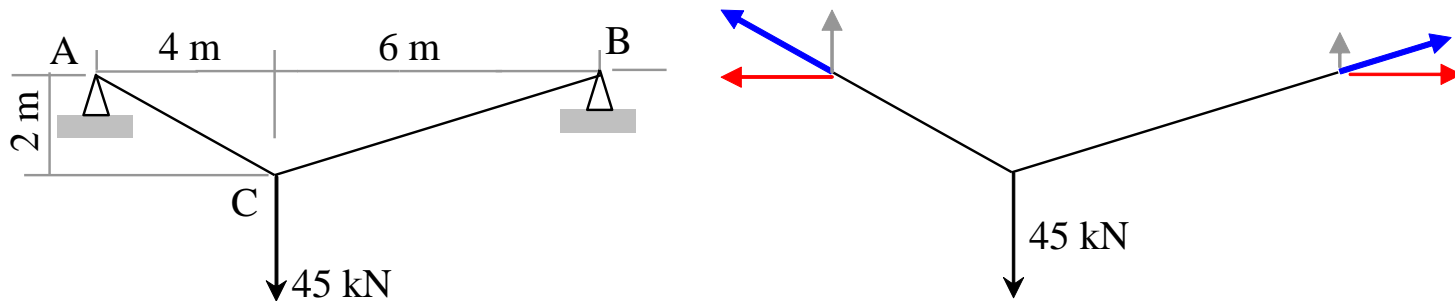 - *statically indeterminate to n^th degree*
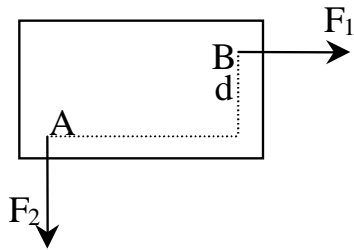
Partial Constraints

Cable Reactions

- *equilibrium:*
  - *more reactions (4) than equations*
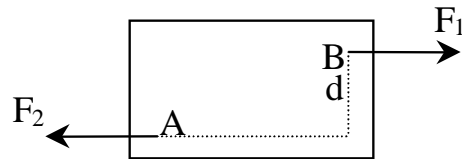 - *but, we have slope relationships*
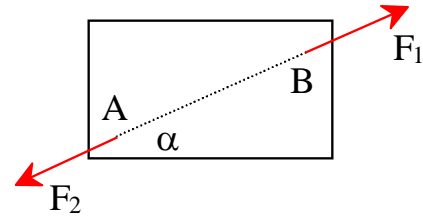 - *x component the same everywhere*

Two Force Rigid Bodies

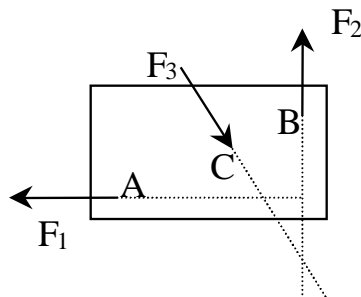- *equilibrium:*
  - *forces in line, equal and opposite*
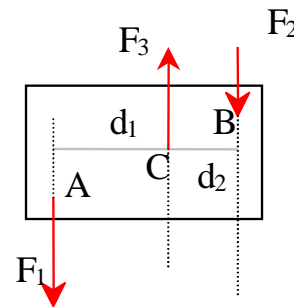

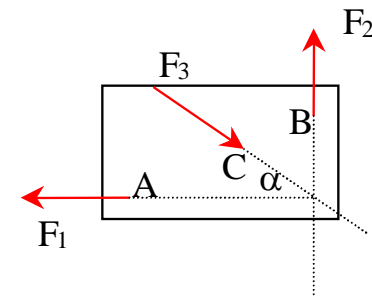A (no)   B (no)   C

Three Force Rigid Bodies

- ## *equilibrium:*

    - *– concurrent or parallel forces*

        beams!

A (no) B C
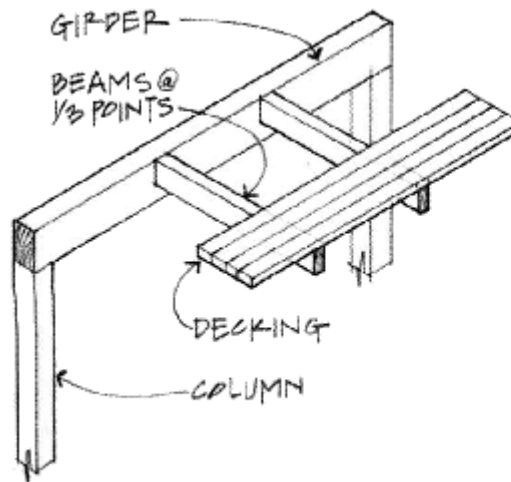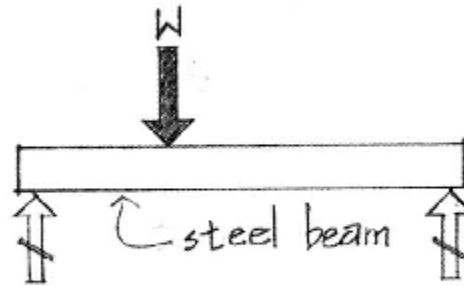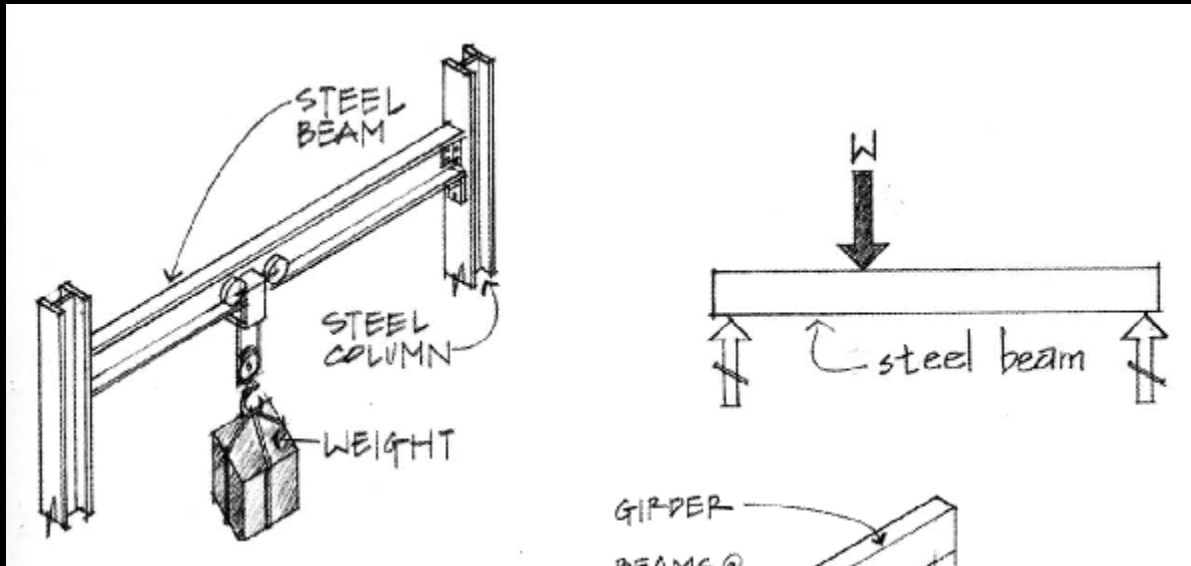
Concentrated Loads
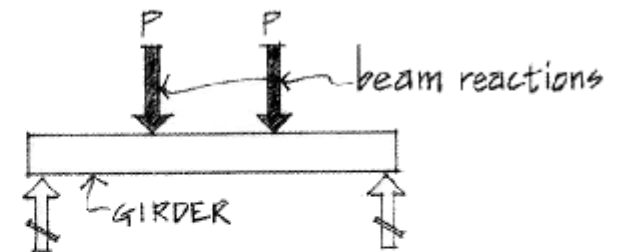
Distributed Loads
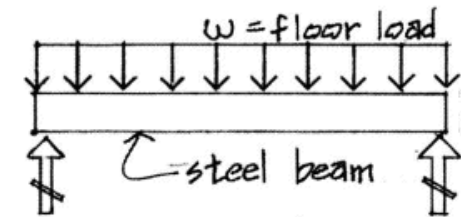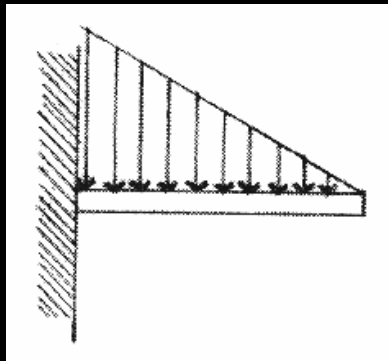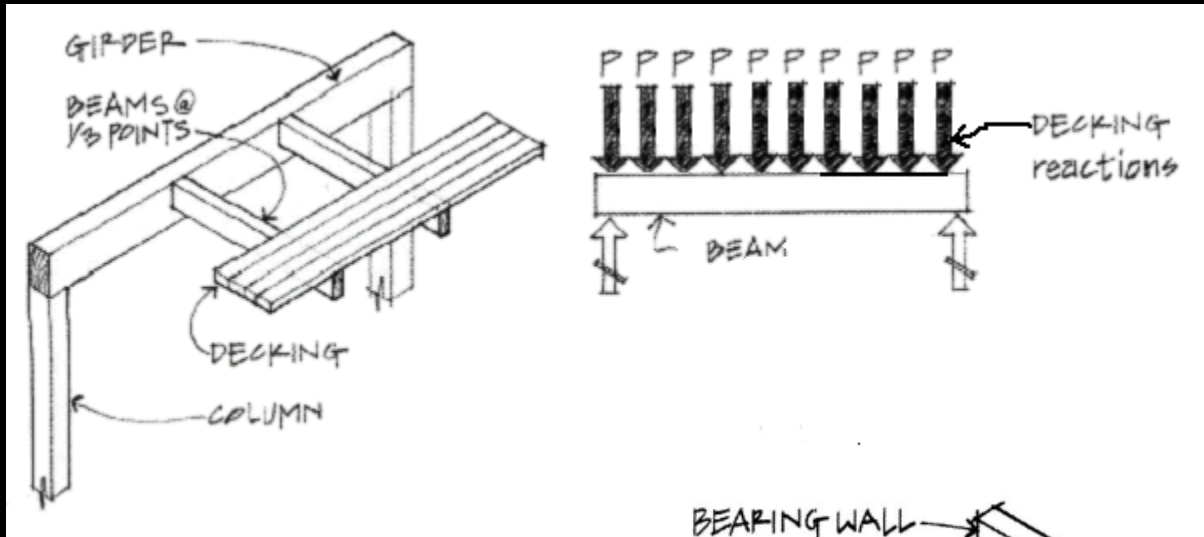
Beam Supports

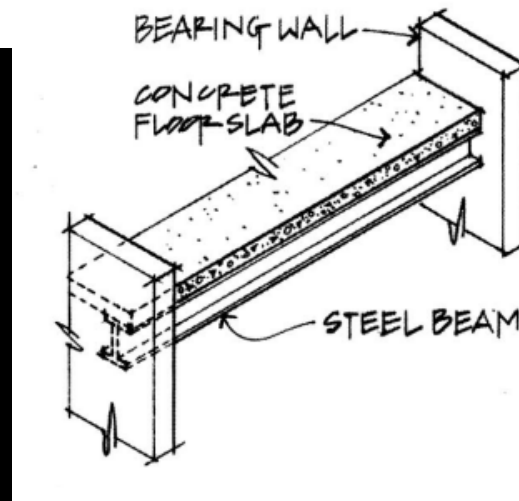- ## *statically determinate*

simply supported
(most common)

overhang

cantilever

- ## *statically indeterminate*

continuous
(most common case when $L_1=L_2$)

Propped

Restrained
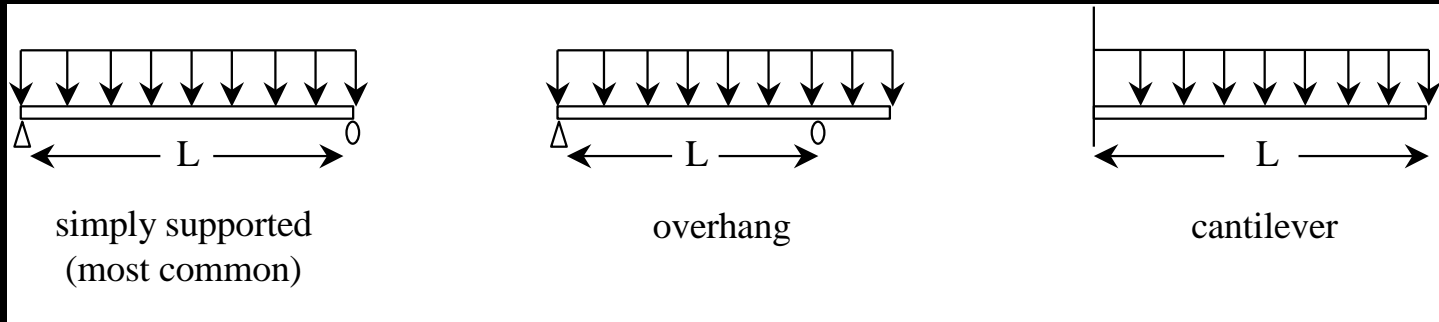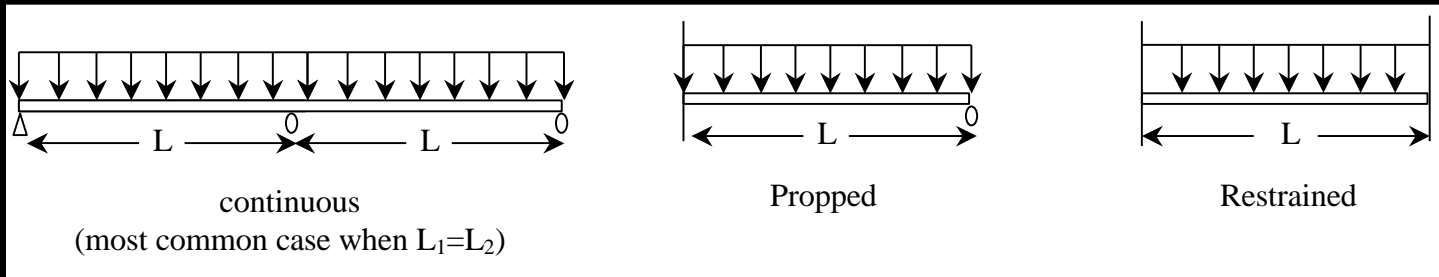
Equivalent Force Systems

- *replace forces by resultant*

- *place resultant where M = 0*

- *using <u>calculus</u> and area centroids*

$$W = \int_0^L w\,dx = \int dA_{loading} = A_{loading}$$
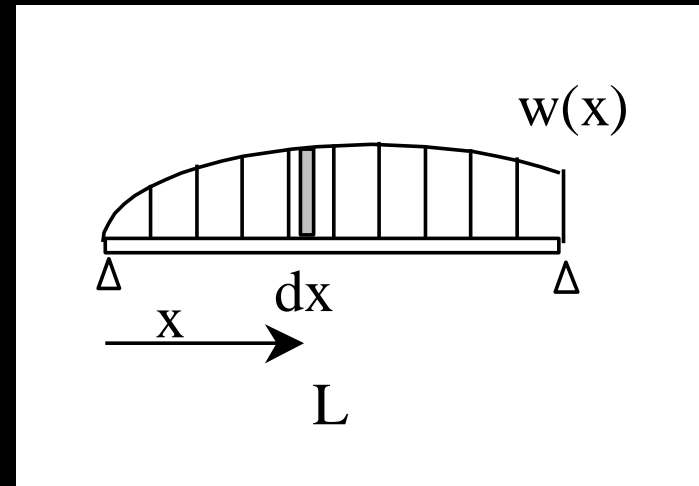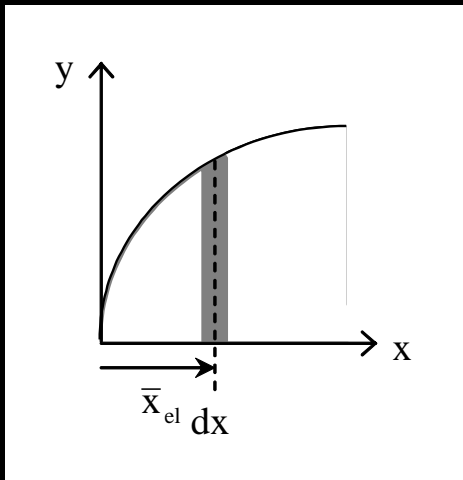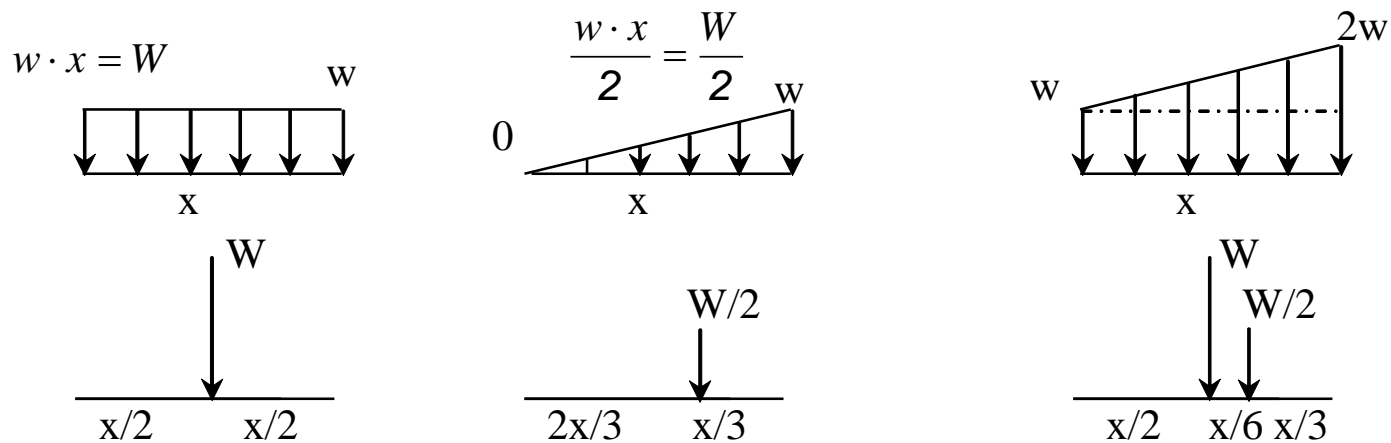
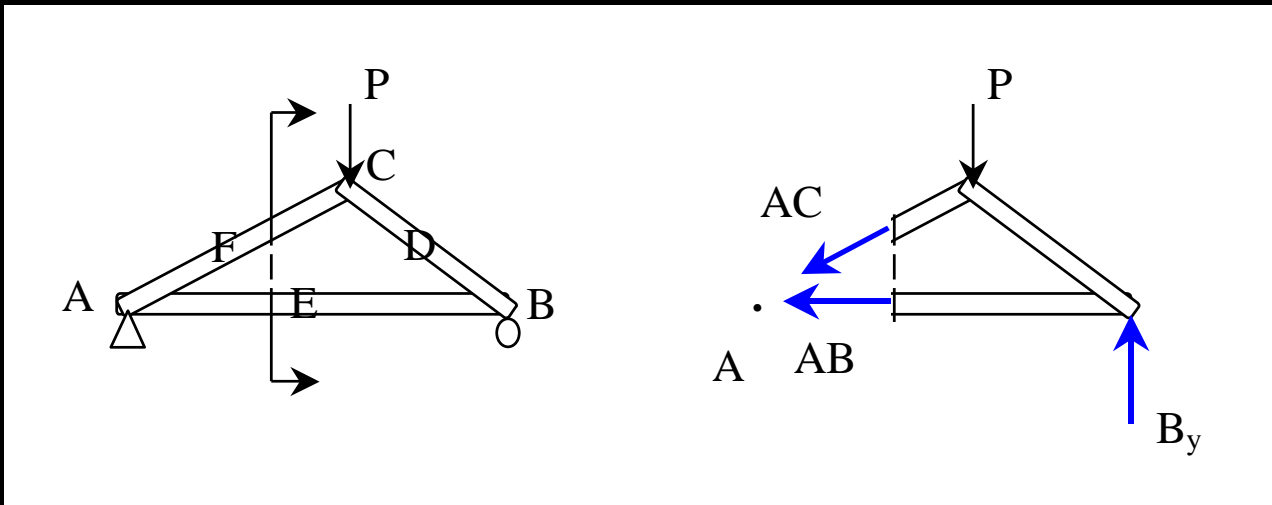# *Load Areas*

- *area is width x "height" of load*

- *w is load per unit length*

- *W is total load*

$w \cdot x = W$ w

x

W

x/2   x/2

$\dfrac{w \cdot x}{2} = \dfrac{W}{2}$ w

0

x

W/2

2x/3   x/3

2w

w

x

W

W/2

x/2   x/6 x/3

Method of Sections

- *relies on internal forces being in equilibrium on a section*

- *cut to expose <u>3 or less</u> members*

- *coplanar forces $\rightarrow$ $\sum M = 0$ too*

Method of Sections

- *joints on or off the section are good to sum moments*

- *quick for few members*

- *not always obvious where to cut or sum*