

**ARCHITECTURAL STRUCTURES:  
FORM, BEHAVIOR, AND DESIGN**

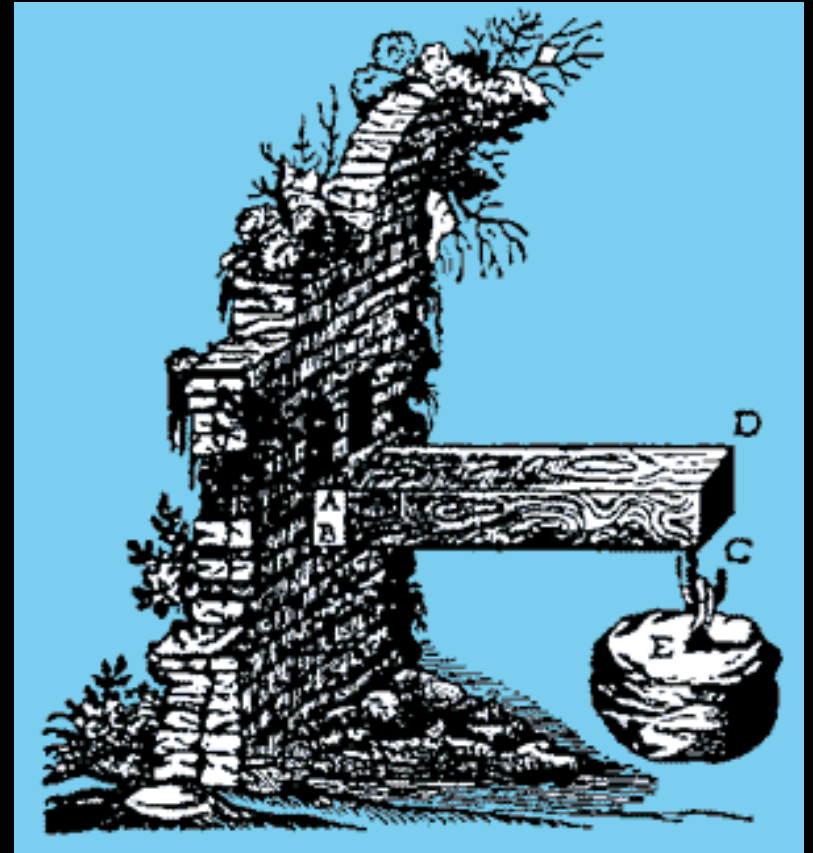
**ARCH 331**

**DR. ANNE NICHOLS**

**FALL 2013**

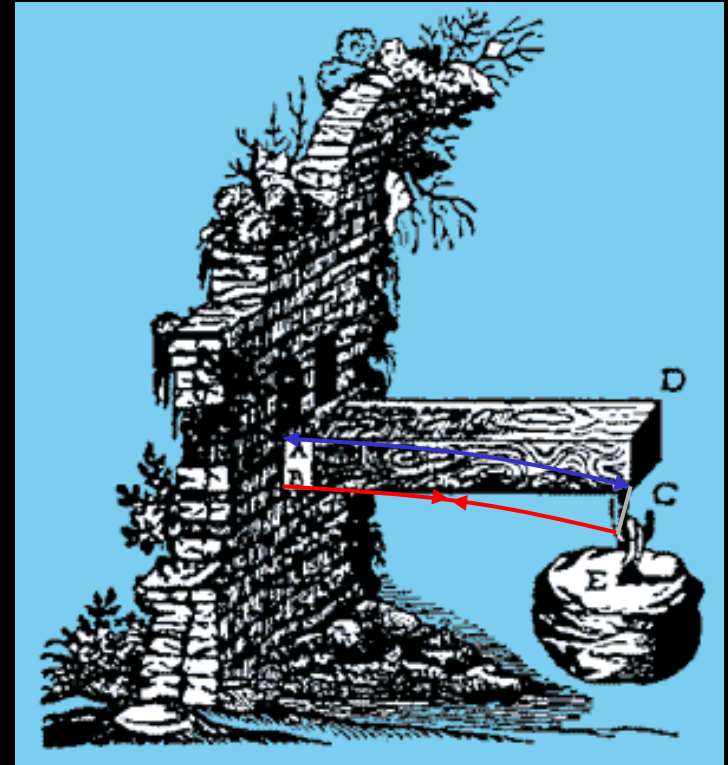
**lecture  
ten**

**beams:  
bending and shear stress**



# Beam Bending

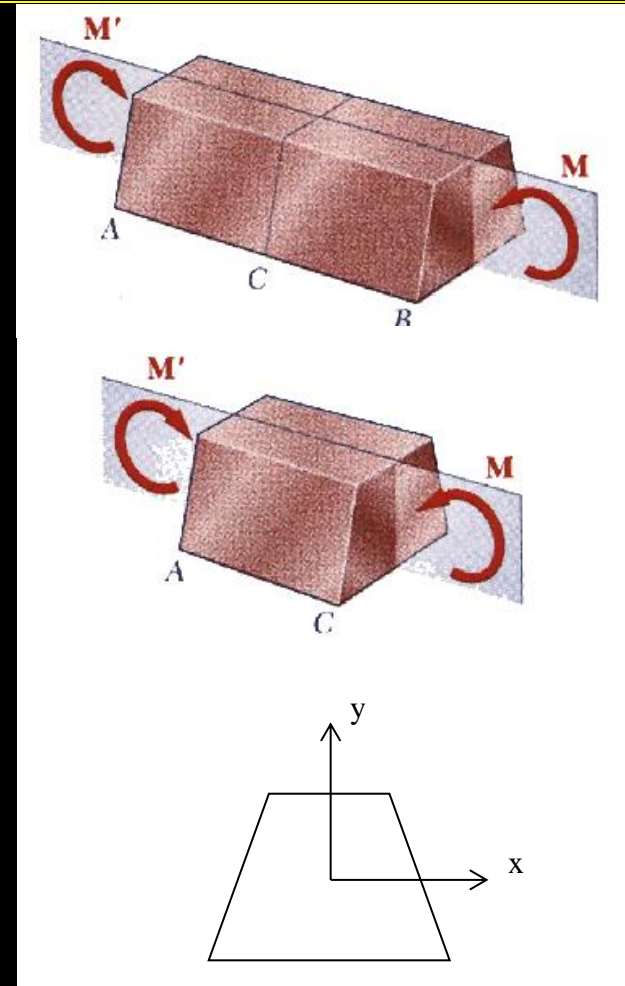
- Galileo
  - relationship between stress and depth<sup>2</sup>
- can see
  - top squishing
  - bottom stretching



- what are the stress across the section?

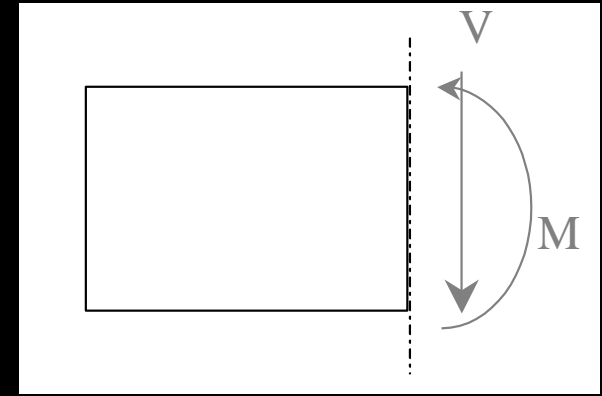
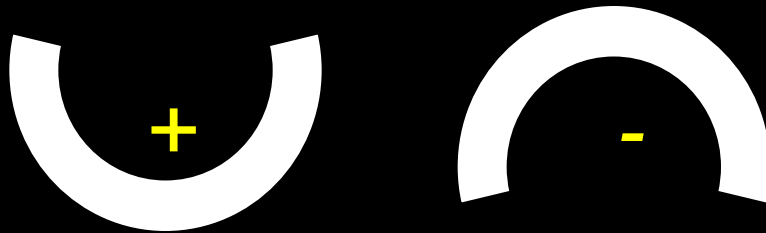
# Pure Bending

- *bending only*
- *no shear*
- *axial normal stresses from bending can be found in*
  - *homogeneous materials*
  - *plane of symmetry*
  - *follow Hooke's law*



# Bending Moments

- *sign convention:*



- *size of maximum internal moment will govern our design of the section*

# Normal Stresses

- *geometric fit*
  - *plane sections remain plane*
  - *stress varies linearly*

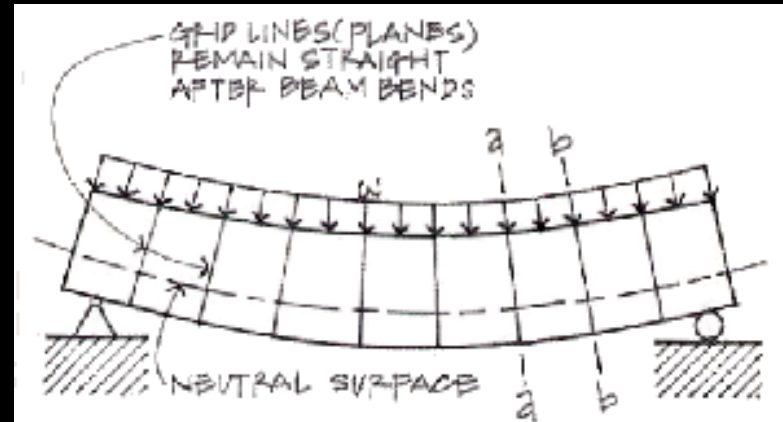


Figure 8.5(b) Beam bending under load.

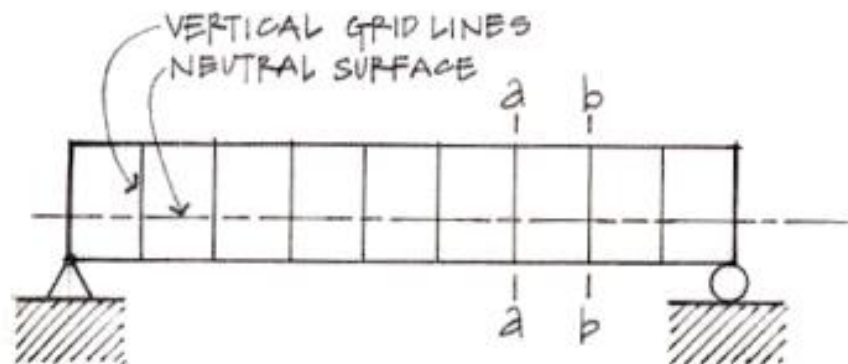
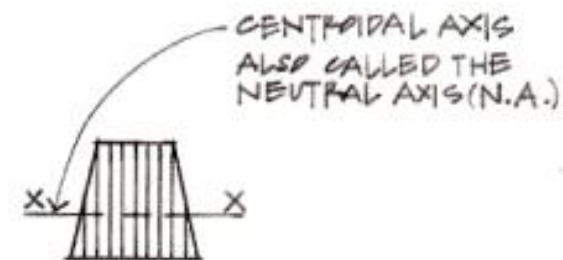


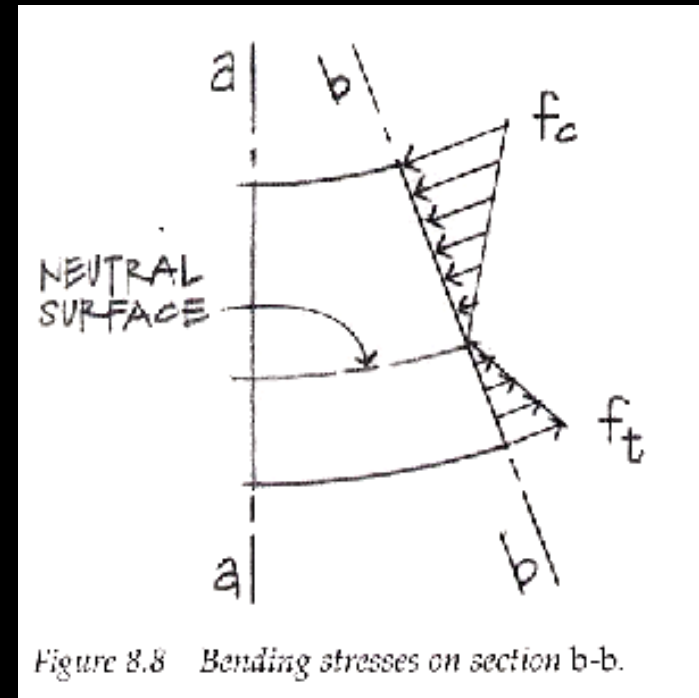
Figure 8.5(a) Beam elevation before loading.



Beam cross section.

# Neutral Axis

- *stresses vary linearly*
- *zero stress occurs at the centroid*
- *neutral axis is line of centroids (n.a.)*

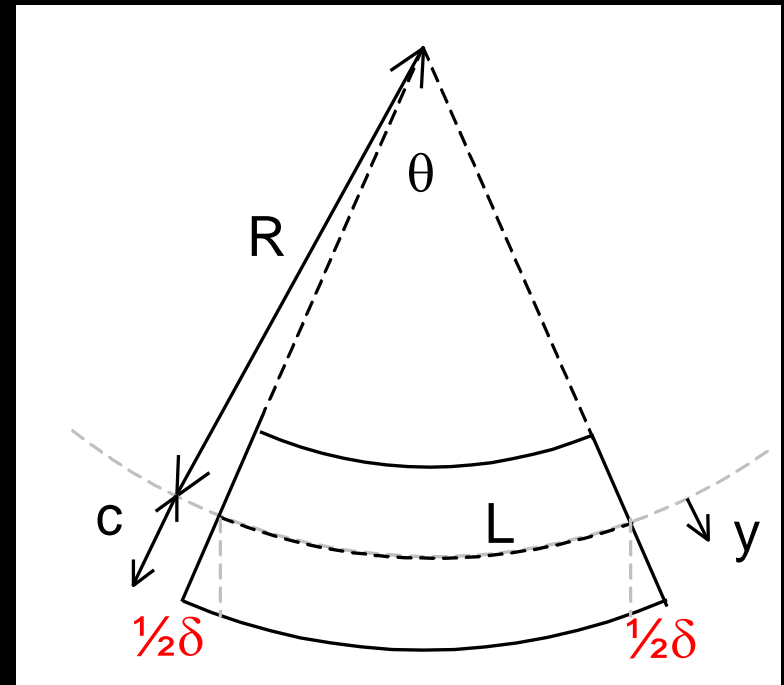


# Derivation of Stress from Strain

- *pure bending = arc shape*

$$L = R\theta$$

$$L_{outside} = (R + y)\theta$$



$$\epsilon = \frac{\delta}{L} = \frac{L_{outside} - L}{L} = \frac{(R + y)\theta - R\theta}{R\theta} = \frac{y}{R}$$

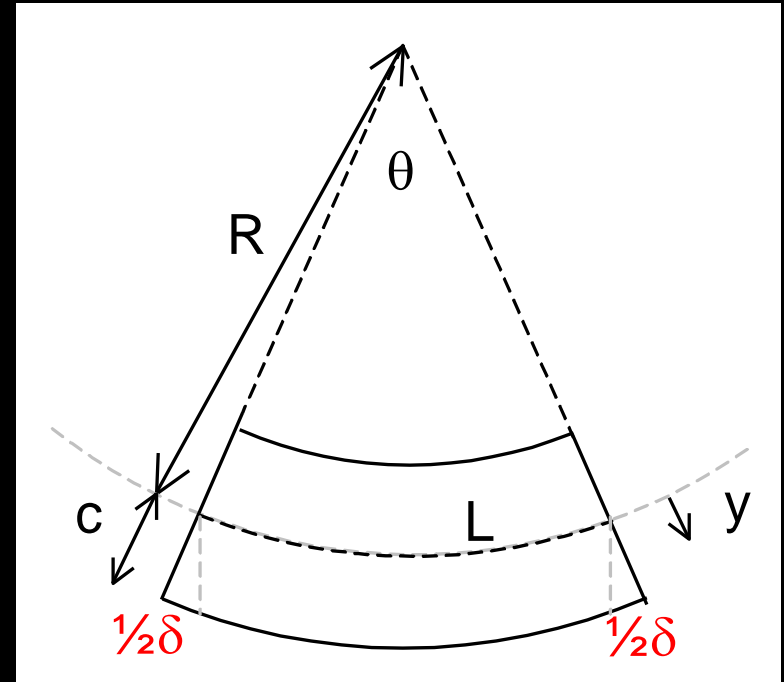
# Derivation of Stress

- zero stress at n.a.

$$f = E\varepsilon = \frac{Ey}{R}$$

$$f_{\max} = \frac{Ec}{R}$$

$$f = \frac{y}{c} f_{\max}$$





# Bending Moment

- *resultant moment from stresses = bending moment!*

$$M = \Sigma f y \Delta A$$

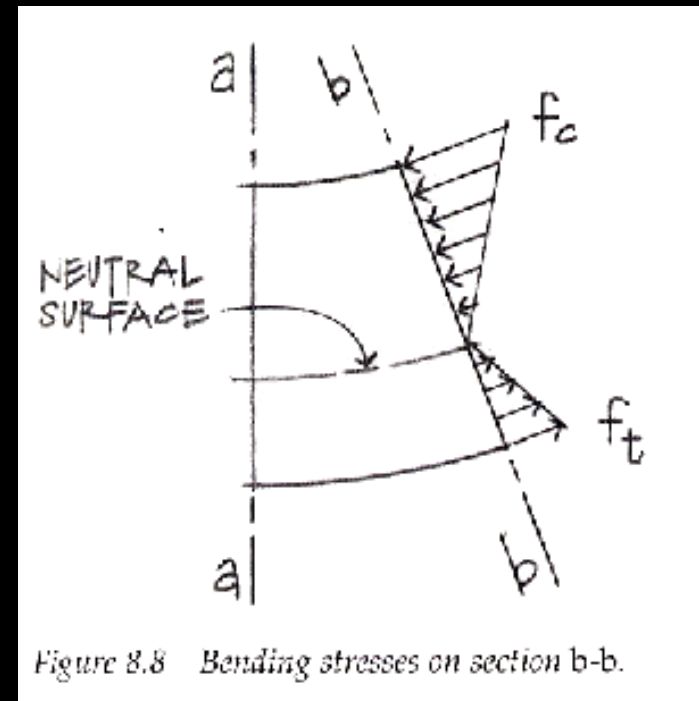
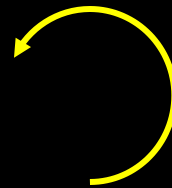


Figure 8.8 Bending stresses on section b-b.

$$= \Sigma \frac{y f_{max}}{c} y \Delta A = \frac{f_{max}}{c} \underline{\Sigma y^2 \Delta A} = \frac{f_{max}}{c} I = f_{max} S$$

# Bending Stress Relations

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$$\frac{1}{R} = \frac{M}{EI}$$

*curvature*

$$f_b = \frac{My}{I}$$

*general bending stress*

$$S = \frac{I}{c}$$

*section modulus*

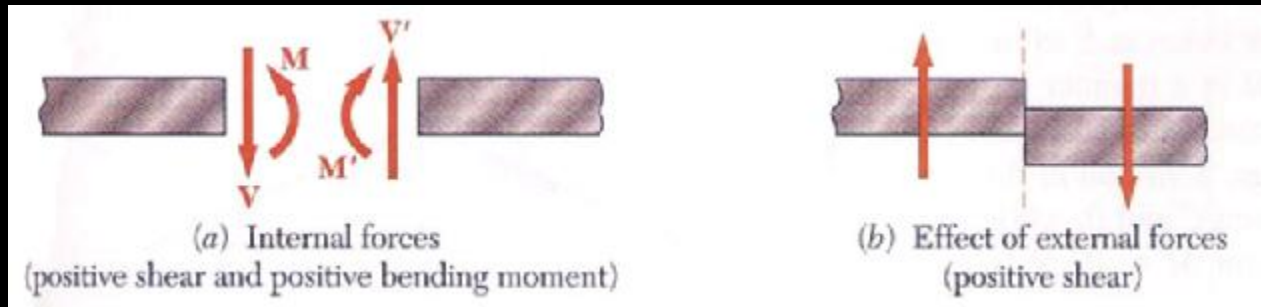
$$f_b = \frac{M}{S}$$

*maximum bending stress*

$$S_{required} \geq \frac{M}{F_b}$$

*required section modulus for design*

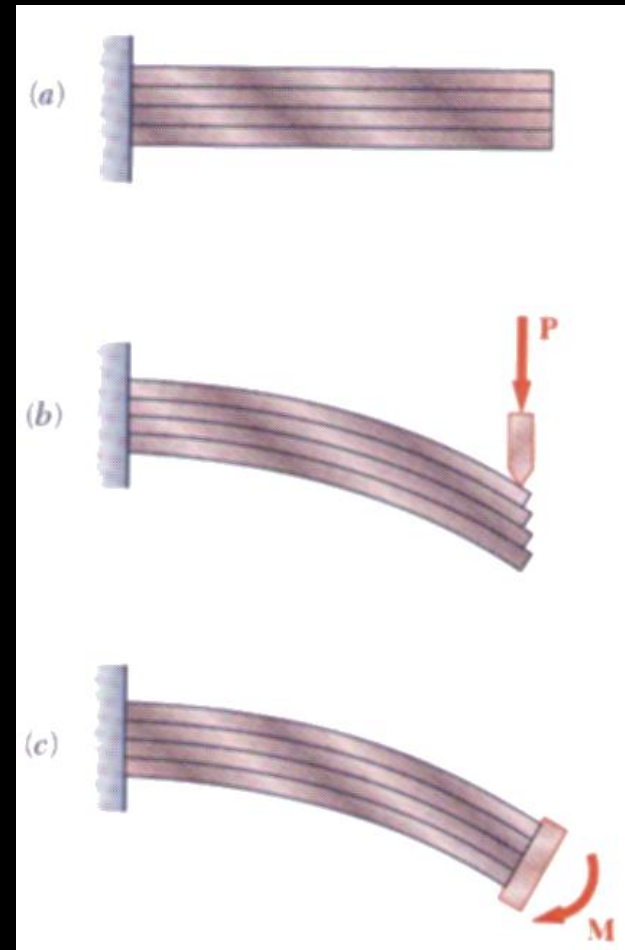
# Transverse Loading and Shear



- *perpendicular loading*
- *internal shear*
- *along with bending moment*

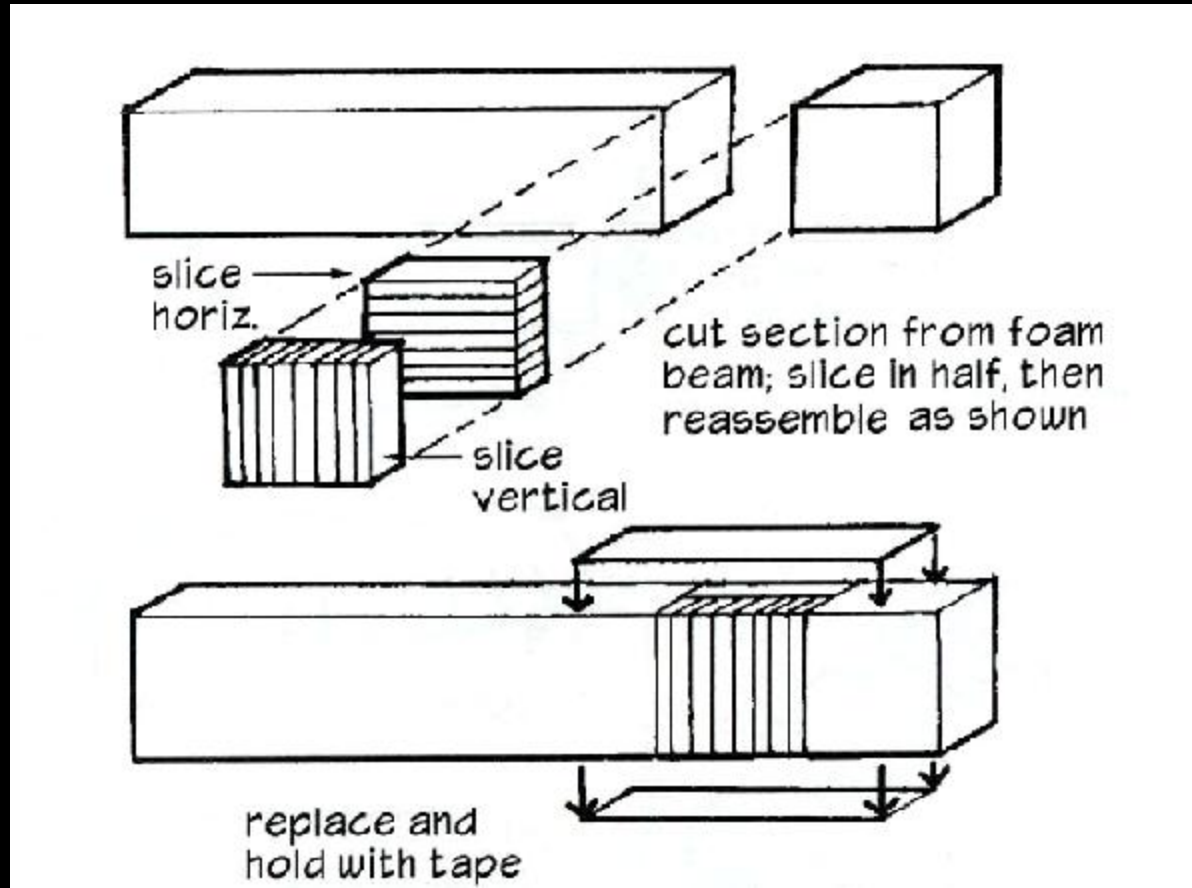
# Bending vs. Shear in Design

- *bending stresses dominate*
- *shear stresses exist horizontally with shear*
- *no shear stresses with pure bending*



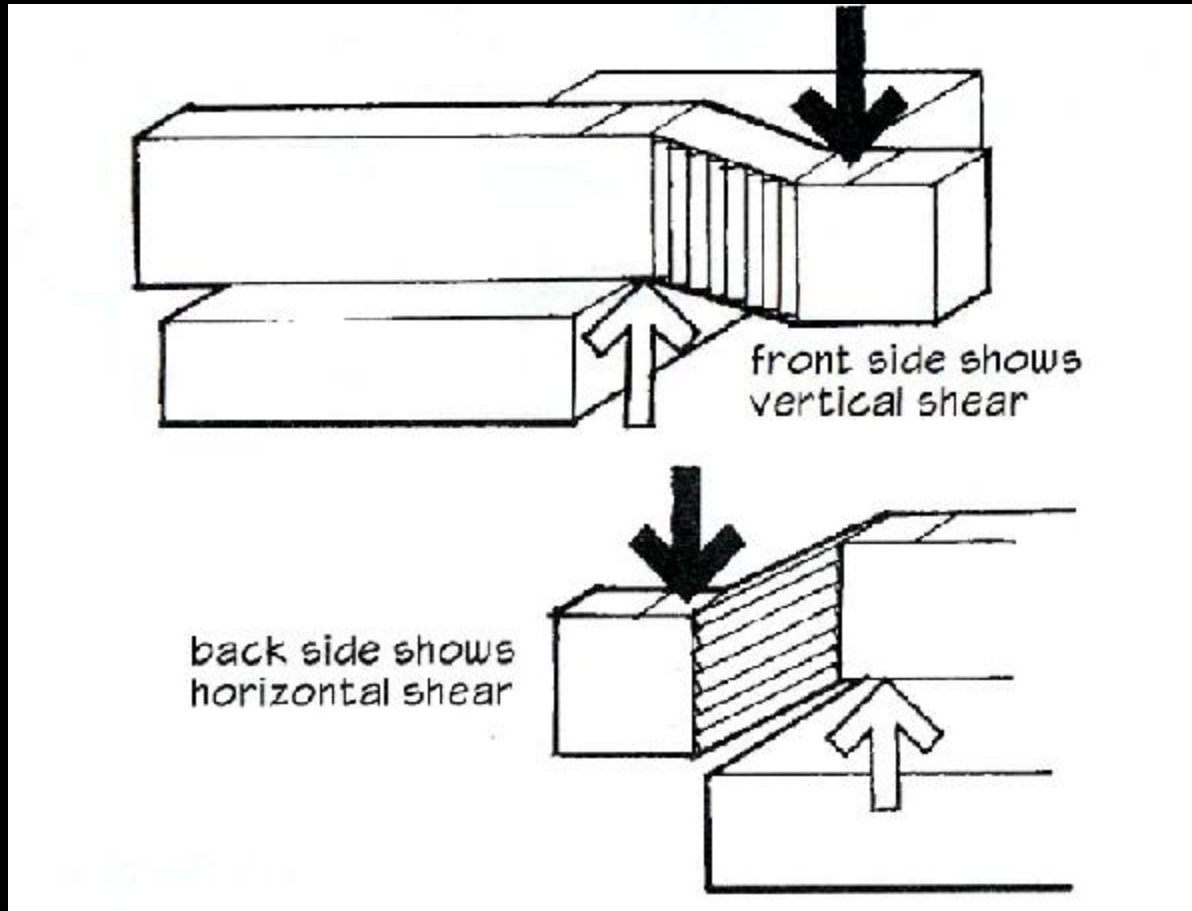
# Shear Stresses

- *horizontal & vertical*



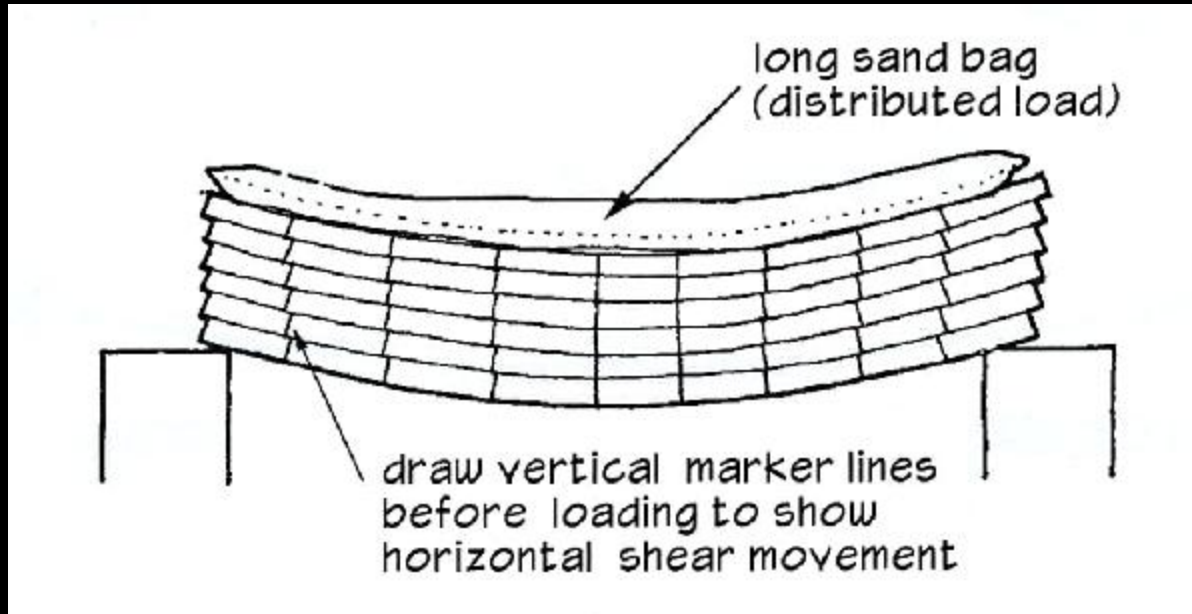
# Shear Stresses

- *horizontal & vertical*



# Beam Stresses

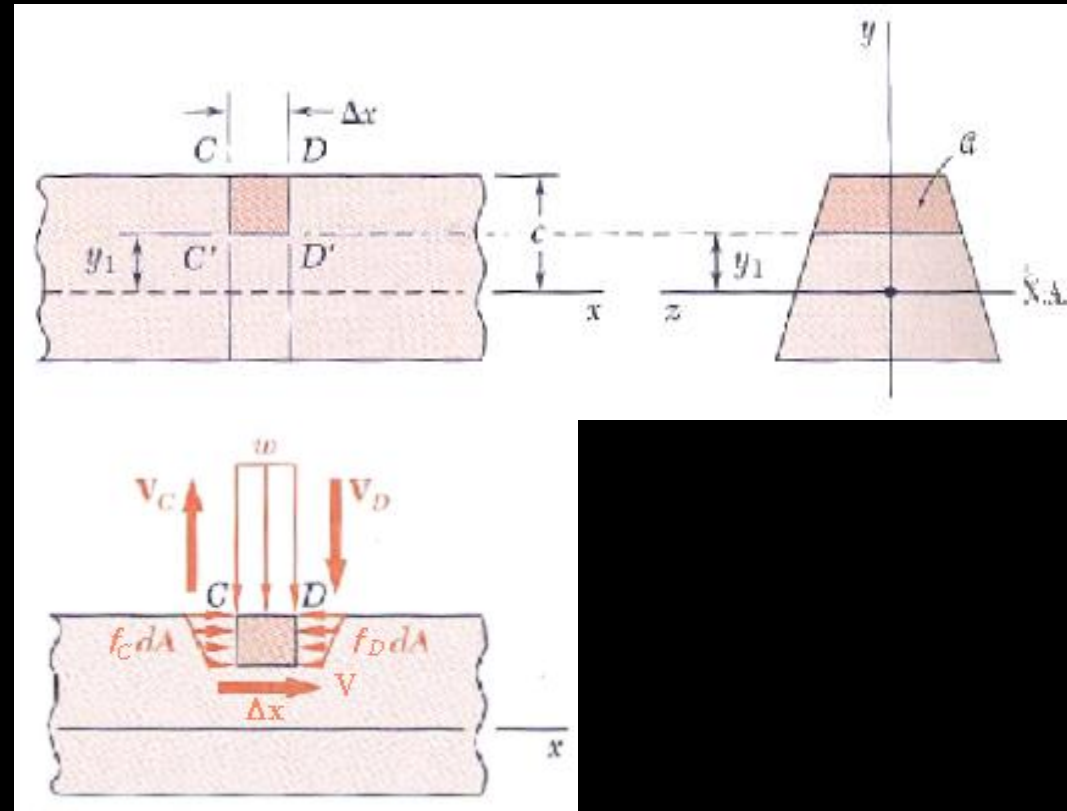
- *horizontal with bending*



# Equilibrium

- horizontal force  $V$  needed

$$V_{longitudinal} = \frac{V_T Q}{I} \Delta x$$



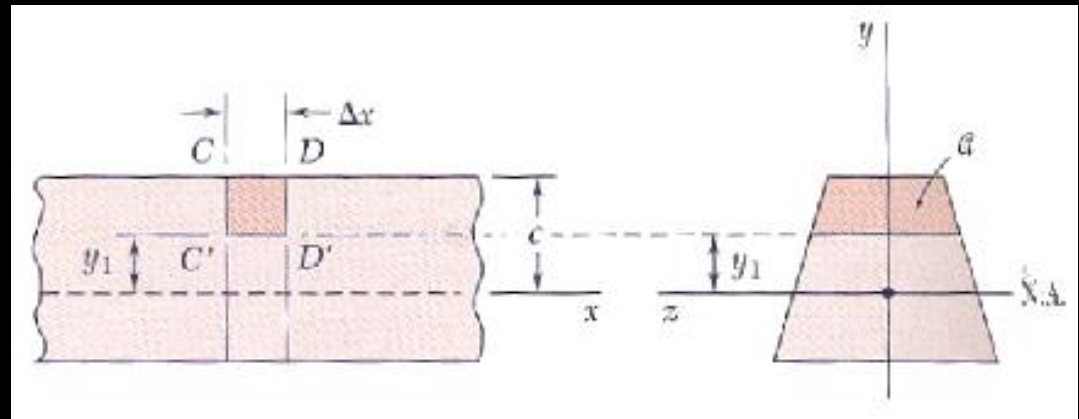
- $Q$  is a moment area



# Moment of Area

- $Q$  is a moment area with respect to the n.a. of area above or below the horizontal

- $Q_{max}$  at  $y=0$   
(neutral axis)



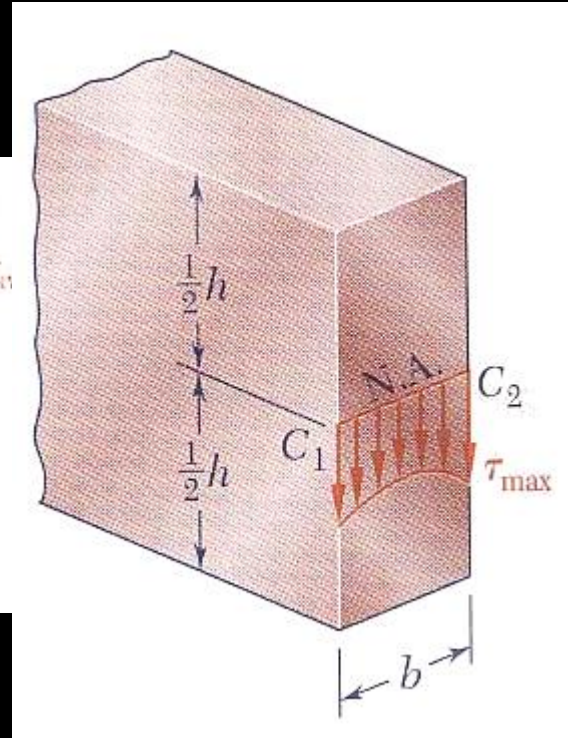
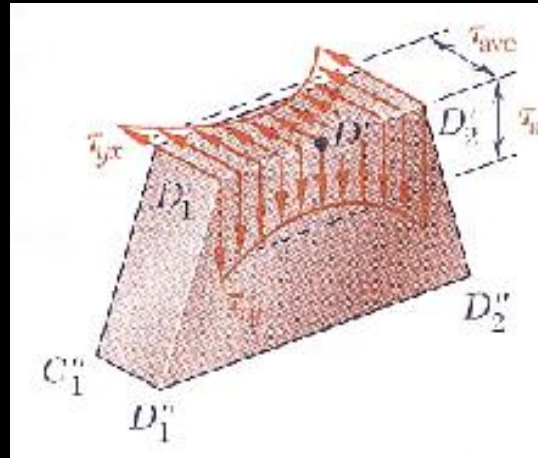
- $q$  is shear flow:

$$q = \frac{V_{longitudinal}}{\Delta x} = \frac{V_T Q}{I}$$

# Shearing Stresses

$$f_v = \frac{V}{\Delta A} = \frac{V}{b \cdot \Delta x}$$

$$f_{v-ave} = \frac{VQ}{Ib}$$



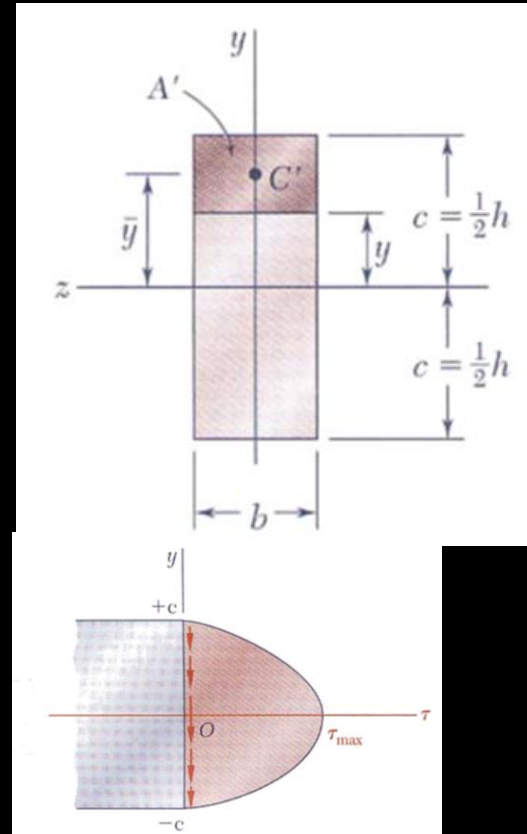
- $f_{v-ave} = 0$  on the top/bottom
- $b$  min may not be with  $Q$  max
- with  $h/4 \geq b$ ,  $f_{v-max} \leq 1.008 f_{v-ave}$

# Rectangular Sections

$$I = \frac{bh^3}{12} \quad Q = A\bar{y} = bh^2/8$$

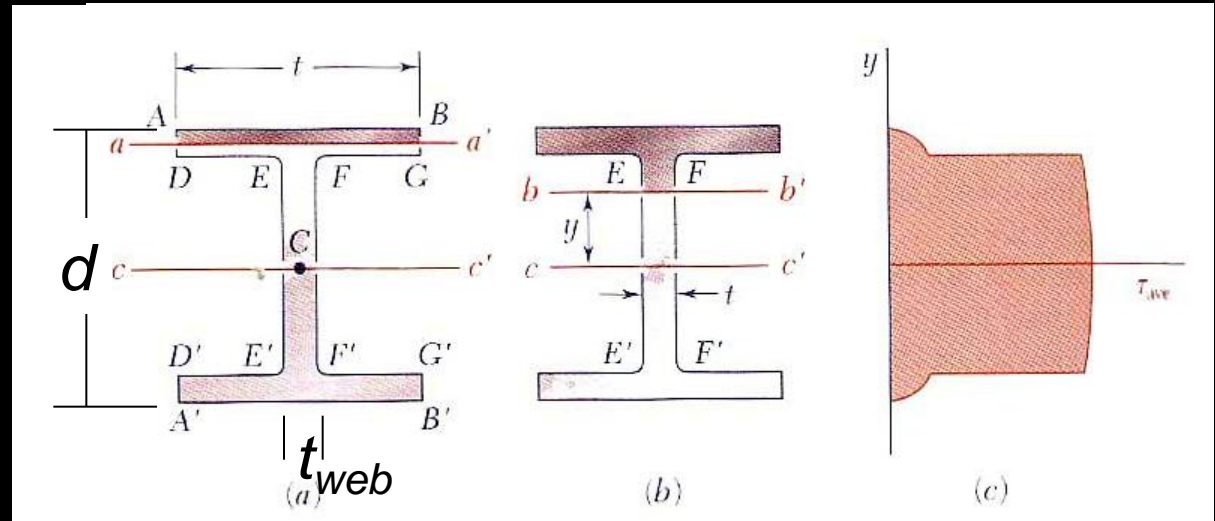
$$f_v = \frac{VQ}{Ib} = \frac{3V}{2A}$$

- $f_{v-max}$  occurs at n.a.



# Steel Beam Webs

- *W and S sections*
  - *b varies*



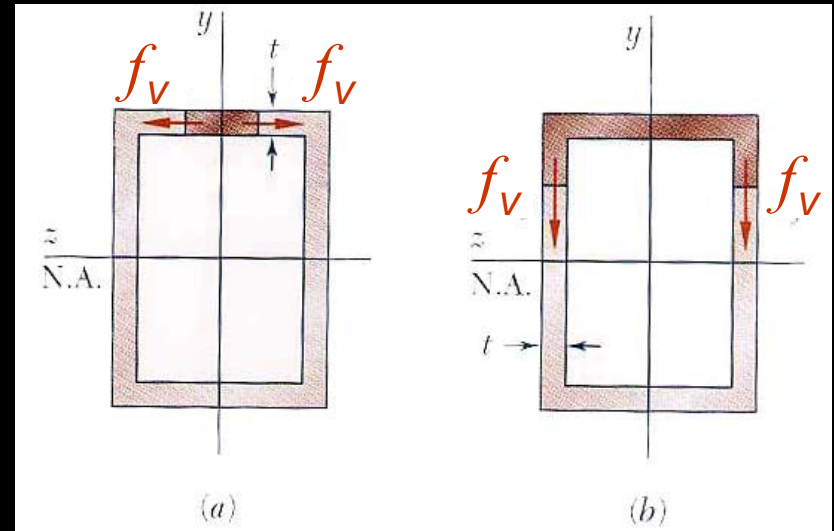
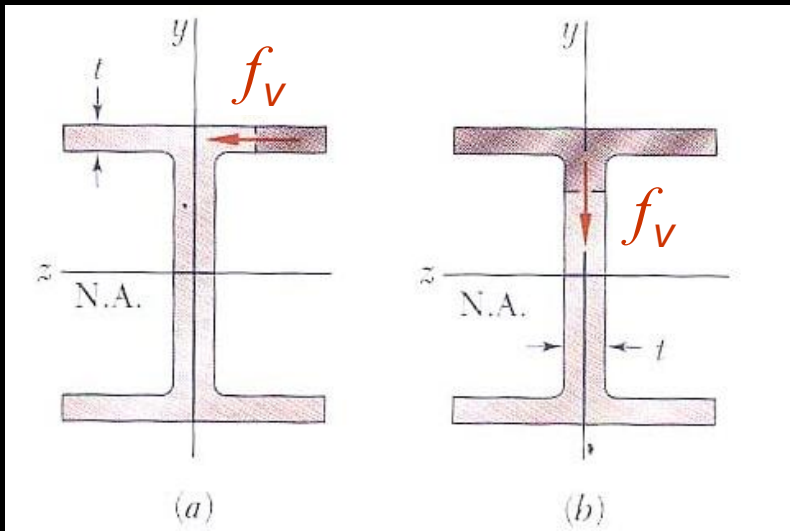
- *stress in flange negligible*
- *presume constant stress in web*

$$f_{v-\max} = \frac{3V}{2A} \approx \frac{V}{A_{web}}$$

# Shear Flow

- loads applied in plane of symmetry
- cut made perpendicular

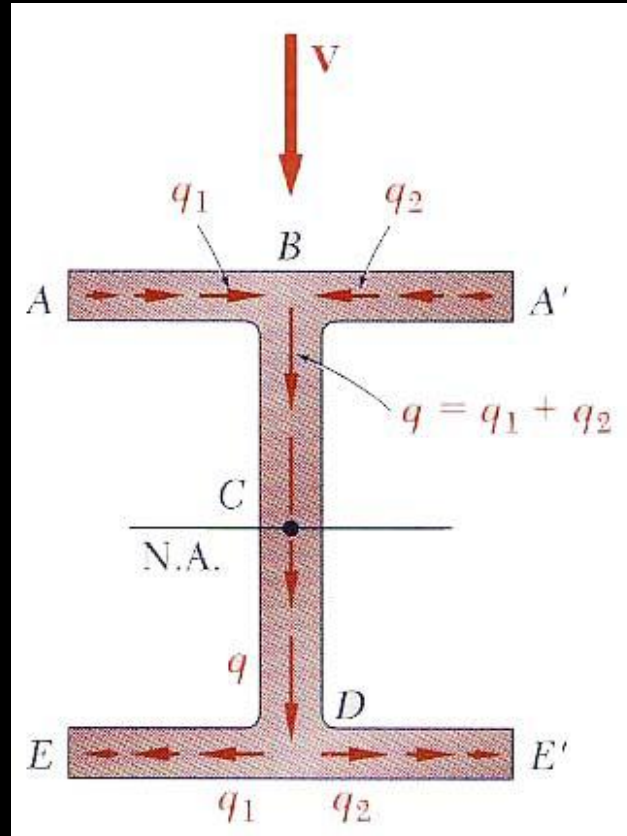
$$q = \frac{VQ}{I}$$



# Shear Flow Quantity

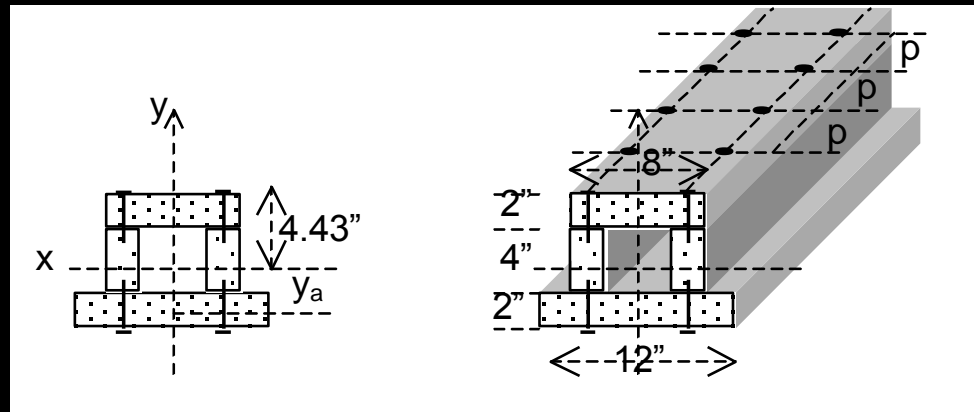
- sketch from  $Q$

$$q = \frac{VQ}{I}$$



# Connectors Resisting Shear

- *plates with*
  - *nails*
  - *rivets*
  - *bolts*
- *splices*



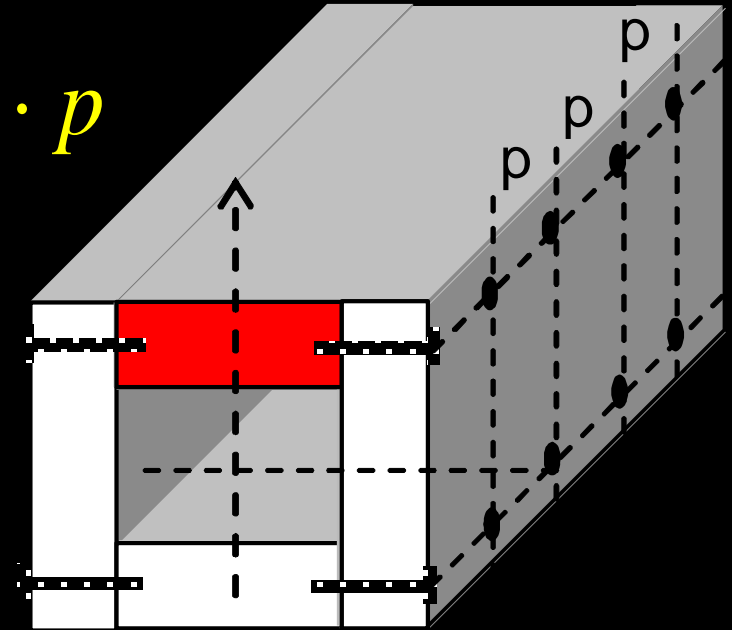
$$\frac{V_{longitudinal}}{p} = \frac{VQ}{I}$$

$$nF_{connector} \geq \frac{VQ_{connected\ area}}{I} \cdot p$$

# Vertical Connectors

- isolate an area with vertical interfaces

$$nF_{connector} \geq \frac{VQ_{connected\ area}}{I} \cdot p$$





# Unsymmetrical Shear or Section

- member can bend and twist
  - not symmetric
  - shear not in that plane
- shear center
  - moments balance

