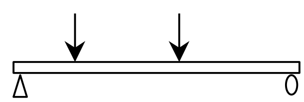
# Beam Structures and Internal Forces

**Notation:**

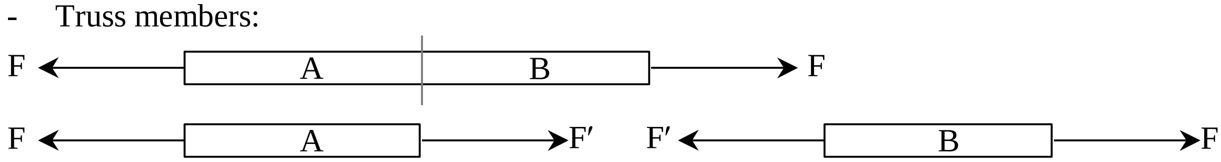
$a$ = algebraic quantity, as is $b$, $c$, $d$
$A$ = name for area
$b$ = intercept of a straight line
$d$ = calculus symbol for differentiation
*(C)* = shorthand for *compression*
$F$ = name for force vectors, as is $P$, $F'$, $P'$
= internal axial force
$F_x$ = force component in the x direction
$F_y$ = force component in the y direction
*FBD* = free body diagram
$L$ = beam span length
$m$ = slope of a straight line
$M$ = internal bending moment
$M(x)$ = internal bending moment as a function of distance $x$
$R$ = name for reaction force vector
*(T)* = shorthand for *tension*
$V$ = internal shear force
$V(x)$ = internal shear force as a function of distance $x$
$w$ = name for distributed load
$W$ = name for total force due to distributed load
$x$ = horizontal distance
$y$ = vertical distance
$^o$ = symbol for order of curve
$\int$ = symbol for integration
$\Delta$ = calculus symbol for small quantity
$\Sigma$ = summation symbol

- BEAMS
  - Important type of structural members (floors, bridges, roofs)
  - Usually long, straight and rectangular
  - Have loads that are usually perpendicular applied at points along the length



Internal Forces 2

- *Internal forces* are those that hold the parts of the member together for equilibrium
  - Truss members:

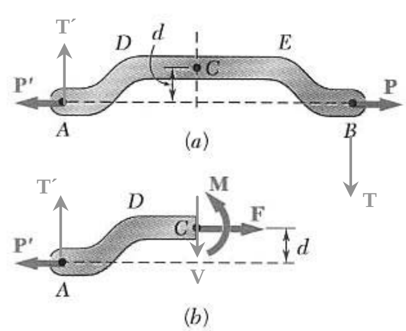



  - For any member:

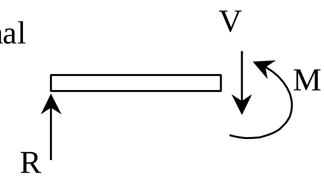
    F = internal *axial force* (perpendicular to cut across section)

    V = internal *shear force* (parallel to cut across section)

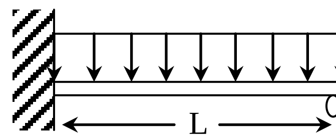
    M = internal *bending moment*



## Support Conditions & Loading




- Most often loads are perpendicular to the beam and cause <u>only</u> internal shear forces and bending moments
- Knowing the internal forces and moments is *necessary* when designing beam size & shape to resist those loads
- Types of loads
  - Concentrated – single load, single moment
  - Distributed – loading spread over a distance, uniform or **non-uniform**.
- Types of supports
  - *Statically determinate*: simply supported, cantilever, overhang (number of unknowns < number of equilibrium equations)
  - *Statically indeterminate*: continuous, fixed-roller, fixed-fixed (number of unknowns < number of equilibrium equations)

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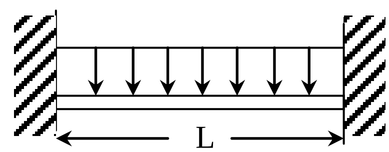
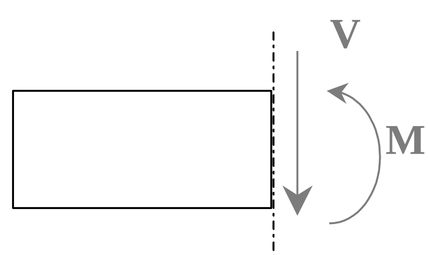



Restrained

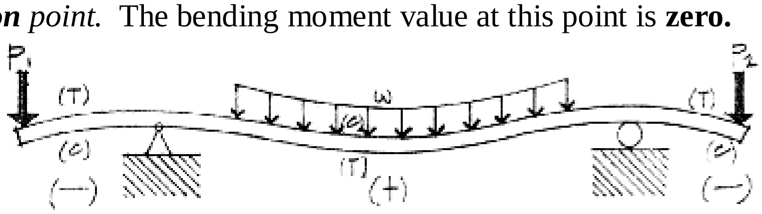
**Sign Conventions for Internal Shear and Bending Moment**
(<u>different</u> from statics and truss members!)




When $\Sigma F_y$ **excluding V** on the left hand side (LHS) section is <u>positive</u>, V will direct <u>down</u> and is considered <u>POSITIVE.</u>

When $\Sigma M$ **excluding M** about the cut on the left hand side (LHS) section causes a smile which could hold water (curl upward), M will be <u>counter clockwise</u> (+) and is considered <u>POSITIVE.</u> 

On the deflected shape of a beam, the point where the shape changes from smile up to frown is called the ***inflection** point.* The bending moment value at this point is **zero.**



**Shear And Bending Moment Diagrams**

The plot of shear and bending moment as they vary across a beam length are *extremely important design tools:* V(x) is plotted on the y axis of the shear diagram, M(x) is plotted on the y axis of the moment diagram.

The *load* diagram is essentially the free body diagram of the beam *with the actual loading (not the equivalent of distributed loads.)*

Maximum Shear and Bending – The maximum *value,* regardless of sign, is important for design.

Method 1: *The Equilibrium Method*

Isolate FDB sections at significant points along the beam and determine V and M at the cut section. The values for V and M can also be written in equation format as functions of the distance to the cut section.

Important Places for FBD cuts
- at supports
- at concentrated loads
- at start and end of distributed loads
- at concentrated moments

Method 2: *The Semigraphical Method*

Relationships exist between the loading and shear diagrams, and between the shear and bending diagrams.

Knowing the *area* of the loading gives the *change in* ***shear (V)***.

Knowing the *area* of the shear gives the *change in* ***bending moment (M)***.

Concentrated loads and moments cause a vertical *jump* in the diagram.

$$\frac{\Delta V}{\Delta x}\underset{\lim\ \to 0}{} = \frac{dV}{dx} = -w$$ (the negative shows it is down because we give *w* a positive value)

$$V_D - V_C = -\int_{x_C}^{x_D} w\,dx =$$ the **area** under the load curve between C & D

***These shear formulas are NOT VALID at discontinuities like concentrated loads*

$$\lim_{\Delta x \to 0} \frac{\Delta M}{\Delta x} = \frac{dM}{dx} = V$$

$$M_D - M_C = \int_{x_C}^{x_D} V dx = \text{ the } \textbf{area} \text{ under the shear curve between C \& D}$$

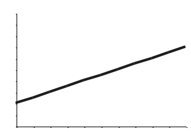
*<u>* These moment formulas ARE VALID even with concentrated loads.</u>*

*<u>*These moment formulas are NOT VALID at discontinuities like applied moments.</u>*

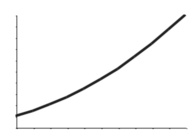
The MAXIMUM BENDING MOMENT from a curve that is <u>continuous</u> can be found when the slope is zero $\left(\frac{dM}{dx} = 0\right)$, which is when the value of the shear is 0.

**Basic Curve Relationships (from calculus) for y(x)**

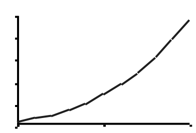
<u>Horizontal Line</u>: $y = b$ *(constant)* and the area (change in shear) = $b \cdot x$, resulting in a:



<u>Sloped Line</u>: $y = mx+b$ and the area (change in shear) = $\frac{\Delta y \cdot \Delta x}{2}$, resulting in a:



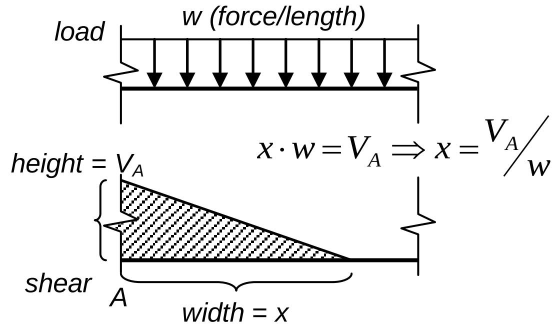
<u>Parabolic Curve</u>: $y = ax^2 + b$ and the area (change in shear) = $\frac{\Delta y \cdot \Delta x}{3}$, resulting in a:



<u>3rd Degree Curve</u>: $y = ax^3 + bx^2 + cx + d$

**Free Software Site**: <u>http://www.rekenwonder.com/atlas.htm</u>

BASIC PROCEDURE:




1. Find all support forces.

*V diagram:*

2. At free ends and at simply supported ends, the shear will have a zero value.
3. At the left support, the shear will equal the reaction force.
4. The shear will not change in x until there is another load, where the shear is reduced if the load is negative. If there is a distributed load, the change in shear is the area under the loading.
5. At the right support, the reaction is treated just like the loads of step 4.
6. At the free end, the shear should go to zero.

*M diagram:*

7. At free ends and at simply supported ends, the moment will have a zero value.
8. At the left support, the moment will equal the reaction moment (if there is one).
9. The moment will not change in x until there is another load or applied moment, where the moment is reduced if the applied moment is negative. If there is a value for shear on the V diagram, the change in moment is the area under the shear diagram.

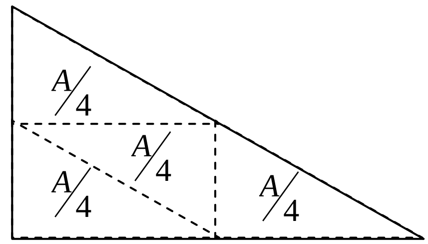
   *For a triangle in the shear diagram, the width will equal the height ÷ w!*

10. At the right support, the moment reaction is treated just like the moments of step 9.
11. At the free end, the moment should go to zero.

**Parabolic Curve Shapes Based on Triangle Orientation**

In order to tell if a parabola curves "up" or "down" from a triangular area in the preceding diagram, the orientation of the triangle is used as a reference.

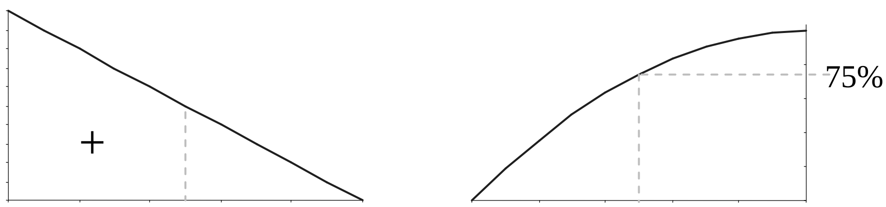
Geometry of Right Triangles



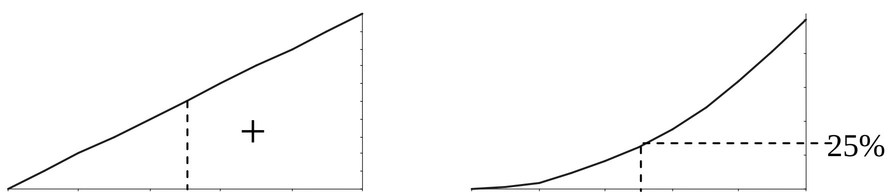

Similar triangles show that four triangles, each with ¼ the area of the large triangle, fit within the large triangle. This means that ¾ of the area is on one side of the triangle, if a line is drawn though the middle of the base, and ¼ of the area is on the other side.

By how a triangle is oriented, we can determine the curve shape in the next diagram.

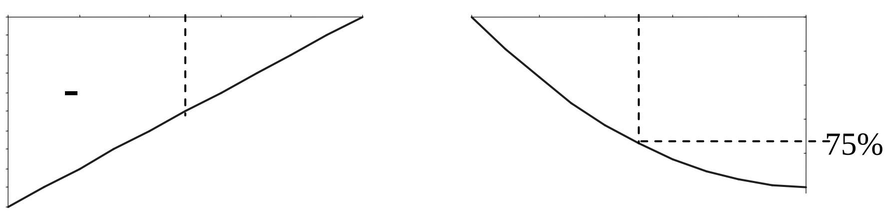
CASE 1: *Positive* triangle with fat side to the *left*.



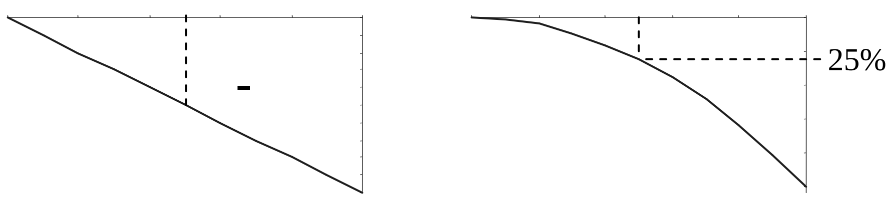

CASE 2: *Positive* triangle with fat side to the *right*.



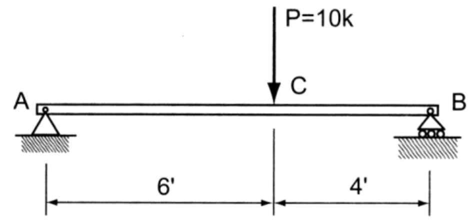

CASE 3: *Negative* triangle with fat side to the *left*.




CASE 4: *Negative* triangle with fat side to the *right*.

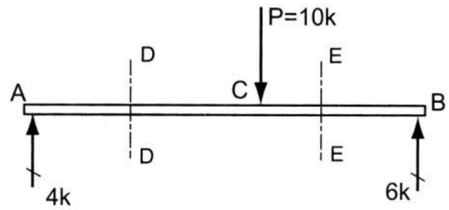


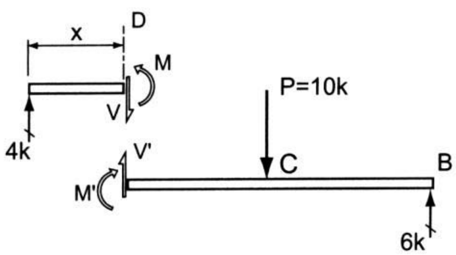
Example 1 (pg 273)

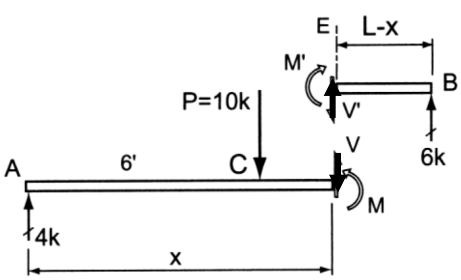


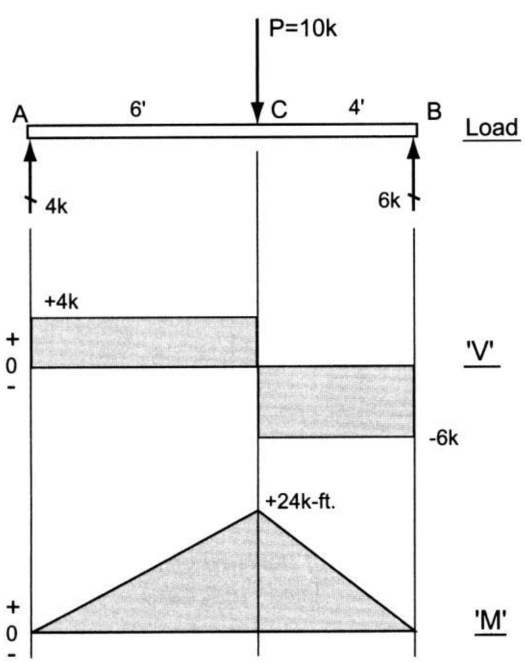

### Example Problem 8.1 (Equilibrium Method)

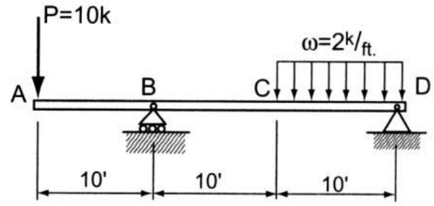
Draw the shear and moment diagram for a simply supported beam with a single concentrated load (Figure 8.8), using the equilibrium method. Verify the general equation from Beam Diagrams & Formulas.



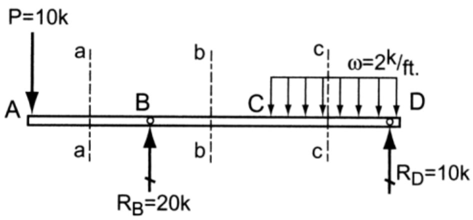



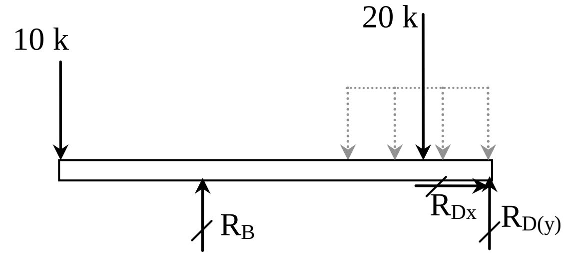


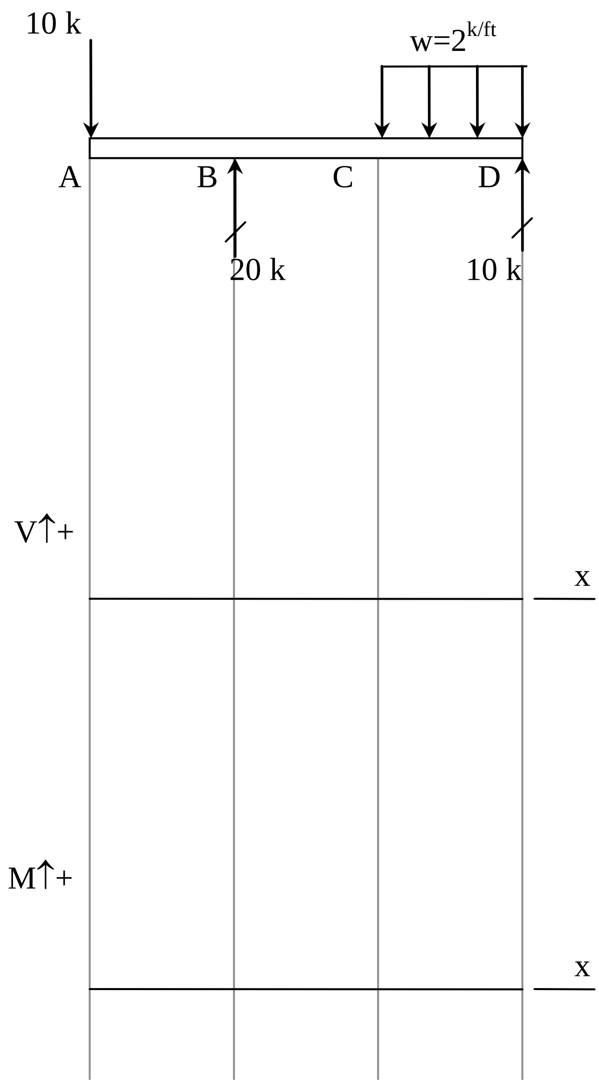

## Example 2 (pg 275)

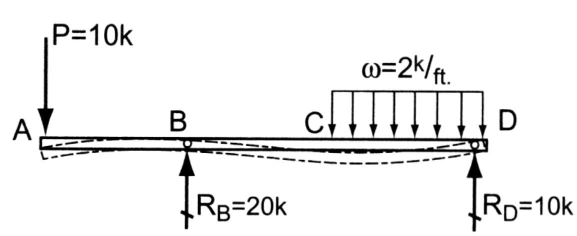
### Example Problem 8.2(Equilibrium Method)

Draw $V$ and $M$ diagrams for an overhang beam (Figure 8.12) loaded as shown. Determine the critical $V_{max}$ and $M_{max}$ locations and magnitudes.



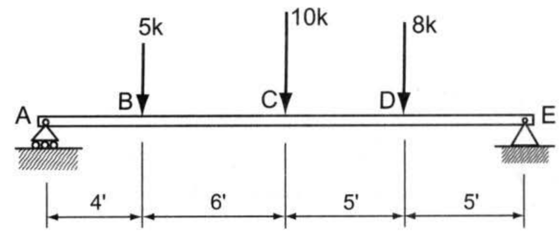



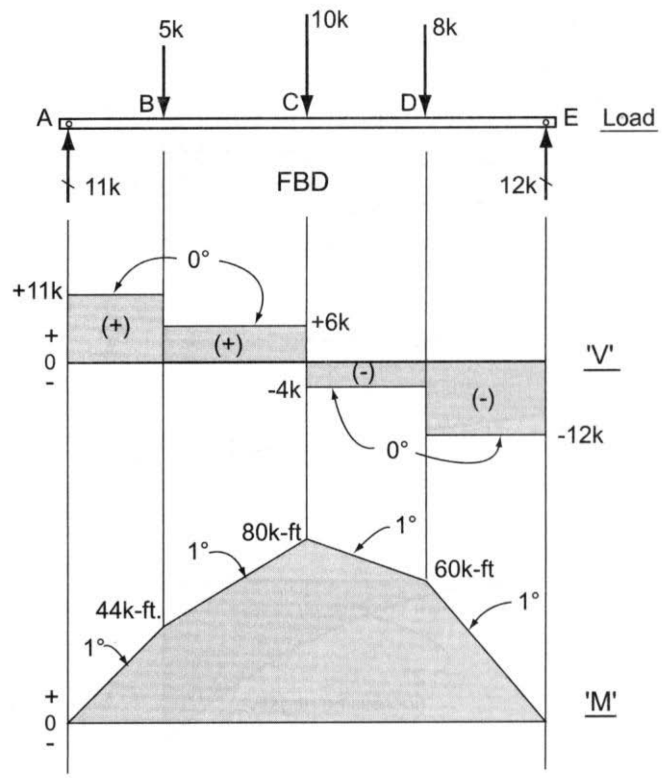



Example 3 (pg 283)

**Example Problem 8.4**

Construct the $V$ and $M$ diagrams for the girder that supports three concentrated loads as shown in Figure 8.28.

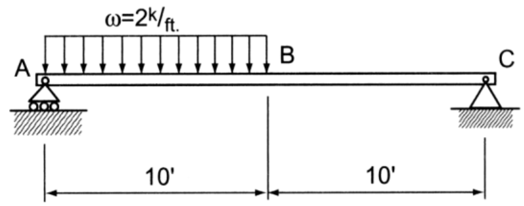


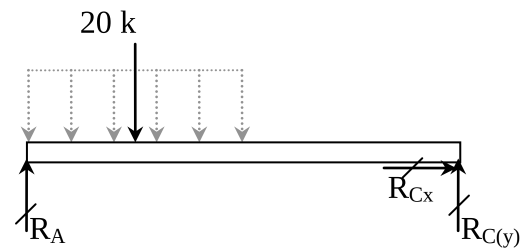



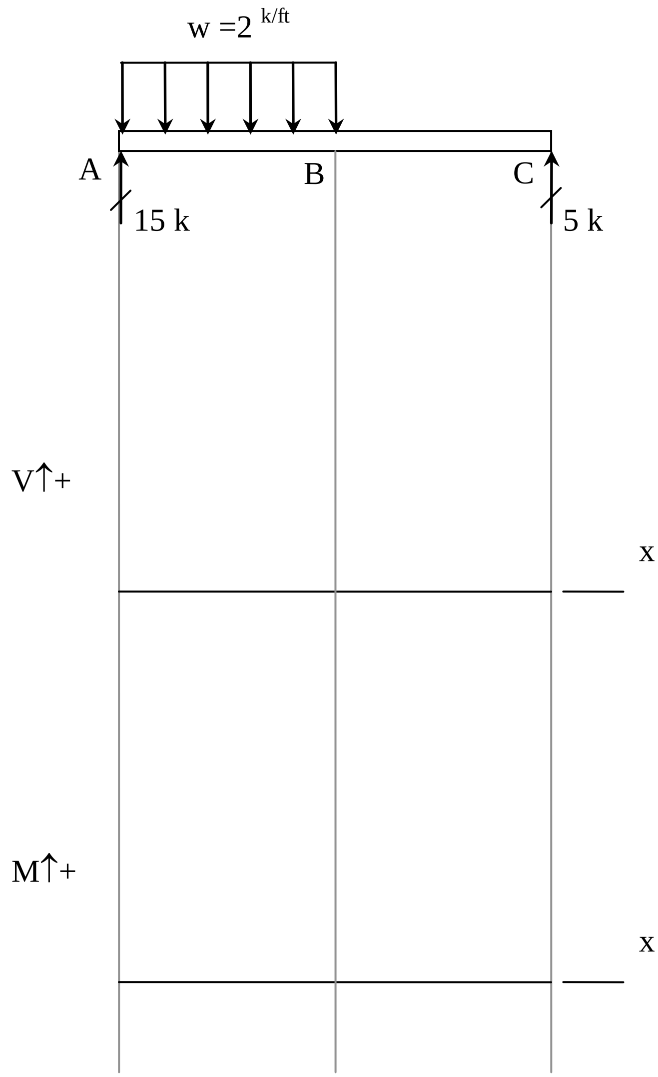
## Example 4 (pg 285)

### Example Problem 8.6 (Semi-Graphical Method)

Construct $V$ and $M$ diagrams for the simply supported beam $ABC$, which is subjected to a partial uniform load (Figure 8.30).



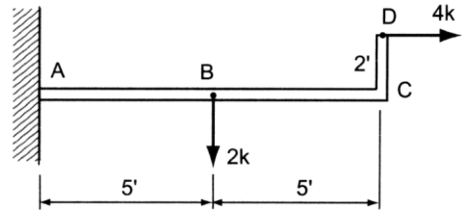



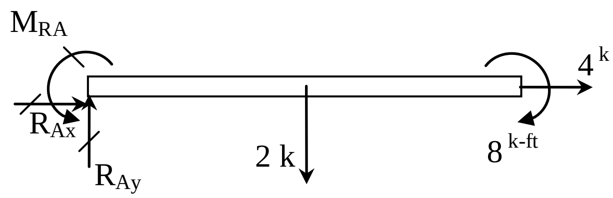



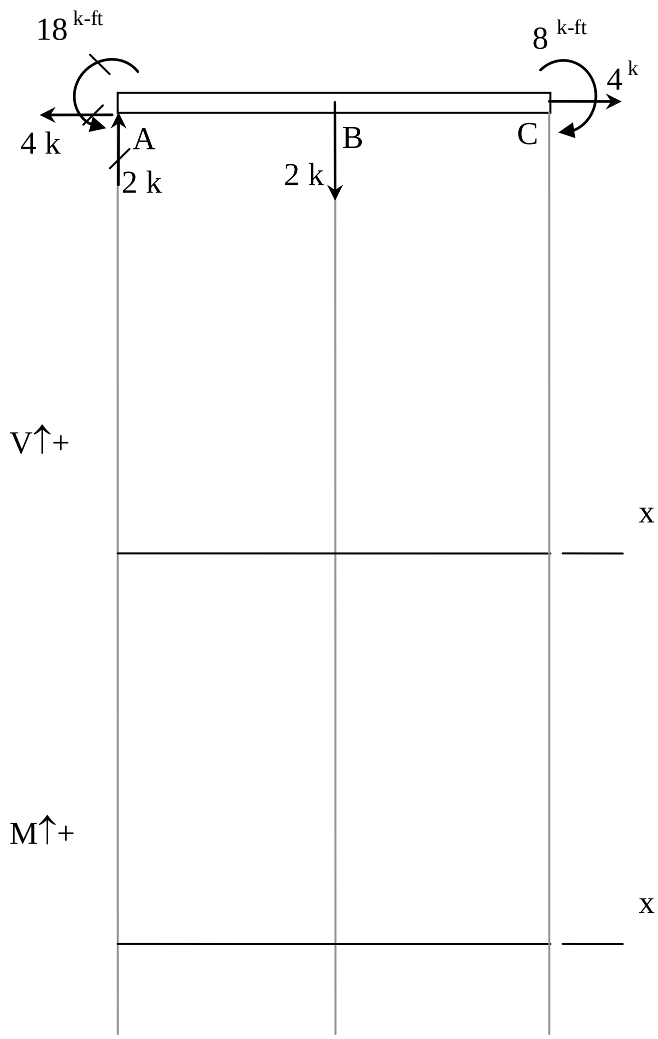
Example 5 (pg 286)

**Example Problem 8.7 (Figure 8.31)**

For a cantilever beam with an upturned end, draw the load, shear, and moment diagrams.



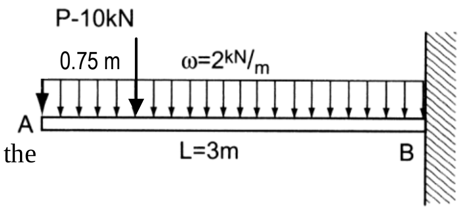



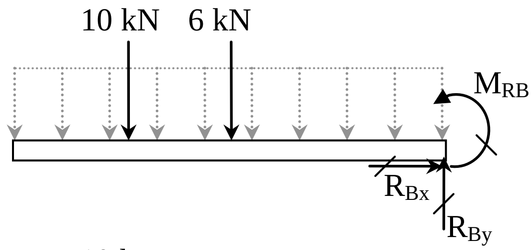
## Example 6 (*changed from* pg 284)

**Example Problem 8.5 (Semi-Graphical Method)**

A cantilever beam supports a uniform load of $\omega = 2\,{}^{kN}\!/_{m}$ over its entire span, plus a concentrated load of 10 kN at 0.75 m from the free end. Construct the *V* and *M* diagrams (Figure 8.29).




SOLUTION:
Determine the reactions:




$\sum F_x = R_{Bx} = 0$ $R_{Bx}$ = 0 kN

$\sum F_y = -10kN - (2\,{}^{kN}\!/_{m})(3m) + R_{By} = 0$ $R_{By}$ = 16 kN

$\sum M_B = (10kN)(2.25m) + (6kN)(1.5m) + M_{RB} = 0$ $M_{RB}$ = -31.5 kN-m

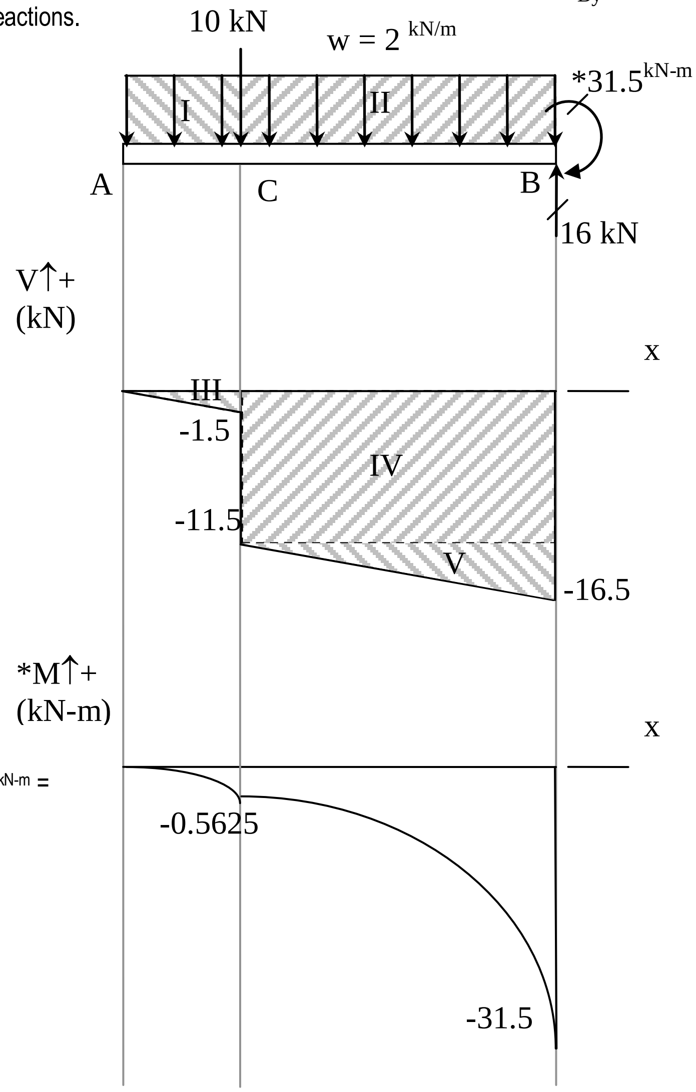
Draw the load diagram *with the distributed load* as given with the reactions.

<u>Shear Diagram:</u>

Label the load areas and calculate:

Area I = (-2 kN/m)(0.75 m) = -1.5 kN
Area II = (-2 kN/m)(2.25 m) = -4.5 kN

$V_A$ = 0
$V_C$ = $V_A$ + Area I = 0 -1.5 kN = -1.5 kN *and*
$V_C$ = $V_C$ + force at C = -1.5 kN -10 kN= -11.5 kN
$V_B$ = $V_C$ + Area II = -11.5 kN -4.5 kN = -16 kN *and*
$V_B$ = $V_B$ + force at B = -16 kN +16 kN= 0 kN

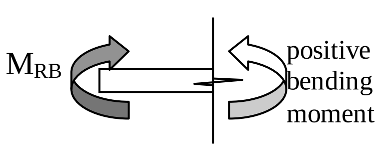



<u>Bending Moment Diagram:</u>

Label the load areas and calculate:

Area III = (-1.5 kN)(0.75 m)/2 = -0.5625 kN-m
Area IV = (-11.5 kN)(2.25m) = -25.875 kN-m
Area V = (-16 – 11.5 kN)(2.25m)/2 = -5.0625 kN-m

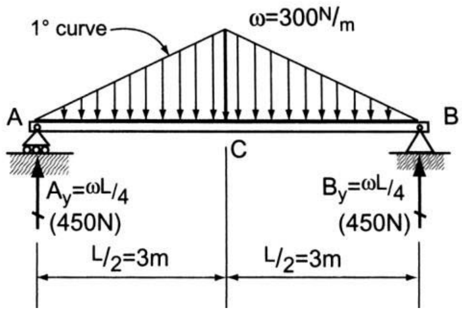
$M_A$ = 0
$M_C$ = $M_A$ + Area III = 0 - 0.5625 kN-m = - 0.5625 kN-m
$M_B$ = $M_C$ + Area IV + Area V = - 0.5625 kN-m - 25.875 kN-m - 5.0625 kN-m =
= -31.5 kN-m *and*
$M_B$ = $M_B$ + moment at B = -31.5 kN-m + 31.5 kN-m = 0 kN-m



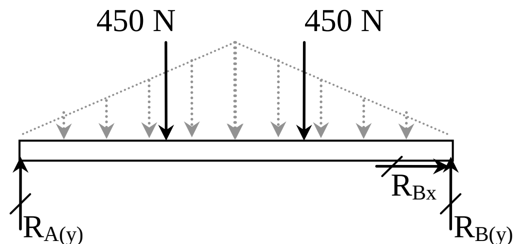
## Example 7 (pg 287)

**Example Problem 8.9 (Figure 8.33)**

A header beam spanning a large opening in an industrial building supports a triangular load as shown. Construct the $V$ and $M$ diagrams and label the peak values.




SOLUTION:
Determine the reactions:




$\sum F_x = R_{Bx} = 0$ $R_{Bx}$ = 0 kN

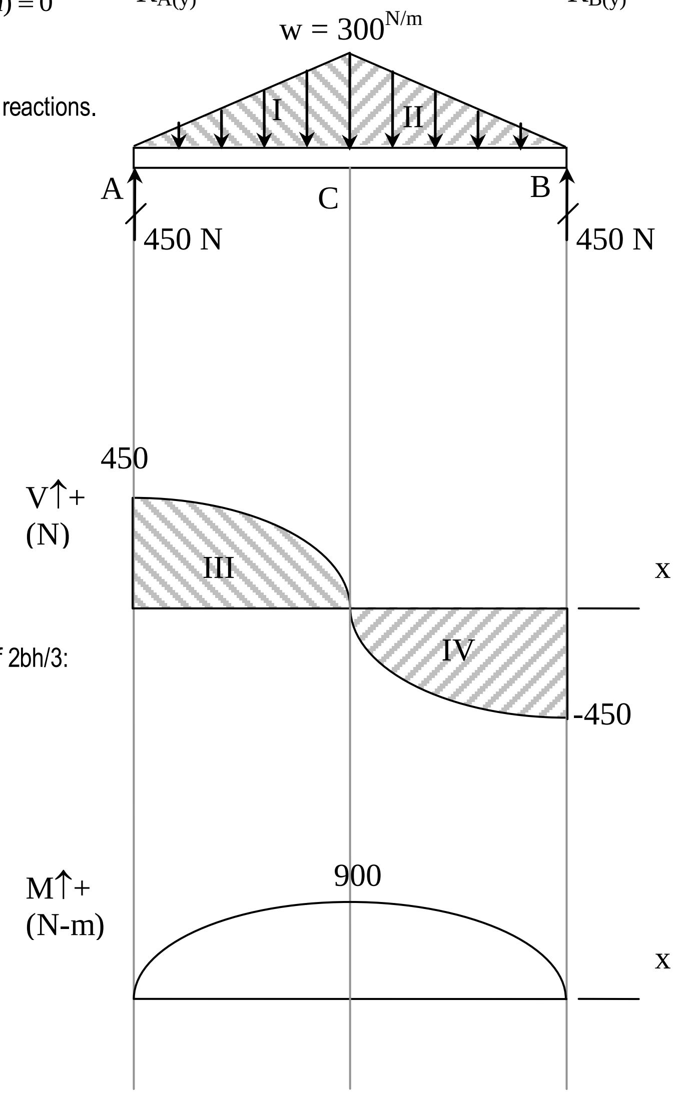
$\sum F_y = R_{Ay} - (300\,{}^{N}\!/_{m})(3m)\,{}^{1}\!/_{2} + -(300\,{}^{N}\!/_{m})(3m)\,{}^{1}\!/_{2} + R_{By} = 0$

or by load tracing $R_{Ay}$ & $R_{By}$ = ( wL/2)/2 = (300 N/m)(6 m)/4 = 450 N

$\sum M_A = -(450N)({}^{2}\!/_{3} \times 3m) - (450N)(3 + {}^{1}\!/_{3} \times 3m) + R_{By}(6m) = 0$

$R_{By}$ = 450 N

Draw the load diagram *with the distributed load* as given with the reactions.




Shear Diagram:

Label the load areas and calculate:

Area I = (-300 N/m)(3 m)/2 = -450 N
Area II = -300 N/m)(3 m)/2 = -450 N

$V_A$ = 0 *and* $V_A$ = $V_A$ + force at A = 0 + 450 N = 450 N
$V_C$ = $V_A$ + Area I = 450 N -450 N = 0 N
$V_B$ = $V_C$ + Area II = 0 N – 450 N = -450 N *and*
$V_B$ = $V_B$ + force at B = -450 N + 450 N = 0 N

Bending Moment Diagram:

Label the load areas and calculate:

Areas III & IV happen to be parabolic segments with an area of 2bh/3:
Area III = 2(3 m)(450 N)/3 = 900 N-m
Area IV = -2(3 m)(450 N)/3 = -900 N-m

$M_A$ = 0
$M_C$ = $M_A$ + Area III = 0 + 900 N-m = 900 N-m
$M_B$ = $M_C$ + Area IV = 900 N-m - 900 N-m = 0

We can prove that the area is a parabolic segment by using the equilibrium method at C:

$\sum M_{section\,cut} = M_C - (450N)(3m) + (450N)({}^{1}\!/_{3} \times 3m) = 0$

so $M_c$ = 900 N-m