ARCH 331: Practice Quiz 5

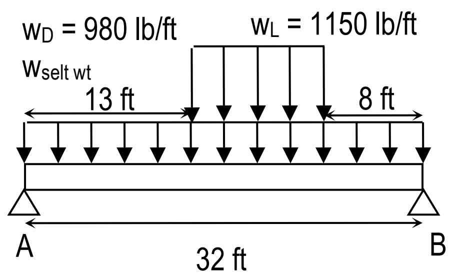
Note: No aids are allowed for part 1. One side of a letter sized paper with notes is allowed during part 2, along with a silent, ***non-programmable*** *calculator. There are reference charts for part 2, shown on pages 2-6.*

Clearly show your work and answer.

Part 1) Worth 5 points (conceptual questions)

Part 2) Worth 45 points

(NOTE: The loading type [ex, live, dead, wind...] and sizes *can and will* *be changed for the quiz with respect to the beam diagrams and formula provided. The support condition, section,* ***and bracing*** *for the column* *can and will* *be changed.)*



A wide flange beam of A992 steel (F_y = 50 ksi, E = 29 x 10^3 ksi) is needed to span 32 ft and support uniformly distributed loads of 980 lb/ft of dead load (from materials), the self weight, and 1150 lb/ft of live load over a length of 11 feet as shown. The beam is simply supported with a maximum unbraced length of 15.5 ft.

a) Select the most economical beam adequate for flexural strength using LRFD design and the chart provided (including self weight). *Assume that the dead load determines the location of the maximum moment and superimpose the live load moment there.*

b) Determine the minimum moment of inertia required such that the dead load deflection does not exceed 1.25 inches assuming a self weight of 60 lb/ft. [*or live load deflection– using* $\Delta_{max} = wl^4/(152EI)$ *because there is no equation – does not exceed 0.8 in; or total deflection assuming that the dead load determines the location of the maximum moment – using* $\Delta_x = 5wx(l^3 - 3lx^2 + 2x^3)/(192EI)$ *because there is no equation for the partial distributed load – does not exceed 1.75 in.]*

A W 250 x 49 metric column is 5.75 m tall of A36 steel (F_y = 250 MPa, E = 200 x 10^3 MPa). The base is fixed and the top is pinned in the weak axis, while the strong axis is considered pinned at the top and bottom (no picture and approximated conditions). The section properties are:

A = 6260 mm^2, I_x = 70.7 x 10^6 mm^4, r_x = 106 mm, I_y = 15.2 x 10^6 mm^4, r_y = 49.3 mm

c) If the column is to support 200 kN of dead load and 600 kN of live load, is it adequate for design using LRFD?

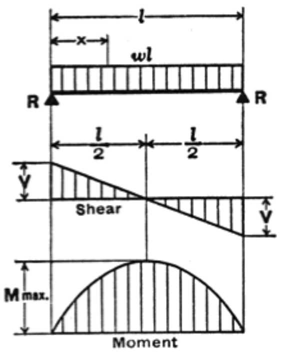
Answers – Not provided on actual quiz!

a) M_u = 279.4 k-ft, use W21x55 (M_u* < 319 k-ft)
b) $I_{req'd}$ = 676.9 in^4 (dead only) [$I_{req'd\text{-}live\ only}$ = 590.9 in^4; $I_{req'd\text{-}total}$ = 750.8 in^4]
c) ϕP_n = 886 kN $\therefore$ Not OK (weak axis governs because $\phi P_{n\text{-}strong}$ = 1199 kN)

Disclaimer: Answers have NOT been painstakingly researched.

REFERENCE CHARTS FOR QUIZ 5

1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load . . . . $= wl$

R = V . . . . $= \frac{wl}{2}$

V_x . . . . $= w\left(\frac{l}{2} - x\right)$

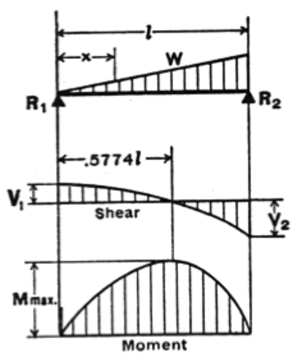
M max. (at center) . . . . $= \frac{wl^2}{8}$

M_x . . . . $= \frac{wx}{2}(l - x)$

Δmax. (at center) . . . . $= \frac{5wl^4}{384EI}$

Δx . . . . $= \frac{wx}{24EI}(l^3 - 2lx^2 + x^3)$

2. SIMPLE BEAM—LOAD INCREASING UNIFORMLY TO ONE END



Total Equiv. Uniform Load . . . . $= \frac{16W}{9\sqrt{3}} = 1.0264W$

$R_1 = V_1$ . . . . $= \frac{W}{3}$

$R_2 = V_2$ max. . . . . $= \frac{2W}{3}$ $W = \frac{wl}{2}$

V_x . . . . $= \frac{W}{3} - \frac{Wx^2}{l^2}$

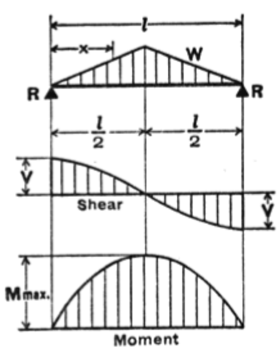
M max. $\left(\text{at } x = \frac{l}{\sqrt{3}} = .5774l\right)$ . . $= \frac{2Wl}{9\sqrt{3}} = .1283\ Wl$

M_x . . . . $= \frac{Wx}{3l^2}(l^2 - x^2)$

Δmax. $\left(\text{at } x = l\sqrt{1 - \sqrt{\frac{8}{15}}} = .5193l\right)$ $= .01304\ \frac{Wl^3}{EI}$

Δx . . . . $= \frac{Wx}{180EIl^2}(3x^4 - 10l^2x^2 + 7l^4)$

3. SIMPLE BEAM—LOAD INCREASING UNIFORMLY TO CENTER



Total Equiv. Uniform Load . . . . $= \frac{4W}{3}$ $W = \frac{wl}{2}$

R = V . . . . $= \frac{W}{2}$

V_x $\left(\text{when } x < \frac{l}{2}\right)$ . . . . . $= \frac{W}{2l^2}(l^2 - 4x^2)$

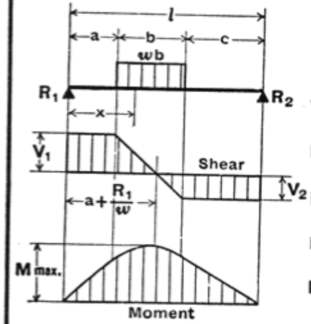
M max. (at center) . . . . . $= \frac{Wl}{6}$

M_x $\left(\text{when } x < \frac{l}{2}\right)$ . . . . . $= Wx\left(\frac{1}{2} - \frac{2x^2}{3l^2}\right)$

Δmax. (at center) . . . . . $= \frac{Wl^3}{60EI}$

Δx $\left(\text{when } x < \frac{l}{2}\right)$ . . . . . $= \frac{Wx}{480EIl^2}(5l^2 - 4x^2)^2$

4. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED



$R_1 = V_1$ (max. when a < c) . . . $= \frac{wb}{2l}(2c + b)$

$R_2 = V_2$ (max. when a > c) . . . $= \frac{wb}{2l}(2a + b)$

V_x (when x > a and < (a + b)) . $= R_1 - w(x - a)$

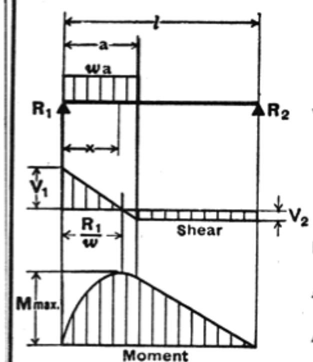
M max. $\left(\text{at } x = a + \frac{R_1}{w}\right)$ . . . . $= R_1\left(a + \frac{R_1}{2w}\right)$

M_x (when x < a) . . . . . . $= R_1x$

M_x (when x > a and < (a + b)) . $= R_1x - \frac{w}{2}(x - a)^2$

M_x (when x > (a + b)) . . . . $= R_2(l - x)$

5. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED AT ONE END



$R_1 = V_1$ max. . . . . $= \frac{wa}{2l}(2l - a)$

$R_2 = V_2$ . . . . $= \frac{wa^2}{2l}$

V_x (when x < a) . . . . . . $= R_1 - wx$

M max. $\left(\text{at } x = \frac{R_1}{w}\right)$ . . . . . . $= \frac{R_1^2}{2w}$

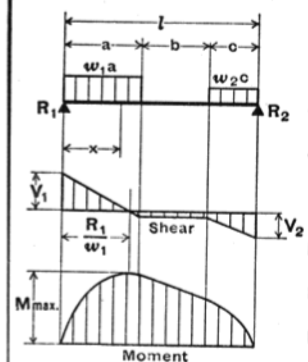
M_x (when x < a) . . . . . . $= R_1x - \frac{wx^2}{2}$

M_x (when x > a) . . . . . . $= R_2(l - x)$

Δx (when x < a) . . . . . . $= \frac{wx}{24EIl}\left(a^2(2l-a)^2 - 2ax^2(2l-a) + lx^3\right)$

Δx (when x > a) . . . . . . $= \frac{wa^2(l-x)}{24EIl}(4xl - 2x^2 - a^2)$

6. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED AT EACH END



$R_1 = V_1$ . . . . $= \frac{w_1a(2l - a) + w_2c^2}{2l}$

$R_2 = V_2$ . . . . $= \frac{w_2c(2l - c) + w_1a^2}{2l}$

V_x (when x < a) . . . . . . $= R_1 - w_1x$

V_x (when x > a and < (a + b)) . $= R_1 - w_1a$

V_x (when x > (a + b)) . . . . $= R_2 - w_2(l - x)$

M max. $\left(\text{at } x = \frac{R_1}{w_1} \text{ when } R_1 < w_1a\right)$. $= \frac{R_1^2}{2w_1}$

M max. $\left(\text{at } x = l - \frac{R_2}{w_2} \text{ when } R_2 < w_2c\right)$ $= \frac{R_2^2}{2w_2}$

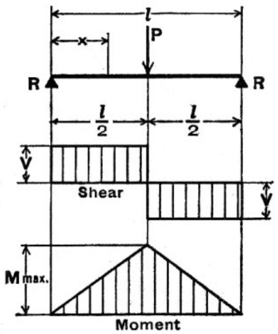
M_x (when x < a) . . . . . . $= R_1x - \frac{w_1x^2}{2}$

M_x (when x > a and < (a + b)) . $= R_1x - \frac{w_1a}{2}(2x - a)$

M_x (when x > (a + b)) . . . . $= R_2(l - x) - \frac{w_2(l - x)^2}{2}$

REFERENCE CHARTS FOR QUIZ 5

7. SIMPLE BEAM—CONCENTRATED LOAD AT CENTER



Total Equiv. Uniform Load . . . . . . $= 2P$

$R = V$ . . . . $= \frac{P}{2}$

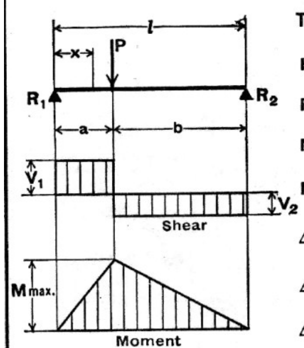
M max. (at point of load) . . . . . . $= \frac{Pl}{4}$

M_x (when $x < \frac{l}{2}$) . . . . . . $= \frac{Px}{2}$

Δmax. (at point of load) . . . . . . $= \frac{Pl^3}{48EI}$

Δ_x (when $x < \frac{l}{2}$) . . . . . . $= \frac{Px}{48EI}(3l^2 - 4x^2)$

8. SIMPLE BEAM—CONCENTRATED LOAD AT ANY POINT



Total Equiv. Uniform Load . . . . . . $= \frac{8Pab}{l^2}$

$R_1 = V_1$ (max. when $a < b$) . . . . . $= \frac{Pb}{l}$

$R_2 = V_2$ (max. when $a > b$) . . . . . $= \frac{Pa}{l}$

M max. (at point of load) . . . . . $= \frac{Pab}{l}$

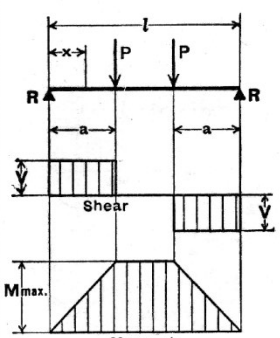
M_x (when $x < a$) . . . . . $= \frac{Pbx}{l}$

Δmax. $\left(\text{at } x = \sqrt{\frac{a(a+2b)}{3}} \text{ when } a > b\right)$ $= \frac{Pab(a+2b)\sqrt{3a(a+2b)}}{27EIl}$

Δa (at point of load) . . . . . $= \frac{Pa^2b^2}{3EIl}$

Δ_x (when $x < a$) . . . . . $= \frac{Pbx}{6EIl}(l^2 - b^2 - x^2)$

9. SIMPLE BEAM—TWO EQUAL CONCENTRATED LOADS SYMMETRICALLY PLACED



Total Equiv. Uniform Load . . . . . . $= \frac{8Pa}{l}$

$R = V$ . . . . $= P$

M max. (between loads) . . . . . . $= Pa$

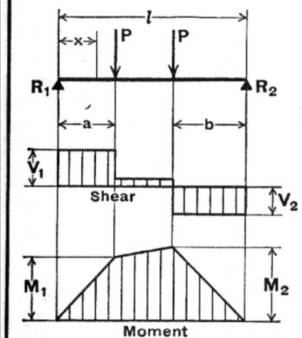
M_x (when $x < a$) . . . . . . $= Px$

Δmax. (at center) . . . . $= \frac{Pa}{24EI}(3l^2 - 4a^2)$

Δ_x (when $x < a$) . . . . $= \frac{Px}{6EI}(3la - 3a^2 - x^2)$

Δ_x (when $x > a$ and $< (l - a)$) . . . $= \frac{Pa}{6EI}(3lx - 3x^2 - a^2)$

10. SIMPLE BEAM—TWO EQUAL CONCENTRATED LOADS UNSYMMETRICALLY PLACED



$R_1 = V_1$ (max. when $a < b$) . . . . . $= \frac{P}{l}(l - a + b)$

$R_2 = V_2$ (max. when $a > b$) . . . . . $= \frac{P}{l}(l - b + a)$

V_x (when $x > a$ and $< (l - b)$) . . . $= \frac{P}{l}(b - a)$

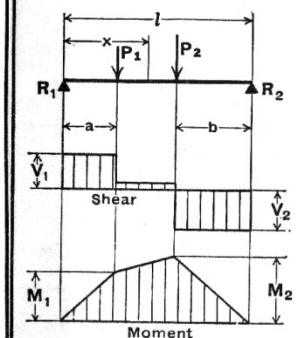
M_1 (max. when $a > b$) . . . . . $= R_1 a$

M_2 (max. when $a < b$) . . . . . $= R_2 b$

M_x (when $x < a$) . . . . $= R_1 x$

M_x (when $x > a$ and $< (l - b)$) . . . $= R_1 x - P(x - a)$

11. SIMPLE BEAM—TWO UNEQUAL CONCENTRATED LOADS UNSYMMETRICALLY PLACED



$R_1 = V_1$ . . . . $= \frac{P_1(l - a) + P_2 b}{l}$

$R_2 = V_2$ . . . . $= \frac{P_1 a + P_2(l - b)}{l}$

V_x (when $x > a$ and $< (l - b)$) . . . $= R_1 - P_1$

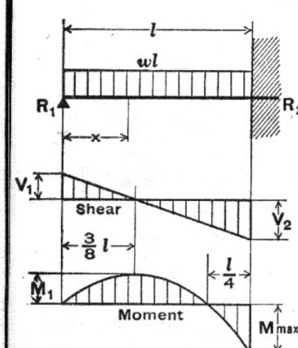
M_1 (max. when $R_1 < P_1$) . . . . $= R_1 a$

M_2 (max. when $R_2 < P_2$) . . . . $= R_2 b$

M_x (when $x < a$) . . . . $= R_1 x$

M_x (when $x > a$ and $< (l - b)$) . . . $= R_1 x - P_1(x - a)$

12. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load . . . . . . $= wl$

$R_1 = V_1$ . . . . $= \frac{3wl}{8}$

$R_2 = V_2$ max. . . . . $= \frac{5wl}{8}$

V_x . . . . $= R_1 - wx$

M max. . . . . $= \frac{wl^2}{8}$

M_1 (at $x = \frac{3}{8}l$) . . . . $= \frac{9}{128}wl^2$

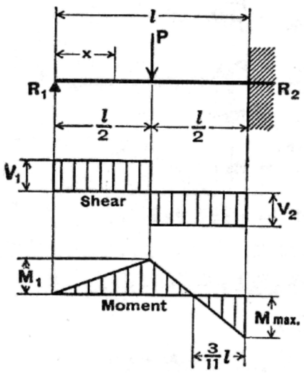
M_x . . . . $= R_1 x - \frac{wx^2}{2}$

Δmax. (at $x = \frac{l}{16}(1 + \sqrt{33}) = .4215l$) . $= \frac{wl^4}{185EI}$

Δ_x . . . . $= \frac{wx}{48EI}(l^3 - 3lx^2 + 2x^3)$

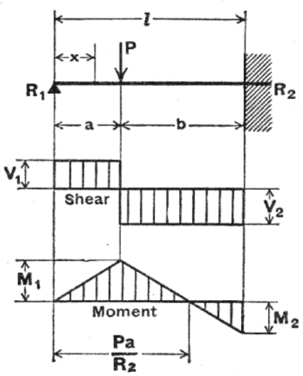
REFERENCE CHARTS FOR QUIZ 5

13. BEAM FIXED AT ONE END, SUPPORTED AT OTHER— CONCENTRATED LOAD AT CENTER



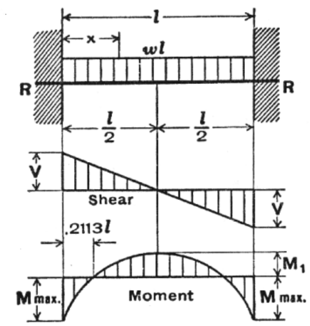
Total Equiv. Uniform Load		$= \frac{3P}{2}$
$R_1 = V_1$		$= \frac{5P}{16}$
$R_2 = V_2$ max.		$= \frac{11P}{16}$
M max.	(at fixed end)	$= \frac{3Pl}{16}$
M_1	(at point of load)	$= \frac{5Pl}{32}$
M_x	$\left(\text{when } x < \frac{l}{2}\right)$	$= \frac{5Px}{16}$
M_x	$\left(\text{when } x > \frac{l}{2}\right)$	$= P\left(\frac{l}{2} - \frac{11x}{16}\right)$
Δmax.	$\left(\text{at } x = l\sqrt{\frac{1}{5}} = .4472l\right)$	$= \frac{Pl^3}{48EI\sqrt{5}} = .009317\frac{Pl^3}{EI}$
Δ_x	(at point of load)	$= \frac{7Pl^3}{768EI}$
Δ_x	$\left(\text{when } x < \frac{l}{2}\right)$	$= \frac{Px}{96EI}(3l^2 - 5x^2)$
Δ_x	$\left(\text{when } x > \frac{l}{2}\right)$	$= \frac{P}{96EI}(x - l)^2(11x - 2l)$

14. BEAM FIXED AT ONE END, SUPPORTED AT OTHER— CONCENTRATED LOAD AT ANY POINT



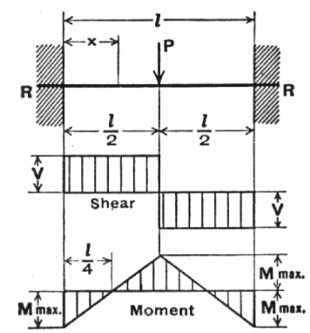
$R_1 = V_1$		$= \frac{Pb^2}{2l^3}(a + 2l)$
$R_2 = V_2$		$= \frac{Pa}{2l^3}(3l^2 - a^2)$
M_1	(at point of load)	$= R_1 a$
M_2	(at fixed end)	$= \frac{Pab}{2l^2}(a + l)$
M_x	(when $x < a$)	$= R_1 x$
M_x	(when $x > a$)	$= R_1 x - P(x - a)$
Δmax.	$\left(\text{when } a < .414l \text{ at } x = l\frac{l^2+a^2}{3l^2-a^2}\right)$	$= \frac{Pa}{3EI}\frac{(l^2 - a^2)^3}{(3l^2 - a^2)^2}$
Δmax.	$\left(\text{when } a > .414l \text{ at } x = l\sqrt{\frac{a}{2l+a}}\right)$	$= \frac{Pab^2}{6EI}\sqrt{\frac{a}{2l+a}}$
Δ_a	(at point of load)	$= \frac{Pa^2b^3}{12EIl^3}(3l + a)$
Δ_x	(when $x < a$)	$= \frac{Pb^2x}{12EIl^3}(3al^2 - 2lx^2 - ax^2)$
Δ_x	(when $x > a$)	$= \frac{Pa}{12EIl^3}(l-x)^2(3l^2x - a^2x - 2a^2l)$

15. BEAM FIXED AT BOTH ENDS—UNIFORMLY DISTRIBUTED LOADS



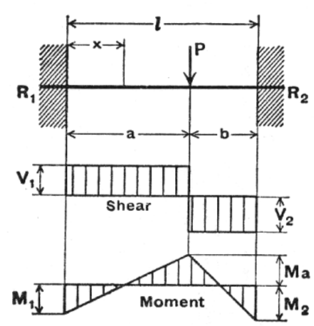
Total Equiv. Uniform Load		$= \frac{2wl}{3}$
$R = V$		$= \frac{wl}{2}$
V_x		$= w\left(\frac{l}{2} - x\right)$
M max.	(at ends)	$= \frac{wl^2}{12}$
M_1	(at center)	$= \frac{wl^2}{24}$
M_x		$= \frac{w}{12}(6lx - l^2 - 6x^2)$
Δmax.	(at center)	$= \frac{wl^4}{384EI}$
Δ_x		$= \frac{wx^2}{24EI}(l - x)^2$

16. BEAM FIXED AT BOTH ENDS—CONCENTRATED LOAD AT CENTER



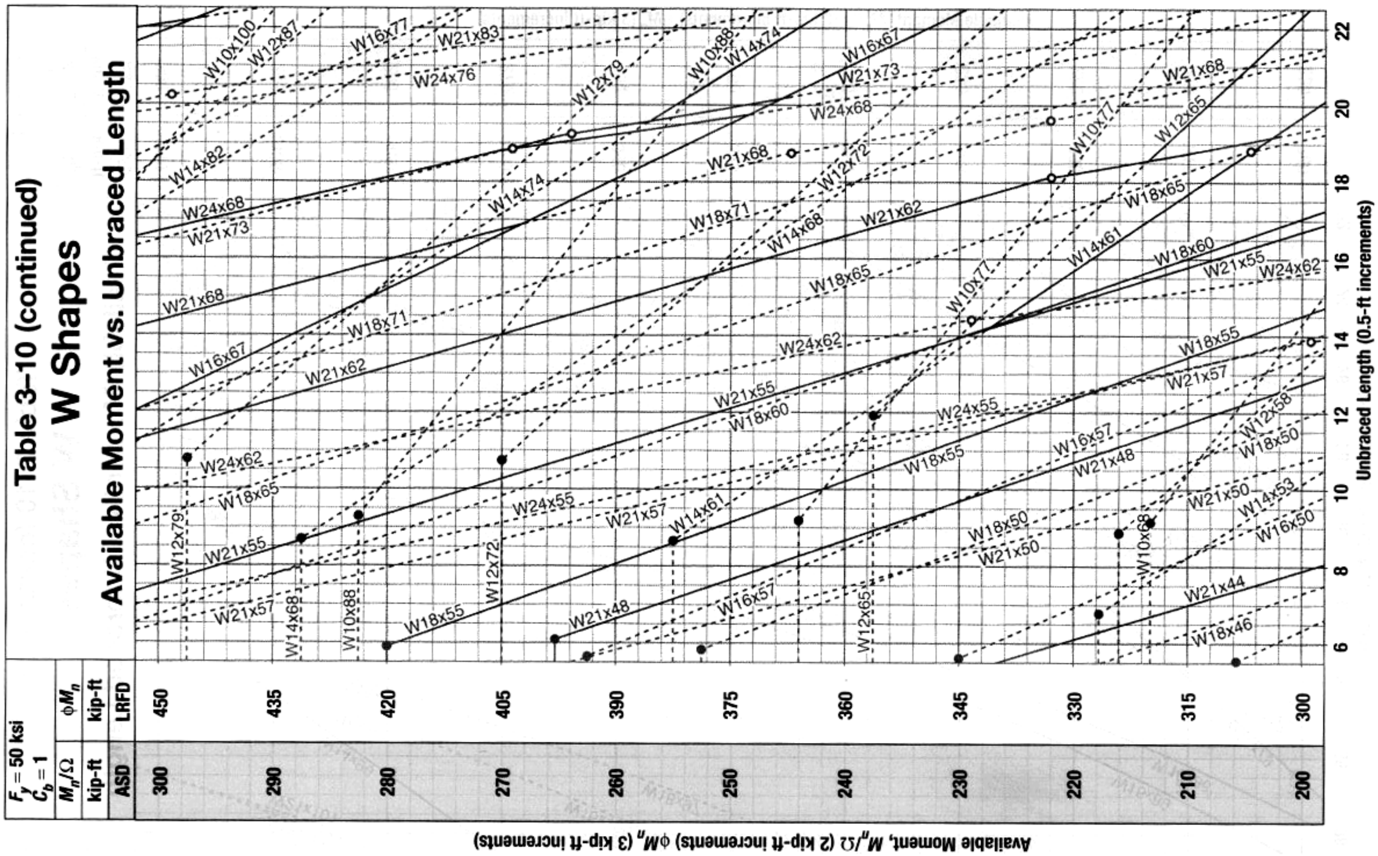
Total Equiv. Uniform Load		$= P$
$R = V$		$= \frac{P}{2}$
M max.	(at center and ends)	$= \frac{Pl}{8}$
M_x	$\left(\text{when } x < \frac{l}{2}\right)$	$= \frac{P}{8}(4x - l)$
Δmax.	(at center)	$= \frac{Pl^3}{192EI}$
Δ_x	$\left(\text{when } x < \frac{l}{2}\right)$	$= \frac{Px^2}{48EI}(3l - 4x)$

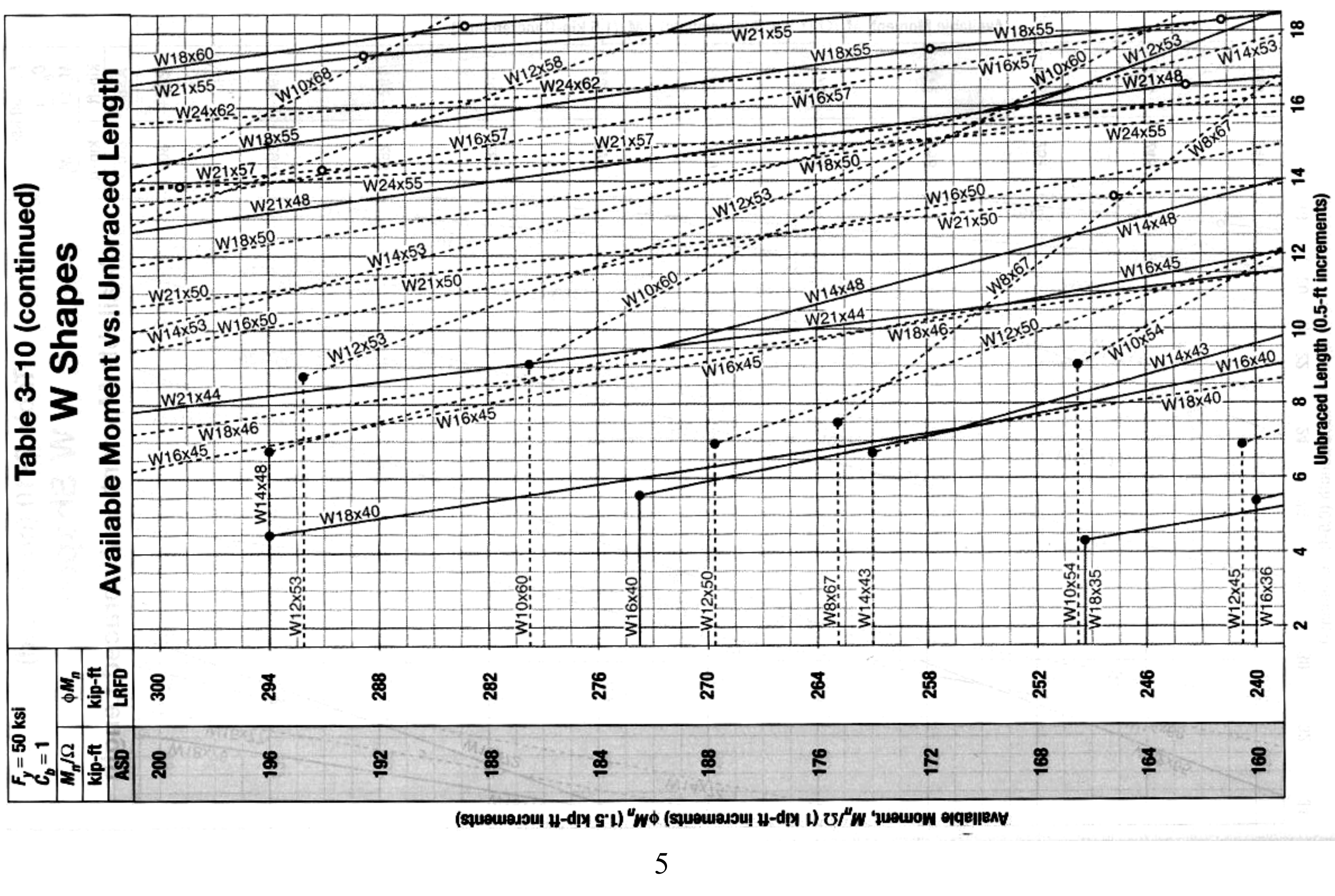
17. BEAM FIXED AT BOTH ENDS—CONCENTRATED LOAD AT ANY POINT



$R_1 = V_1$	(max. when $a < b$)	$= \frac{Pb^2}{l^3}(3a + b)$
$R_2 = V_2$	(max. when $a > b$)	$= \frac{Pa^2}{l^3}(a + 3b)$
M_1	(max. when $a < b$)	$= \frac{Pab^2}{l^2}$
M_2	(max. when $a > b$)	$= \frac{Pa^2b}{l^2}$
M_a	(at point of load)	$= \frac{2Pa^2b^2}{l^3}$
M_x	(when $x < a$)	$= R_1 x - \frac{Pab^2}{l^2}$
Δmax.	$\left(\text{when } a > b \text{ at } x = \frac{2al}{3a + b}\right)$	$= \frac{2Pa^3b^2}{3EI(3a + b)^2}$
Δ_a	(at point of load)	$= \frac{Pa^3b^3}{3EIl^3}$
Δ_x	(when $x < a$)	$= \frac{Pb^2x^2}{6EIl^3}(3al - 3ax - bx)$

REFERENCE CHARTS FOR QUIZ 5



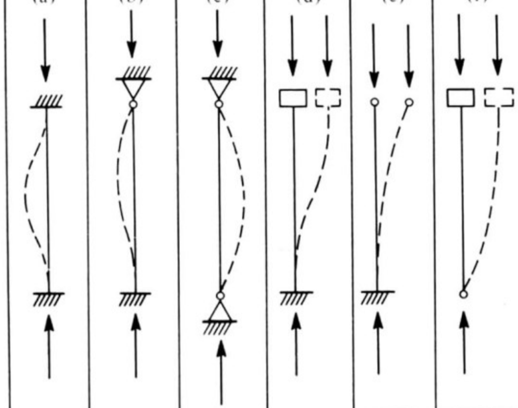


REFERENCE CHARTS FOR QUIZ 5

Available Critical Stress, $\phi_c F_{cr}$, for Compression Members, MPa (F_y = *250* MPa and ϕ_c = 0.90)

KL/r	$\phi_c\ F_{cr}$	*KL/r*	$\phi_c\ F_{cr}$	*KL/r*	$\phi_c\ F_{cr}$	*KL/r*	$\phi_c\ F_{cr}$	*KL/r*	$\phi_c\ F_{cr}$
1	223.4	41	204.5	81	158.2	121	103.4	161	60.1
2	223.4	42	203.6	82	156.8	122	102.0	162	59.4
3	223.3	43	202.7	83	155.4	123	100.7	163	58.6
4	223.2	44	201.8	84	154.1	124	99.4	164	57.9
5	223.1	45	200.8	85	152.7	125	98.1	165	57.2
6	223.0	46	199.8	86	151.4	126	96.9	166	56.5
7	222.8	47	198.9	87	150.0	127	95.6	167	55.9
8	222.6	48	197.9	88	148.6	128	94.3	168	55.2
9	222.4	49	196.9	89	147.2	129	93.0	169	54.5
10	222.2	50	195.8	90	145.8	130	91.8	170	53.9
11	222.0	51	194.8	91	144.5	131	90.5	171	53.3
12	221.7	52	193.8	92	143.1	132	89.3	172	52.7
13	221.4	53	192.7	93	141.7	133	88.0	173	52.0
14	221.1	54	191.6	94	140.3	134	86.7	174	51.4
15	220.8	55	190.5	95	138.9	135	85.5	175	50.9
16	220.4	56	189.4	96	137.5	136	84.2	176	50.3
17	220.0	57	188.3	97	136.1	137	83.0	177	49.7
18	219.6	58	187.1	98	134.7	138	81.8	178	49.2
19	219.2	59	186.0	99	133.4	139	80.6	179	48.6
20	218.7	60	184.8	100	132.0	140	79.5	180	48.1
21	218.3	61	183.7	101	130.6	141	78.3	181	47.5
22	217.8	62	182.5	102	129.2	142	77.2	182	47.0
23	217.3	63	181.3	103	127.8	143	76.2	183	46.5
24	216.7	64	180.1	104	126.4	144	75.1	184	46.0
25	216.2	65	178.8	105	125.0	145	74.1	185	45.5
26	215.6	66	177.6	106	123.6	146	73.1	186	45.0
27	215.0	67	176.4	107	122.3	147	72.1	187	44.5
28	214.4	68	175.1	108	120.9	148	71.1	188	44.1
29	213.7	69	173.9	109	119.5	149	70.2	189	43.6
30	213.1	70	172.6	110	118.2	150	69.2	190	43.1
31	212.4	71	171.3	111	116.8	151	68.3	191	42.7
32	211.7	72	170.0	112	115.4	152	67.4	192	42.3
33	211.0	73	168.7	113	114.1	153	66.5	193	41.8
34	210.2	74	167.4	114	112.7	154	65.7	194	41.4
35	209.4	75	166.1	115	111.4	155	64.8	195	41.0
36	208.7	76	164.8	116	110.0	156	64.0	196	40.5
37	207.9	77	163.5	117	108.7	157	63.2	197	40.1
38	207.0	78	162.2	118	107.3	158	62.4	198	39.7
39	206.2	79	160.8	119	106.0	159	61.6	199	39.3
40	205.4	80	159.5	120	104.7	160	60.8	200	38.9

REFERENCE CHARTS FOR QUIZ 5

	(a)	(b)	(c)	(d)	(e)	(f)
Buckled shape of column shown by dashed line						
Theoretical *K* value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design values when ideal conditions are approximated	0.65	0.80	1.0	1.2	2.10	2.0
End conditions code	 Rotation fixed, Translation fixed; Rotation free, Translation fixed; Rotation fixed, Translation free; Rotation free, Translation free					