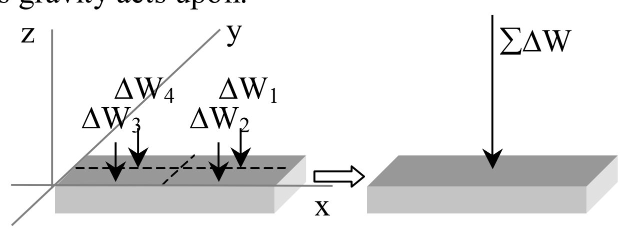
Centers of Gravity - Centroids

Notation:

A = name for area
C = designation for channel section
= name for centroid
F_z = force component in the z direction
L = name for length
O = name for reference origin
Q_x = first moment area about an x axis (using y distances)
Q_y = first moment area about an y axis (using x distances)
t = name for thickness
t_w = thickness of web of wide flange
W = name for force due to weight
= designation for wide flange section
x = horizontal distance
$\bar{x}$ = the distance in the x direction from a reference axis to the centroid of a shape
$\hat{x}$ = the distance in the x direction from a reference axis to the centroid of a composite shape
y = vertical distance
$\bar{y}$ = the distance in the y direction from a reference axis to the centroid of a shape
$\hat{y}$ = the distance in the y direction from a reference axis to the centroid of a composite shape
z = distance perpendicular to *x-y* plane
$\int$ = symbol for integration
Δ = calculus symbol for small quantity
γ = density of a material (unit weight)
Σ = summation symbol

- The cross section shape and how it resists bending and twisting is important to understanding beam and column behavior.

- The *center of gravity* is the location of the equivalent force representing the total weight of a body comprised of particles that each have a mass gravity acts upon.



Resultant force: Over a body of constant thickness in x and y

$$\sum F_z = \sum_{i=1}^{n} \Delta W_i = \boldsymbol{W} \qquad \text{W} = \int \text{dW}$$

Location: $\bar{x}$, $\bar{y}$ is the equivalent location of the force W from all ΔW_i's over all x & y locations (with respect to the moment from each force) from:

$$\sum M_y = \sum_{i=1}^{n} x_i \Delta W_i = \bar{x}\boldsymbol{W} \qquad \bar{x}\boldsymbol{W} = \int x dW \Rightarrow \bar{x} = \frac{\int x dW}{\boldsymbol{W}} \text{ OR } \boxed{\bar{x} = \frac{\Sigma(x\Delta W)}{\boldsymbol{W}}}$$

$$\sum M_x = \sum_{i=1}^{n} y_i \Delta W_i = \bar{y}\boldsymbol{W} \qquad \bar{y}\boldsymbol{W} = \int y dW \Rightarrow \bar{y} = \frac{\int y dW}{\boldsymbol{W}} \text{ OR } \boxed{\bar{y} = \frac{\Sigma(y\Delta W)}{\boldsymbol{W}}}$$

- The *centroid of an area* is the average x and y locations of the area particles

For a discrete shape (ΔA_i) of a uniform thickness and material, the weight can be defined as:

$\Delta W_i = \gamma t \Delta A_i$ where:
γ is weight per unit **volume** (= specific weight) with units of N/m^3 or lb/ft^3
$t\Delta A_i$ is the volume

So if $\boldsymbol{W} = \gamma t \boldsymbol{A}$:

$$\bar{x}\gamma t\boldsymbol{A} = \int x\gamma t dA \Rightarrow \bar{x}\boldsymbol{A} = \int x dA \quad \text{OR} \quad \boxed{\bar{x} = \frac{\Sigma(x\Delta A)}{\boldsymbol{A}}} \quad \text{and similarly} \quad \boxed{\bar{y} = \frac{\Sigma(y\Delta A)}{\boldsymbol{A}}}$$

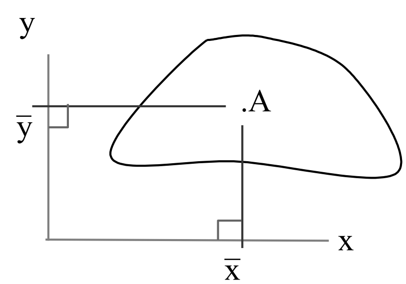
Similarly, for a line with constant cross section, a ($\Delta W_i = \gamma a \Delta L_i$):

$$\bar{x}\boldsymbol{L} = \int x dL \quad \text{OR} \quad \boxed{\bar{x} = \frac{\Sigma(x\Delta L)}{\boldsymbol{L}}} \quad \text{and} \quad \bar{y}\boldsymbol{L} = \int y dL \quad \text{OR} \quad \boxed{\bar{y} = \frac{\Sigma(y\Delta L)}{\boldsymbol{L}}}$$

- $\bar{x}$, $\bar{y}$ **with respect to an x, y coordinate system** is the centroid of an area AND the center of **gravity** for a body of uniform material and thickness.

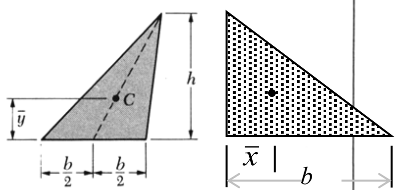
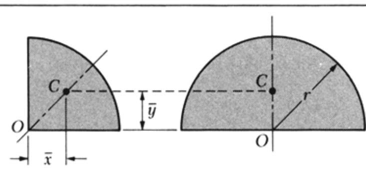
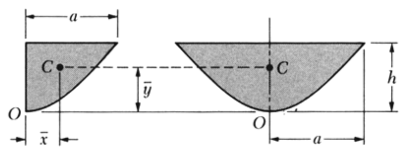
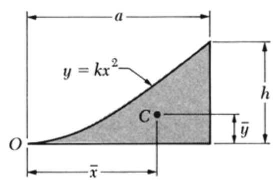
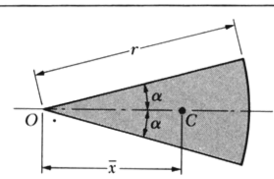
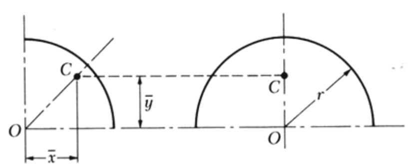
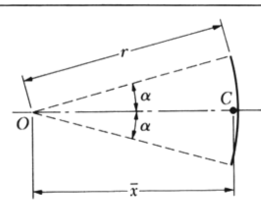
- The *first moment of the area* is like a force moment: and is the **area** multiplied by the perpendicular distance to an axis.

$$Q_x = \int y dA = \bar{y}A \qquad Q_y = \int x dA = \bar{x}A$$



- Centroids of Common Shapes

Centroids of Common Shapes of Areas and Lines

Shape		$\bar{x}$	$\bar{y}$	Area
Triangular area		$\frac{b}{3}$	$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
Circular sector		$\frac{2r \sin \alpha}{3\alpha}$	0	αr^2
Quarter-circular arc		$\frac{2r}{\pi}$	$\frac{2r}{\pi}$	$\frac{\pi r}{2}$
Semicircular arc		0	$\frac{2r}{\pi}$	πr
Arc of circle		$\frac{r \sin \alpha}{\alpha}$	0	$2\alpha r$

- Symmetric Areas

  - An area is symmetric with respect to a line when every point on one side is mirrored on the other. The line divides the area into equal parts and the centroid will be on that axis.

  - An area can be symmetric to a *center point* when every (x,y) point is matched by a (-x,-y) point. It does not necessarily have an axis of symmetry. The center point is the *centroid.*

  - If the symmetry line is on an axis, the centroid location is on that axis (value of 0). With double symmetry, the centroid is at the intersection.

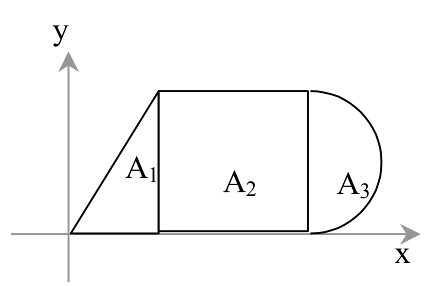
  - Symmetry can also be defined by areas that match across a line, but are 180° to each other.

Basic Steps

1. Draw a reference origin.
2. Divide the area into basic shapes
3. Label the basic shapes (components)
4. Draw a table with headers of *Component*, *Area,* $\bar{x}$, $\bar{x}A$, $\bar{y}$, $\bar{y}A$
5. Fill in the table value
6. Draw a summation line. Sum all the areas, all the $\bar{x}A$ terms, and all the $\bar{y}A$ terms
7. Calculate $\hat{x}$ and $\hat{y}$

- Composite Shapes

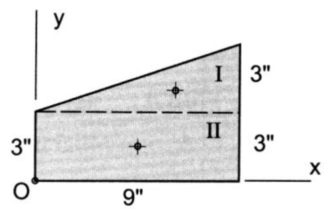
  If we have a shape made up of basic shapes that we know centroid locations for, we can find an "average" centroid of the areas.

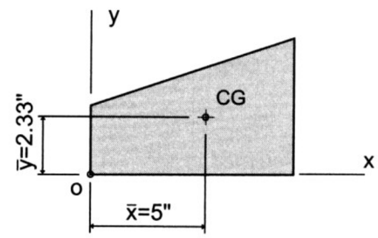


$$\hat{x}\boldsymbol{A} = \hat{x}\sum_{i=1}^{n} A_i = \sum_{i=1}^{n} \bar{x}_i A_i \qquad \hat{y}\boldsymbol{A} = \hat{y}\sum_{i=1}^{n} A_i = \sum_{i=1}^{n} \bar{y}_i A_i$$

OR $$\hat{x} = \frac{\Sigma \bar{x}A}{\boldsymbol{A}} \qquad \hat{y} = \frac{\Sigma \bar{y}A}{\boldsymbol{A}}$$

Centroid values can be negative.
Area values can be negative (holes)

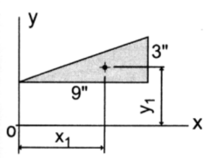
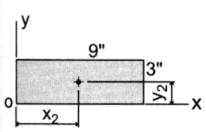




Example 1 (pg 243)

Example Problem 7.1: Centroids (Figures 7.5 and 7.6)

Determine the centroidal x and y distances for the composite area shown. Use the lower left corner of the trapezoid as the reference origin.

Component	Area (ΔA) (in.^{2})	$\boxed{\bar{x}\ (in.)}$	$\boxed{\bar{x}\Delta A\ (in.^3)}$	$\boxed{\bar{y}\ (in.)}$	$\boxed{\bar{y}\Delta A\ (in.^3)}$
 (a)	$\frac{9''(3'')}{2} = 13.5$ in.^{2}	6"	81 in.^{3}	4"	54 in.^{3}
 (b)	9" (3") = 27 in.^{2}	4.5"	121.5 in.^{3}	1.5"	40.5 in.^{3}
	$A = \sum \Delta A = 40.5$ in.^{2}		$\sum \boxed{\bar{x}\Delta A} = 202.5$ in.^{3}		$\sum \boxed{\bar{y}\Delta A} = 94.5$ in.^{3}

$$\hat{x} = \frac{202.5in^3}{40.5in^2}$$
$$= 5in$$

$$\hat{y} = \frac{94.5in^3}{40.5in^2}$$
$$= 2.33in$$

Example 2 (pg 245)

Example Problem 7.3b (Figure 7.13)

An alternate method that can be employed in solving this problem is referred to as the *negative area method.*

A 6" thick concrete wall panel is precast to the dimensions as shown. Using the lower left corner as the reference origin, determine the center of gravity (centroid) of the panel.