

Thermal Effects and Indeterminacy

Notation:

A = area E = modulus of elasticity or Young's modulus f = stress L = length P = name for axial force vector P' = name of reaction force α = coefficient of thermal expansion for a material	ε_t = thermal strain (no units) δ = elongation or length change δ_P = elongation due to axial load δ_{restr} = restrained length change δ_T = elongation due or length change due to temperature ΔT = change in temperature
--	--

Thermal Strains

Physical restraints limit deformations to be the same, or sum to **zero**, or be proportional with respect to the rotation of a rigid body.

We know axial stress relates to axial strain: $\delta = \frac{PL}{AE}$ which relates δ to P

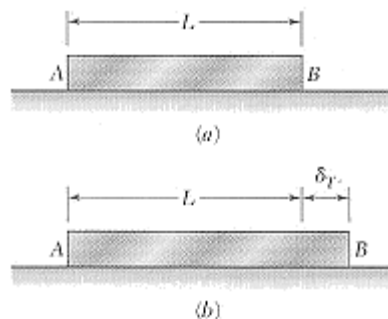
Deformations can be caused by the *material* reacting to a change in energy with temperature. In general (there are some exceptions):

- Solid materials can **contract** with a decrease in temperature.
- Solid materials can **expand** with an increase in temperature.

The change in length per unit temperature change is the *coefficient of thermal expansion*, α .

It has units of $1/^\circ F$ or $1/^\circ C$ and the deformation is related by: $\delta_T = \alpha(\Delta T)L$

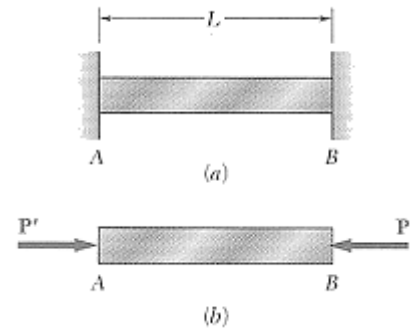
Thermal Strain: $\varepsilon_T = \alpha\Delta T$



There is **no stress** associated with the length change with free movement, BUT if there are restraints, thermal deformations or strains *can cause internal forces and stresses*.

How A Restrained Bar Feels with Thermal Strain

1. Bar pushes on supports because the material needs to expand with an increase in temperature.
2. Supports push *back*.
3. Bar is restrained, can't move and the reaction causes internal *stress*.



Superposition Method

If we want to solve a statically indeterminate problem that has extra support forces:

- We can remove a support or supports that *makes the problem look statically determinate*
- Replace it with a reaction and treat it like it is an applied force
- Impose geometry restrictions that the support imposes

For Example:

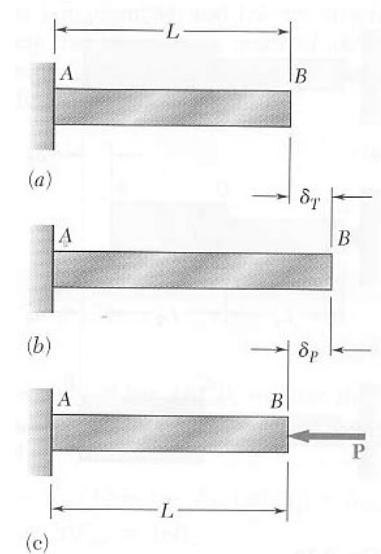
$$\delta_T = \alpha(\Delta T)L$$

$$\delta_p = -\frac{PL}{AE}$$

$$\delta_p + \delta_T = 0$$

$$-\frac{PL}{AE} + \alpha(\Delta T)L = 0$$

$$P = \alpha(\Delta T)L \frac{AE}{L} = \alpha(\Delta T)AE \quad f = -\frac{P}{A} = -\alpha(\Delta T)E$$



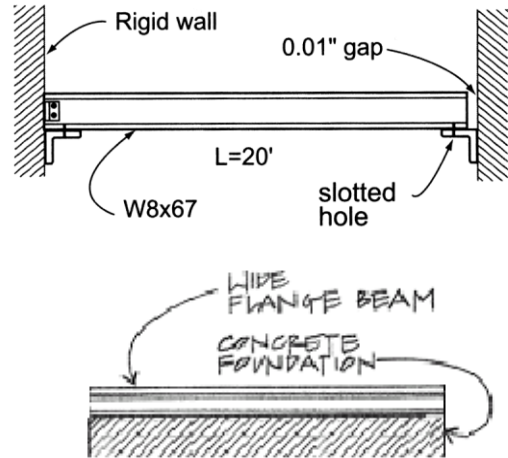
Example 1 (pg 228)

Example Problem 6.24 (Figures 6.58 and 6.59)

A W8x67 steel beam, 20 ft. in length, is rigidly attached at one end of a concrete wall. If a gap of 0.010 in. exists at the opposite end when the temperature is 45°F, what results when the temperature rises to 95°F?

ALSO: If the beam is anchored to a concrete slab, and the steel sees a temperature change of 50° F while the concrete only sees a change of 30° F, determine the compressive stress in the beam.

$$\begin{aligned} \alpha_c &= 5.5 \times 10^{-6} / ^\circ \text{F} & E_c &= 3 \times 10^6 \text{ psi} \\ \alpha_s &= 6.5 \times 10^{-6} / ^\circ \text{F} & E_s &= 29 \times 10^6 \text{ psi} \end{aligned}$$



Example 2

5.21 A short concrete column measuring 12 in. square is reinforced with four #8 bars ($A_s = 4 \times 0.79 \text{ in.}^2 = 3.14 \text{ in.}^2$) and supports an axial load of 250k. Steel bearing plates are used top and bottom to ensure equal deformations of steel and concrete. Calculate the stress developed in each material if:

$$E_c = 3 \times 10^6 \text{ psi and}$$

$$E_s = 29 \times 10^6 \text{ psi}$$

Solution:

From equilibrium:

$$[\Sigma F_y = 0] - 250 \text{ k} + f_s A_s + f_c A_c = 0$$

$$A_s = 3.14 \text{ in.}^2$$

$$A_c = (12'' \times 12'') - 3.14 \text{ in.}^2 \cong 141 \text{ in.}^2$$

$$3.14 f_s + 141 f_c = 250 \text{ k}$$

From the deformation relationship:

$$\delta_s = \delta_c; L_s = L_c$$

$$\therefore \frac{\delta_s}{L} = \frac{\delta_c}{L}$$

and

$$\epsilon_s = \epsilon_c$$

Since

$$E = \frac{f}{\epsilon}$$

and

$$\frac{f_s}{E_s} = \frac{f_c}{E_c}$$

$$f_s = f_c \frac{E_s}{E_c} = \frac{29 \times 10^3 (f_c)}{3 \times 10^3} = 9.67 f_c$$

Substituting into the equilibrium equation:

$$3.14 (9.67 f_c) + 141 f_c = 250$$

$$30.4 f_c + 141 f_c = 250$$

$$171.4 f_c = 250$$

$$f_c = 1.46 \text{ ksi}$$

$$\therefore f_s = 9.67 (1.46) \text{ ksi}$$

$$f_s = 14.1 \text{ ksi}$$

