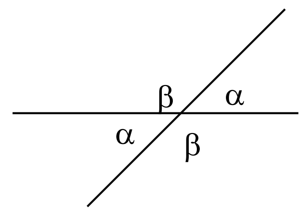
Math for Structures I

1. Parallel lines never intersect.

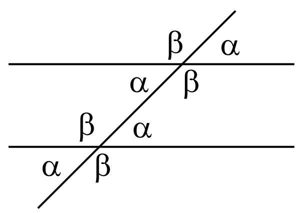
2. Two lines are *perpendicular* (or *normal*) when they intersect at a right angle = 90°.

3. *Intersecting* (or *concurrent)* lines cross or meet at a point.

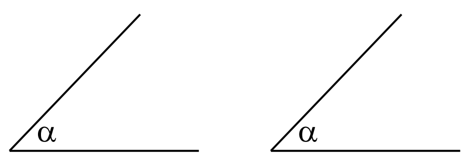
4. If two lines cross, the opposite angles are identical:



5. If a line crosses two parallel lines, the intersection angles with the same orientation are identical:

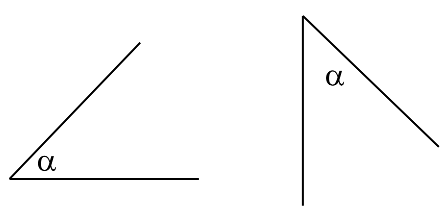


6. If the sides of two angles are parallel and intersect in the same fashion, the angles are identical.

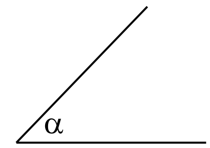


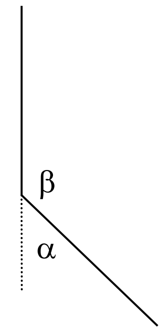
7. If the sides of two angles are parallel, but intersect in the opposite fashion, the angles are *supplementary*: $\alpha+\beta = 180°$.

8. If the sides of two angles are perpendicular and intersect in the same fashion, the angles are identical.

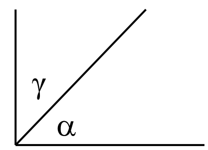


9. If the sides of two angles are perpendicular, but intersect in the opposite fashion, the angles are *supplementary*: $\alpha+\beta = 180°$.

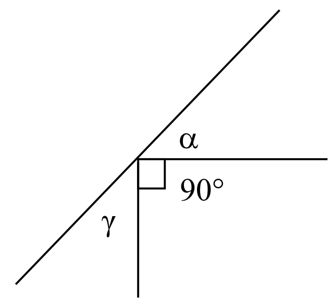




10. If the side of two angles bisects a right angle, the angles are *complimentary*: $\alpha+\gamma = 90°$.

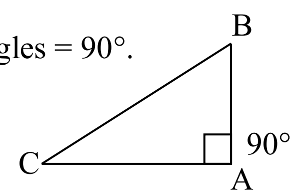


11. If a right angle bisects a straight line, the remaining angles are *complimentary*: $\alpha+\gamma = 90°$.



12. The sum of the interior angles of a triangle = 180°.

13. For a right triangle, that has one angle of 90°, the sum of the other angles = 90°.

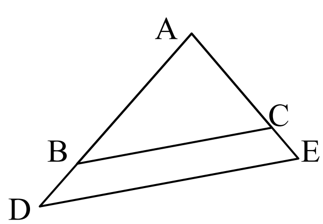


14. For a right triangle, the sum of the squares of the sides equals the square of the hypotenuse:

$$AB^2 + AC^2 = CB^2$$

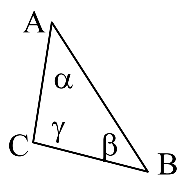
15. Similar triangles have identical angles in the same orientation. Their sides are related by:

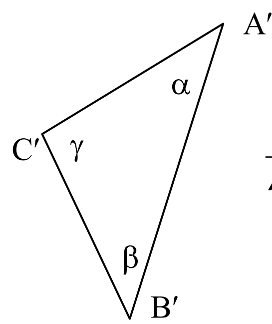
Case 1:



$$\frac{AB}{AD} = \frac{AC}{AE} = \frac{BC}{DE}$$

Case 2:





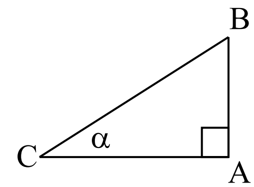
$$\frac{AB}{A'B'} = \frac{AC}{A'C'} = \frac{BC}{B'C'}$$

16. For right triangles:

$$sin = \frac{opposite side}{hypotenuse} = sin\alpha = \frac{AB}{CB}$$

$$cos = \frac{adjacent side}{hypotenuse} = cos\alpha = \frac{AC}{CB}$$

$$tan = \frac{opposite side}{adjacent side} = tan\alpha = \frac{AB}{AC}$$



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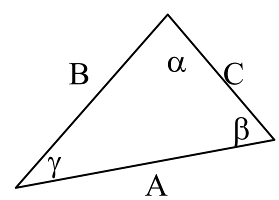
17. If an angle is greater than 180° and less than 360°, *sin* will be less than 0.
If an angle is greater than 90° and less than 270°, *cos* will be less than 0.
If an angle is greater than 90° and less than 180°, *tan* will be less than 0.
If an angle is greater than 270° and less than 360°, *tan* will be less than 0.

18. LAW of SINES (any triangle)

$$\frac{\sin\alpha}{A} = \frac{\sin\beta}{B} = \frac{\sin\gamma}{C}$$



19. LAW of COSINES (any triangle)

$$A^2 = B^2 + C^2 - 2BC\cos\alpha$$

20. Surfaces or areas have dimensions of width and length and units of length *squared* (ex. in^2 or inches x inches).

21. Solids or volumes have dimension of width, length and height or thickness and units of length *cubed (*ex. m^3 or m x m x m)

22. Force is defined as mass times acceleration. So a weight due to a mass is accelerated upon by gravity: $F = m \cdot g$ $\quad g = 9.81\, {}^{m}/_{sec^2} = 32.17\, {}^{ft}/_{sec^2}$

23. Weight can be determined by volume if the unit weight or *density* is known: $W = V\cdot\gamma$
where ·V is in units of $length^3$ and γ is in units of force/unit volume

24. Algebra: If $a \cdot b = c \cdot d$ then it can be rewritten:

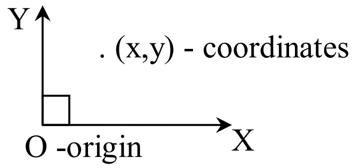
$a \cdot b + k = c \cdot d + k$ add a constant

$c \cdot d = a \cdot b$ switch sides

$a = \frac{c \cdot d}{b}$ divide both sides by *b*

$\frac{a}{c} = \frac{d}{b}$ divide both sides by *b·c*

25. Cartesian Coordinate System



26. Solving equations with one unknown:

1st order polynomial: $2x-1=0\cdots$ $2x=1\cdots$ $x=\frac{1}{2}$

$ax+b=0\cdots$ $x=\dfrac{-b}{a}$

2nd order polynomial $ax^2+bx+c=0\cdots$ $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ two answers (radical *cannot* be negative)

$x^2-1=0\cdots$
$(a=1,b=0,c=-1)$ $x=\dfrac{-0\pm\sqrt{0^2-4(-1)}}{2\cdot 1}\cdots$ $x=\pm 1$

27. Solving 2 linear equations simultaneously: *also see item #31*

One equation consisting only of variables can be rearranged and then substituted into the second equation:

ex: $5x-3y=0$
$4x-y=2$

add 3y to both sides to rearrange	$5x=3y$
divide both sides by 5	$x=\frac{3}{5}y$
substitute x into the other equation	$4(\frac{3}{5}y)-y=2$
add like terms	$\frac{7}{5}y=2$
simplify	$\boxed{y=\frac{10}{7}=1.43}$

Equations can be added and factored to eliminate one variable:

ex: $2x+3y=8$
$4x-y=2$

	$2x+3y=8$
multiply both sides by 3	$12x-3y=6$
and add	$14x+0=14$
simplify	$\boxed{x=1}$
put x=1 in an equation for y	$2\cdot 1+3y=8$
simplify	$3y=6$
	$\boxed{y=2}$

28. Derivatives of polynomials

$$y = constant \qquad \frac{dy}{dx} = 0$$

$$y = x \qquad \frac{dy}{dx} = 1$$

$$y = ax \qquad \frac{dy}{dx} = a$$

$$y = x^2 \qquad \frac{dy}{dx} = 2x$$

$$y = x^3 \qquad \frac{dy}{dx} = 3x^2$$

29. The minimum and maximum of a function can be found by setting the derivative = 0 and solving for the unknown variable.

30. Calculators (and software) process equations by an "order of operations", which typically means they process functions like exponentials and square roots before simpler functions such as + or -. BE SURE to specify with parenthesis what order you want, or you'll get the wrong answers. It is also important to have degrees set in your calculator for trig functions.

    For instance, Excel uses – for sign (like -1) first, then will process exponents and square roots, times and divide, followed by plus and minus. If you type 4x10^2 and really mean (4x10)^2 you will get an answer of 400 instead of 1600.

31. <u>Using a TI-83 to solve a system of linear equations in a matrix form with **rref(**</u>:

    Matrices of linear equations expect the coefficients in front of variable to be put in the same order in each row, and the numerical solution (= to) as the last value. So for the 2nd set of equations in item #27 ($2x + 3y = 8$ and $4x - y = 2$), the matrix to enter would look like

$$\begin{bmatrix} 2 & 3 & 8 \\ 4 & -1 & 2 \end{bmatrix}$$

1. Press [2nd] [MATRIX]. Press [▶] [▶] to display the MATRIX EDIT menu. Press **1** to select **1:[A],**

2. Press **2** [ENTER] **3** [ENTER] to define a 2 x 3 matrix. The rectangular cursor indicates the current element. Ellipses (...) indicate additional columns beyond the screen.

```
MATRIX[A]   2 x3
[0      0     0      ]
[0      0     0      ]

1 , 1=0
```

3. Press **2** [ENTER] to enter the first element. The rectangular cursor moves to the second column of the first row.

4. Press **3** [ENTER] **8** [ENTER] to complete the first row for $2x + 3y = 8$

5. Press **4** [ENTER] **-1** [ENTER] **2** [ENTER] to enter the second row for $4x - y = 2$

```
MATRIX[A]   2 x3
[2      0     0      ]
[0      0     0      ]

1 , 2=0
```

6. Press [2nd] [QUIT] to return to the home screen. If necessary, press [CLEAR] to clear the home screen. Press [2nd] [MATRIX] [▶] to display the MATRIX MATH menu. Press [▲] to wrap to the end of the menu. Select **B:rref(** to copy **rref(** to the home screen.

```
MATRIX[A]  2 x3
[2    3    8    ]
[4   −1    2    ]

2 , 3=2
```

```
rref(
```

7. Press [2nd] [MATRIX] **1** to select **1:[A]** from the MATRIX NAMES menu. Press [)] [ENTER]. The reduced row-echelon form of the matrix is displayed and stored in **Ans**.

$1x + 0y = 1$ therefore $x = 1$

$0x + 1y = 2$ therefore $y = 2$

```
rref([A])
[[1    0    1
  0    1    2]]
```